Reconciling neutrino oscillations with SO(10) leptogenesis

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We study the link between neutrino oscillations and leptogenesis in the minimal framework assuming an SO(10) see-saw mechanism with 3 families. The solar and atmospheric data then generically induce a large mass-hierarchy and a small mixing between the lightest right-handed neutrinos, which fails to produce sufficient lepton asymmetry by 5 orders of magnitudes at least. This conclusion can only be evaded in the case of solar vacuum oscillations and for a very specific value of the mixing $\sin^2 2\theta_{e3} = 0.1$, which interestingly lies at the boundary of the CHOOZ exclusion region, but should be accessible to future long baseline experiments.

1 Introduction

The evidence for a total asymmetry between baryon and anti-baryon densities in our surroundings is an indisputable fact of life. This could easily be accounted for by assuming that the initial conditions for our universe are such that this asymmetry is of order one. However, there is every reason to believe that the thermal history of the universe can be traced back in time using known particle physics up to temperatures of maybe 100 Gev, certainly high enough to massively produce quark-antiquark pairs. At these temperatures, big-bang nucleosynthesis requires[17] that the adiabatic invariant ratio

$$Y_B \doteq \frac{n_B - n_{\bar{B}}}{s} = 2 \to 5\,10^{-10}$$

meaning that there be about one extra quark for 10^{10} quark-antiquark pairs. Avoiding such extreme fine-tuning by a dynamical mechanism able to produce this number out of an initially symmetric configuration is the purpose of baryogenesis.

Many mechanisms of baryogenesis have been proposed in the past 30 years (see e.g. Dolgov[7] for a review including the most ingenious and exotic ones), but most rely on ad-hoc new physics. We certainly would prefer to explain the baryon asymmetry from solid, experimentally tested physics. The standard model of particle physics satisfy this criterion at a desperately high level of precision, and it was shown[24] (see also the nice review[22]) to satisfy in principle all the Sakharov[23] conditions required to produce a baryon asymmetry. However, what looked like a very small number for initial conditions, now appears too large for the pure standard model to achieve. At least, CP violation beyond the known CKM phase must be added to resist the strong GIM cancellations in the hot plasma.[9,10]

Furthermore, the only possibility to change baryon number in the standard model is via sphaleron processes which should freeze abruptly below the electroweak phase transition to leave a net asymmetry.[4] Since the transition gets weaker when rising the mass of a single scalar doublet, extra non-standard scalars are invoked to counter the rising of the experimental lower bound on the lightest scalar. Supersymmetry naturally provides a lot of well-motivated extra scalars, but getting a strong enough transition with $m_H \approx 100\mathrm{GeV}$ requires a huge gap between the right-handed stop and all other sfermions, which may seem contrived, and even so, could not protect E-W baryogenesis against a $\approx 110\mathrm{GeV}$ Higgs bound.[6] Unless a scalar is soon discovered, we thus seem to be lead back into the pre-sphaleron[16] situation, where baryogenesis was a footprint from extreme high-energy physics, with little chance of an experimental cross-check.

However, another experimental signal for non-standard physics has since then developed into a quantitative and solid field, namely the evidence for neutrino oscillations which strongly points towards small but non-zero neutrino masses.[18] If these masses are of the Dirac type, right-handed neutrinos must be added, but we are left with the puzzle: why are neutrinos 10^{10} times lighter than charged leptons? In our mind, the smallest theoretical price to pay for resolving this puzzle is to keep the right-handed neutrinos, but give them large Majorana masses $\approx 10^{10-16}\mathrm{GeV}$: this is the celebrated see-saw mechanism.[11,25]. Such a high Majorana mass breaks lepton number at a slow enough rate, and in a very indirect way, one can say that neutrino oscillations provide a leptonic analogue of the baryon number violation looked for unsuccessfully in proton decay searches during the last decades.

With this theoretical prejudice, we are thus lead to take the existence of heavy right-handed neutrinos for granted. This makes leptogenesis an extremely natural mechanism to consider for producing the baryon asymmetry. Indeed, the decay of right-handed neutrinos can easily leave a CP-odd lepton asymmetry, which standard sphaleron processes can convert into a baryon asymmetry. The only questions are 1) how closely can this asymmetry be related to tested or testable neutrino oscillation physics, and 2) how much asymmetry can be produced.

2 Leptogenesis

These questions can only have a definite answer for definite Dirac neutrino masses. In this work, we assume that neutrino Yukawa couplings are fixed by naive SO(10) relations to the up-quarks Yukawa couplings:

$$m_{\nu D} = m_u/3. \tag{1}$$

For light neutrino oscillations,[3] we will use the conventional mixing matrix $m_\nu = U.\mathrm{D}(m_1, m_2, m_3).U^T$ where

$$U = \underbrace{U_{23}}_{atm} \cdot \underbrace{U_{13}}_{Chooz} \cdot \underbrace{U_{12}}_{sun} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \tag{2}$$

and we will take the following experimental data for granted:

- Atmospheric neutrinos:[8] $\Delta m^2_{atm} \approx 2\,10^{-3}\mathrm{eV}^2$ with nearly maximal mixing: $\sin^2 2\theta_{atm} = 0.7 \to 1$ so that $|s_{23}| = 0.48 \to 0.7$.

- CHOOZ reactor experiment:[1] for the above value of Δm^2_{atm}, $|U_{e3}|^2 < 5.10^{-2}$ ($|U_{e3}|^2 > 0.95$ would leave too little room for solar oscillations) so that $|s_{e3}| < 0.2$

- Solar neutrinos:[2] we will consider 3 possible resolutions of the solar neutrino puzzle: 1) vacuum oscillations (V.O.) with $\Delta m^2_{sun} \approx 10^{-10}\mathrm{eV}^2$ and $0.8 < \sin^2 2\theta_{sun}$ so that either $|s_{12}| = 0.5 \to 0.7$ or $|c_{12}| = 0.5 \to 0.7$; 2) small mixing angle MSW oscillations (SMA) with $\Delta m^2_{sun} \approx 10^{-5}\mathrm{eV}^2$ $\sin^2 2\theta_{sun} = 10^{-3} \to 10^{-2}$ corresponding to $|s_{12}| = 0.0158 \to 0.05$ or $|c_{12}| = 0.0158 \to 0.05$; 3) large mixing angle MSW oscillations (LMA) with $\Delta m^2_{sun} \approx 10^{-5}\mathrm{eV}^2$ and $0.42 \le \sin^2 2\theta_{sun} \le 0.75$ corresponding to $|s_{12}| = 0.35 \to 0.5$ or $|c_{12}| = 0.35 \to 0.5$.

Combining these with the naive $SO(10)$ relations (1) and the see-saw mechanism gives a failry restrictive expression for right-handed neutrino masses:

$$M_R = \frac{1}{9(v^u)^2}\mathrm{D}(m_u, m_c, m_t).U_{eff}.\mathrm{D}(\frac{1}{m_1}, \frac{1}{m_2}, \frac{1}{m_3}).U_{eff}^T.\mathrm{D}(m_u, m_c, m_t) \tag{3}$$

$$= V_R.\mathrm{D}(M_1, M_2, M_3).V_R^T\ , \tag{4}$$

where

$$U_{eff} \doteq V^{CKM}.U \tag{5}$$

contains the known CKM matrix for quarks, following our $SO(10)$ assumption.

The lightest of these heavy neutrinos decays to scalars and charged leptons, generating a lepton asymmetry Y_L which is transferred to baryons by sphaleron processes (requiring $Y_B \approx -Y_L/3$ at equilibrium). The net result (see e.g. Pilaftsis[19] and references therein) can be compared to the baryon asymmetry required for nucleosynthesis to proceed:

$$Y_{B10} \doteq 10^{10} Y_B \approx \frac{2.46 \times 10^{-10}}{\sqrt{g^*}} \frac{\mathrm{Im}(A_{21}^2)}{A_{11}^2} . \frac{M_1}{M_2} . \frac{M_1}{10^{10} GeV} \approx 0.2 \to 0.5. \qquad (6)$$

The matrix

$$A_{ij} = \frac{1}{9(v^u)^2} V_R^\dagger . \mathrm{D}(m_u^2, m_c^2, m_t^2) . V_R \qquad (7)$$

is simply the hermitian squared Yukawa matrix, and is known once M_R is known. However, neutrino oscillations fail to constrain one mass, say m_1, and the phases of U and hence V_R. In what follows, we will assume a maximal phase for A_{21} and leave m_1 as a free parameter.

By working out a 2 flavors see-saw exercise, it is relatively easy to see that

1. the right neutrinos mass ratio M_1/M_2 is maximized for a ratio of light neutrino masses $m = m_1/m_2$ which is either the squared ratio of Dirac masses $r^2 = (m_u/m_c)^2$ or the light neutrinos mixing angle squared sine $s^2 = \sin^2 \theta_L$, whichever is largest,

2. for most of parameter space, the baryon asymmetry is bounded by

$$Y_{B10}^{max} \propto \frac{|A_{21}|^2}{A_{11}^2} \cdot \frac{M_1}{M_2} \cdot M_1 \approx \frac{m s^2 r^2}{(m + r^2 + s^2)^5}, \qquad (8)$$

and can thus be maximized for a similar or slightly larger value of m,

3. the inverted hierarchy $m_1 \gg m_2$ gives much smaller results than the standard hierarchy $m_1 \ll m_2$.

3 Results

Let us now discuss the numerical results for 3 light neutrino flavors. It turns out the baryon asymmetry is generically too small to be useful. Before concluding that $SO(10)$ see-saw leptogenesis is excluded, we must look for regions of parameters where the asymmetry can be maximized. There are three cases to investigate, one for each of the solar neutrinos solutions. Let us start by vacuum oscillations (V.O.).

In view of the previous section, a good starting point to maximize the baryon asymmetry is to try and make the lightest right-handed masses M_1 and

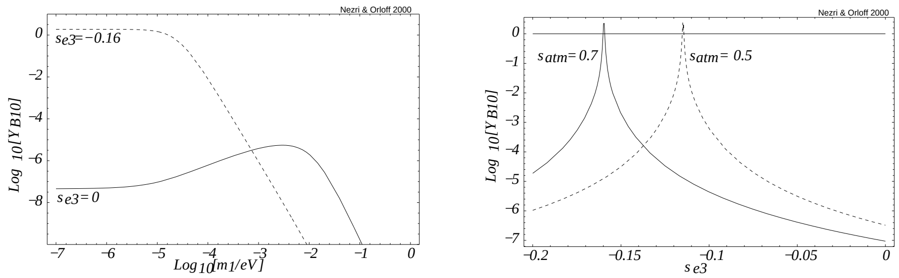

Figure 1. The baryon asymmetry Y_{B10} for vacuum oscillations of solar neutrinos ($m_{sun} = 10^{-5}$eV) of as a function of the lightest neutrino mass m_1 (left) or the mixing angle s_{e3} (right). The plain curve on the left assumes CKM mixing, maximal atmospheric and solar mixing but vanishing s_{e3} while the dashed curve tunes s_{e3} to maximize M_1/M_2 and the asymmetry. The plot on the right shows the sensitivity of the baryon asymmetry on the mixing angle s_{e3}, most constrained by CHOOZ. For $m_1 = m_{sun} = 10^{-5}$eV, a sharp peak is obtained around $s_{e3} \approx -0.22\, s_{atm}$.

M_2 as close as possible to each other. For this, let us first watch the evolution of M_R eigenvalues as functions of m_1 in the absence of mixing ($U_{eff} = 1$ in (3)). For large m_1(degenerate limit) they are separated by large Dirac mass hierarchies. With decreasing m_1, all M_R's grow until $m_1 = m_{atm} \approx 10^{-1}$eV, below which M_3 levels off at $m_t^2/9m_{atm} \approx 10^{14}$GeV. Meanwhile $M_{1,2}$ continue growing together until $m_1 = m_{sun} \approx 10^{-5}$eV, where M_2 stops at about the same value. Further decreasing m_1 then only affects M_1, which becomes degenerate with M_2 for extremely low $m_1 \approx 10^{-11}$eV.

If we now turn on the largest allowed atmospheric and solar mixing, there is no effect in the degenerate limit of large m_1. With decreasing m_1, M_3 starts again levelling off at $m_1 = m_{atm}$. But the maximal atmospheric mixing immediately induces a type of "level crossing" , which effectively exchanges M_2 and M_3. M_2 thus levels off at a much lower value $\approx 10^{10}$GeV which offers a better possibility for M_1 to catch up. However, despite keeping $U_{e3} = 0$, CKM mixing in eq. (5) still induces a non-trivial $s_{13eff} \approx 0.16$ which stops the growth of M_1 below $m_1 = s_{13eff}^2 m_{atm} \approx 10^{-3}$eV. M_2 on the contrary starts growing again until it is hit by the solar mixing at $m_1 \approx m_{sun}$. We thus expect a maximum baryon asymmetry around $m_1 \approx 10^{-3}$eV in this case. A plot of the full 3 flavor asymmetry (plain curve in figure 1) confirms this expectation, and further shows that this maximum is at least 5 orders of magnitude too low. Notice that the shape of the asymmetry nicely fits the picture derived in the previous section if we recall that for $m_1 < m_{sun}$ both M_1 and M_2 stay constant.

To get larger results, we may use the freedom left by the CHOOZ experiment to play with s_{e3}. Indeed, for $s_{e3} \approx -0.16$, $s_{13eff} \approx 0$ so that M_1 is decoupled from the value of m_3, and continues rising up to $m_u^2/9m_{sun} \approx 10^8$GeV. This may give a correct asymmetry as soon as m_1 is smaller that $\approx 10^{-4}$eV. Notice that for such a value, $M_1 \approx 10^8$GeV which is safe w.r.t. gravitino bounds on inflation, and $M_3 \approx 10^{16}$GeV which is not yet too high w.r.t. the GUT scale. So we have an interesting leptogenesis candidate, but only for a very specific value of $s_{e3} \approx -0.22\, s_{atm}$. It is worth detailing where this special value comes from. In the expression of U_{eff}, we may neglect all CKM mixing except the Cabbibo V_{12}^{CKM}. Then keeping terms at most linear in s_{12}^{CKM} and $s_{13} (\doteq s_{e3})$, we may write:

$$\begin{aligned} U_{eff} &\approx V_{12}^{CKM}\, U_{23} U_{13} U_{12}^{sun} &(9)\\ &= U_{23}\, [U_{23}^{\dagger} V_{12}^{CKM} U_{23}]\, U_{13} U_{12}^{sun} &(10)\\ &\approx U_{23}\, U_{13} V_{13}'\, V_{12}' U_{12}^{sun} &(11)\\ &\doteq U_{23eff}\, U_{13eff}\, U_{12eff} &(12)\end{aligned}$$

with the V' matrices given by $s_{12}' \approx c_{23}\, s_{12}^{CKM}$ and $s_{13}' \approx s_{23}\, s_{12}^{CKM}$. Canceling s_{13eff} then indeed requires $s_{13} \approx -s_{23}\, s_{12}^{CKM} \approx -0.16$, as found graphically.

If we now turn from vacuum to MSW solutions of the solar neutrino puzzle, we get a value of m_{sun} at least 2 orders of magnitude larger, and there is too little room for M_1 to grow between $m_1 \approx m_{atm}$ and $m_1 \approx m_{sun}$. However, it will grow for smaller m_1, provided both $s_{13eff} = s_{12eff} = 0$. This is in principle possible, but now requires very special values for both $s_{e3} \approx -0.16$ (as previously) and $s_{sun} \approx -s_{12}^{CKM}\, c_{23} \approx -0.16$ (see eq. 11). This last value of s_{sun} is incompatible with any possible MSW solution. This is illustrated as a function of s_{sun} on figure 2.

With a standard mass hierarchy, we thus see that only vacuum oscillations are able to account both for the solar neutrinos deficit and the baryon asymmetry of the universe. With the same line of reasoning, it is easy to see that with inverted hierarchy ($m_3 \ll m_1 \ll m_2$) the asymmetry is even smaller, because of a larger $M_1 - M_2$ gap coming from the bound $M_1 <\sim m_u^2/6m_{atm}$. Indeed, as the free parameter m_3 is lowered below m_{atm}, two M_R eigenvalues must now stop growing instead of one. There are then two possibilities: if M_1 is the only one that grows in the absence of mixing, then maximal atmospheric mixing stops its growth once $m_3 < s_{atm} m_{atm}$. If on the other hand M_1 stops growing without mixing, then mixing won't help, as it can only induce "levels repulsion", not attraction. In both cases, M_1 is at most 10^4GeV, and M_1/M_2 cannot exceed $m_u^2/m_c^2 \approx 10^{-6}$ which makes the asymmetry too small.

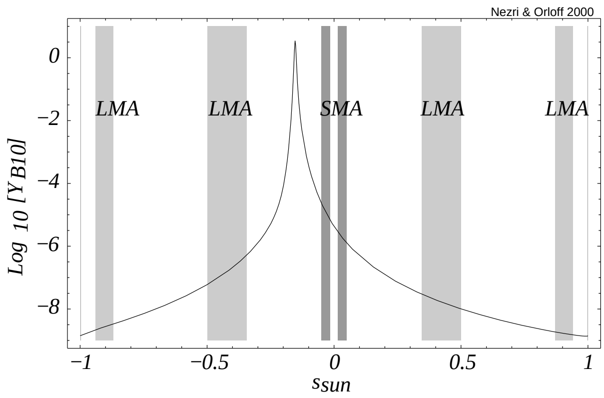

Figure 2. The baryon asymmetry Y_{B10} for MSW oscillations of solar neutrinos ($m_{sun} = 10^{-5}$), as a function of s_{sun} displays a sharp peak. The maximum is just large enough if $m_1 = 10^{-5}$ and $s_{e3} = -0.16$ (sharply peaked also, like in figure 1), but lies just between the small and large mixing angle solutions.

4 Conclusions and perspectives

We have studied the possible relations between neutrino oscillations and the baryon asymmetry of the universe produced by equilibrium decays of right handed neutrinos, assuming an $SO(10)$ see-saw mechanism for neutrino masses. This is the minimal predictive set of assumptions to study such relations. We find that the produced asymmetry is generically six orders of magnitude too small because of the huge Dirac masses hierarchy assumed ($r^2 = m_u^2/m_c^2 \approx 10^{-6}$). The only possibility to explain the observed asymmetry in this framework requires vacuum oscillations of solar neutrinos, with standard hierarchy and very specific values for the least constrained neutrino parameters: 1) the lightest mass ($m_1 \approx m_{sun} \approx 10^{-5}$eV), and 2) the heavy component of the electron neutrino ($|\sin(\theta_{e3})| = |\sin(\theta_{atm})|.|\sin(\theta_{Cabbibo})| \approx 0.16$).

This result calls for several extensions and refinements. It would first of all be more satisfactory to get these parameters out of some theoretical mass model for all particles. This seems quite challenging at the moment. A nice anomalous $U(1)$ model[14] for instance, generically has a large M_1/M_2 hierarchy.

Second, one may question our crude implementation of the $SO(10)$ relations between quarks and leptons Dirac-masses. We simply used $m_{lepton} =$

$m_{quark}/3$, which fails by about a factor 3 for the muon-strange mass ratio. Since the asymmetry goes like r^2 (see equ. 8), such small factors should not qualitatively change our conclusions. However, the correct running of neutrino Yukawa couplings may have important effects on the mixing, which could significantly change the interesting value of $\sin(\theta_{e3})$, and for instance make it inaccessible to long baseline experiments, or on the contrary, already excluded by CHOOZ.

Third, we have only considered decays of right-handed neutrinos produced in thermal equilibrium processes. This is consistent, as the value of M_1 we found is smaller than the maximal reheat temperature allowed by the gravitino bound. However, non-equilibrium production might be competitive for smaller reheat temperatures.[12]

Finally, we only computed an upper bound, assuming maximal CP violation. This is plausible, as phases in the right-handed mixing matrix V_R are completely free at the moment. Given the puzzles around the origin of CP violation, it would however be interesting to see how much asymmetry can be achieved if the mixing matrix U for light neutrinos is CP-invariant, and all CP violation comes from the CKM matrix where it has been experimentally tested.

Our conclusions seemingly contradict existing literature on the subject, which usually leaves the impression that leptogenesis works without restrictions. This comes from our insisting on both $SO(10)$ relations, and up-to-date neutrino oscillations data. Let us review some of these discrepancies. In a thorough study,[5,20] the authors use $SO(10)$-inspired see-saw relations like eq. 3 and conclude that leptogenenesis is generically possible. However, they admit the values taken in the final analysis are hard to reconcile with maximal atmospheric mixing, which at the time was not as firmly established as today. In reference,[13] it is shown that leptogenesis can be reconciled with SMA solar neutrinos. But the Dirac masses needed badly violate $SO(10)$ relations. Finally, another way to loosen the strong $SO(10)$ constraints is to invoke what could be coined "scalar see-saw", namely a small v.e.v. for a scalar triplet that directly contributes to the left neutrino Majorana mass.[21] This however introduces a new parameter, and non-trivial constraints on the scalar sector[15].

References

1. M. Apollonio et al, *Phys. Lett.*, B466:415, 1999.
2. J. N. Bahcall, P. I. Krastev, and A. Yu. Smirnov, *Phys. Rev.*, D58:096016, 1998.
3. S. M. Bilenkii, C. Giunti, and W. Grimus, *Prog. Part. Nucl. Phys.*, 43:1, 1999.
4. A. I. Bochkarev, S. V. Kuzmin, and M. E. Shaposhnikov, *Phys. Lett.*, B244:275, 1990.
5. W. Buchmuller and M. Plumacher, *Phys. Lett.*, B389:73–77, 1996.
6. M. Carena, M. Quiros, and C. E. M. Wagner, *Nucl. Phys.*, B524:3, 1998.
7. A. D. Dolgov, *Phys. Rept.*, 222:309–386, 1992.
8. Y. Fukuda et al, *Phys. Rev. Lett.*, 81:1562–1567, 1998.
9. M. B. Gavela, P. Hernandez, J. Orloff, and O. Pene, *Mod. Phys. Lett.*, A9:795–810, 1994.
10. M. B. Gavela, P. Hernandez, J. Orloff, O. Pene, and C. Quimbay, *Nucl. Phys.*, B430:382–426, 1994.
11. Murray Gell-Mann, Pierre Ramond, and Richard Slansky, In P. van Nieuwenhuizen and D.Z. Freedman, editors, *Supergravity*. North-Holland, 1979.
12. G. F. Giudice, M. Peloso, A. Riotto, and I. Tkachev, *JHEP*, 08:014, 1999.
13. Haim Goldberg, *Phys. Lett.*, B474:389, 2000.
14. Nikolaos Irges, Stephane Lavignac, and Pierre Ramond, *Phys. Rev.*, D58:035003, 1998.
15. Anjan S. Joshipura and Emmanuel A. Paschos, preprint hep-ph/9906498.
16. V. A. Kuzmin, V. A. Rubakov, and M. E. Shaposhnikov, *Phys. Lett.*, B155:36, 1985.
17. Keith A. Olive, Gary Steigman, and Terry P. Walker, preprint astro-ph/9905320.
18. S. T. Petcov, preprint hep-ph/9907216.
19. Apostolos Pilaftsis, *Int. J. Mod. Phys.*, A14:1811, 1999.
20. Michael Plumacher, *Nucl. Phys.*, B530:207, 1998.
21. Palash B. Pal Rabindra N. Mohapatra, *Massive Neutrinos in Physics and Astrophysics*, volume 60 of *Lecture Notes in Physics*, World Scientific, 1998.
22. V. A. Rubakov and M. E. Shaposhnikov, *Usp. Fiz. Nauk*, 166:493–537, 1996.
23. A. D. Sakharov, *Pisma Zh. Eksp. Teor. Fiz.*, 5:32–35, 1967.
24. M. E. Shaposhnikov, *Nucl. Phys.*, B287:757, 1987.
25. Tsutomu Yanagida, *Prog. Theor. Phys.*, 64:1103, 1980.