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LYCEN 2000-44

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Abstract

The modified additive quark model proposed recently by authors allowed to improve agreement of the standard additive quark model with the data on the pp, $\bar{p}p$, $\pi^{\pm}p$, γp and $\gamma\gamma$ total cross-sections as well as on the ratios of real to imaginary part of pp and $\bar{p}p$ amplitudes at $t = 0$. Here we extend this model to non forward elastic scattering of protons and antiprotons. A very high qualitative reproduction of data up to maximal measured momenta transferred at 19.4 GeV $\leq \sqrt{s} \leq$ 630 Gev is obtained. A zero at small $|t|$ in the real part of even amplitude in accordance with a recently proved high energy general theorem is found.

1 Introduction

Small angle elastic scattering of hadrons has always been a crucial source of information about the dynamics of strong interaction. As a rule, these processes, being outside the realm of applications of the theory of the strong interactions (QCD), are described in approaches based on the S-matrix theory. In particular, the various Regge models seem most successful in this direction. However, in the framework of a Regge approach it is impossible to calculate many ingredients of the amplitudes. Therefore, some additional arguments are used to construct the main objects: Regge trajectories and residue functions. Some time they are derived from the fundamental theory, but usually they are based on an intuitive physical picture and on the analysis of the available experimental data.

The additive quark model (AQM) [1] is an example of such a line of arguments. An amplitude of composite particle interaction is constructed as a sum of the elementary amplitudes of the interaction among the constituent quarks. This leads to remarkable relations (counting rules) between various hadronic cross-sections in rather good agreement with the experimental data.

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In a recent paper [2], this standard AQM and the ensuing counting rules were modified to take into account not only the quark-gluonic content of the Pomeron but also of the secondary Reggeons as well as the fact that the soft Pomeron is not just a gluonic ladder. The resulting model, which we called Modified Additive Quark Model (MAQM), was successfully applied to describe various total cross-sections (nucleon-nucleon, π-meson-nucleon and γp, $\gamma\gamma$ interactions) and the ρ-ratio of the real to the imaginary part of the forward pp, $p\bar{p}$ scattering amplitudes. The next step : to consider the differential cross-sections is the object of this paper *i.e.* we extend our model for $t \neq 0$. We deal only with pp and $p\bar{p}$ elastic scattering because, for these processes, are available the richest and most precise experimental data in a wide region of energy and momentum transfer.

Many models (for instance [3, 4, 5, 6, 7]) of pp and $p\bar{p}$ elastic scattering amplitudes describe quite well the available data[4]. A supercritical Pomeron with the intercept $\alpha_{\mathcal{P}}(0) > 1$ is used at the Born level for most of them [3, 4, 5]. Such a Pomeron must then be unitarized because it does not satisfy the unitarity constraints. Usually a method of eikonalization [9] is used to this aim.

An alternative way is to construct a model that from the beginning does not violate the requirements for the analyticity and unitarity of the scattering amplitude [5]. An example of such a model is the model of so called maximal Pomeron and maximal Odderon [6]. However some arguments against of the maximal Odderon exist [9, 10]. Besides, this model is formulated only for pp, $p\bar{p}$ processes and it can not be extended for other processes without a large number of additional free parameters.

In [2], we stick to the second kind of approaches and we use an extended AQM that can be applied not only to nucleon-nucleon scattering. As an explicit choice, for the Pomeron contribution we choose a simple model, namely a special case of the model of Dipole Pomeron, with a unit intercept [7, 11] which leads to a high quality description of the experimental data, both at $t = 0$ [12, 13, 14, 15] as well as at $t \neq 0$ (see Sect. 4).

In the present paper we will not consider any eikonalization and work with the Born amplitudes since in the MAQM they do not violate unitarity (although with the restricted sense set above). However, it is not obvious that eikonalization should not carried out since the latter corresponds to taking into account the physical processes of rescattering corrections. We will not discuss this point here.

It should be stressed that the task of reproducing well the entire set of high energy data (at all values of the momentum transfer t), though far from simple, as a long (and direct) experience teaches us [8], may seem to have a poor theoretical content. This is indeed so in the sense that we have not yet any means of actually knowing soft amplitudes from the first principles. However, just because of this, we believe that it is important to explore all the approaches yielding a good agreement with the existing data. To the extent, in fact, that they may lead to different extrapolations which will, hopefully and foreseeably, be checked in future experiments, we will *a posteriori* have the means of establishing a hierarchy among them.

In Section 2 we recall the main assumptions and results of [2] focalizing on a few arguments in favor of the chosen Pomeron used in the MAQM at $t = 0$. In Section 3 we formulate the MAQM for $t \neq 0$. The results of the fit of the MAQM to the experimental data as well as a comparison with a fit in the framework of the standard AQM (which we

[4] See also the reviews [8] and references therein.

[5] By this, we mean only that the model should not violate grossly and explicitly the constraints of unitarity. This, unfortunately, does not guarantee that unitarity is satisfied.

abbreviate in this old version as SAQM) are presented and discussed in Section 4. We examine also if the amplitude that fits very well the data exhibits automatically a zero in the real part of its even component as required by a general theorem due to A. Martin [16]. Some items are also discussed in this Section (concerning in particular the Odderon and the logarithmic trajectory).

2 The modified additive quark model at $t = 0$

Let us review the main properties of the MAQM, formulated for the forward scattering amplitudes (for details see [2]).

2.1 Pomeron

The Pomeron contribution to the pp and $\bar{p}p$ scattering amplitude at $t = 0$ is written as [6]

$$A_{\mathcal{P}}^{(pp)}(s,0) = 9P_p^2[A_{\mathcal{P}}^{(1)}(s/9,0) + 2A_{\mathcal{P}}^{(2)}(2s/9,0) + A_{\mathcal{P}}^{(3)}(4s/9,0)] \ , \tag{1}$$

The choice of the elementary Pomeron amplitudes $A_{\mathcal{P}}^{(i)}$ is very important from the phenomenological point of view. It is known from a comparison of various Pomeron models [12, 15] that the most economical best fit to the $t = 0$ data on the ρ-ratios of the real to imaginary parts of the forward amplitude $\rho = \Re eA(s,0)/\Im mA(s,0)$ and on the total elastic cross sections σ_{tot} for meson-nucleon and nucleon-nucleon interactions is achieved if $\sigma_{tot} \propto \ln(s/s_0)$ at $s \to \infty$ [7]. Such a behavior corresponds to a "Dipole Pomeron" *i.e.* to a double pole of the amplitude in the complex angular momenta plane j and, at $t = 0$, lies exactly at $j = 1$. In other words the trajectory of this Dipole Pomeron has an intercept equal to one, $\alpha_{\mathcal{P}}(0) = 1$.

It is interesting to note that such a Dipole Pomeron is a singularity of the partial amplitude, $\phi(j,t) \propto (j - \alpha_{\mathcal{P}}(t))^{-\gamma}$ with the maximal hardness, that does not violate the evident inequality $\sigma_{elastic}(s) \leq \sigma_{tot}(s)$. The inequality $\gamma \leq 2$ must be satisfied if the Pomeron trajectory at small t is linear, $\alpha_{\mathcal{P}}(t) \approx 1 + \alpha'_{\mathcal{P}}t$, but $\gamma = 2$ corresponds to the Dipole Pomeron.

The contribution of the Dipole Pomeron to the forward scattering elastic hh amplitude is written in the form

$$A_{\mathcal{P}}^{hh}(s,0) = C_1 + C_2 \ln(-is/s_0) \ ,$$

where C_1, as it follows from the fit, is a negative constant (we take consistently $s_0 = 1$ GeV^2). This may be surprising because at small enough energies the contribution of Dipole Pomeron to σ_{tot} would be negative [8]. However this negative term can be treated [18] as an effective contribution of the Pomeron rescatterings and it is straightforward to demonstrate that its sign may be negative. The above arguments as well as those given

[6]The Pomeron contributions to the amplitudes πp, γp and $\gamma\gamma$ are given in [2]

[7]A combination of a $\ln(s/s_0)$ with a $\ln^2(s/s_0)$ gives comparably good results but it involves more parameters; furthermore, as far as only pp and $\bar{p}p$ are concerned, existing data do not allow to discriminate [13] between behaviors in $\ln^2(s/s_0)$, $\ln(s/s_0)$ and $(s/s_0)^\epsilon$.

[8]It is noted also in [15]

in [2] suggest that we may write the Pomeron amplitudes in MAQM at $t = 0$ as follows

$$\begin{aligned} A^{(1)}_{\mathcal{P}}(s,0) =& \; ig_1^2[-\zeta_{\mathcal{P}} + L(s)] \;, \\ A^{(2)}_{\mathcal{P}}(s,0) =& \; ig_1 g_2[-\zeta_{\mathcal{P}} + L(s)] \;, \\ A^{(3)}_{\mathcal{P}}(s,0) =& \; ig_2^2[-\zeta_{\mathcal{P}} + L(s)] \end{aligned} \tag{2}$$

where

$$L(s) = \ln(-is/s_0).$$

2.2 Secondary Reggeons

In pp and $\bar{p}p$ the secondary Reggeons are numerous, however, for the energy range involved here we can choose to keep only f and ω Reggeons, two non degenerate C=+1 and C=-1 meson trajectories. As it is argued in [2] instead of nine identical diagrams for the f-Reggeon in the SAQM, leading to the factor 9, one obtains

$$A_f^{(pp)}(s,0) = P_p^2(5 + 4\lambda_f)A_f^{(qq)}(s/9,0), \tag{3}$$

where

$$A_f^{(qq)}(s,0) = ig_f^2\Big(-is/s_0\Big)^{\alpha_f(0)-1} \tag{4}$$

and λ_f is a constant taking into account a mixing of $u\bar{u}$ and $d\bar{d}$ quark states in f-Reggeon. Similarly for ω-Reggeon we set

$$A_\omega^{(pp)}(s,0) = P_p^2(5 + 4\lambda_\omega)A_\omega^{(qq)}(s/9,0) \;, \tag{5}$$

$$A_\omega^{(qq)}(s,0) = g_\omega^2\Big(-is/s_0\Big)^{\alpha_\omega(0)-1} \;. \tag{6}$$

An important property of the Dipole Pomeron model is that all fits give a high value of the f-Reggeon intercept, $\alpha_f(0) \approx 0.8 \div 0.82$ [12, 13, 14, 15]. Does such an intercept contradict the data on the f-trajectory known from the resonance region? The answer is "yes" if the trajectory is assumed to be linear. However, aside from general theoretical arguments against linear trajectories, the experimental data on the resonances lying on the f-trajectory indicate its nonlinearity (see Fig.1). A more realistic trajectory such as $\alpha_f(t) = \alpha_f(0) + \gamma_1(\sqrt{4m_\pi^2} - \sqrt{4m_\pi^2 - t}\,) + \gamma_2(\sqrt{t_1} - \sqrt{t_1 - t}\,)$, gives $0.77 < \alpha_f(0) < 0.87$.

Generally, there is an evident correlation between the intercept of the f-Reggeon and the model for the Pomeron. This is due to the fact that in all known processes, Pomeron and f-Reggeon contribute additively. As a rule a higher f-intercept is associated with a slower growth with energy due to Pomeron contribution (as an example see also [15]). In Fig.2 we illustrate this observation and show how $\alpha_f(0)$ is correlated with a power of $\ln s$ in the behaviour of the total cross-section, if the forward scattering amplitudes are parameterized in the form

$$A^{hh}(s,0) = i[C_1 + C_2\ln^\gamma(-is/s_0)] + R(s,0),$$

where the explicit form of secondary Reggeons contribution $R(s,0)$, depends on the kind of interacting hadrons (see [12, 15] for details). As a consequence, we have no serious reason to fix the intercept $\alpha_f(0)$.

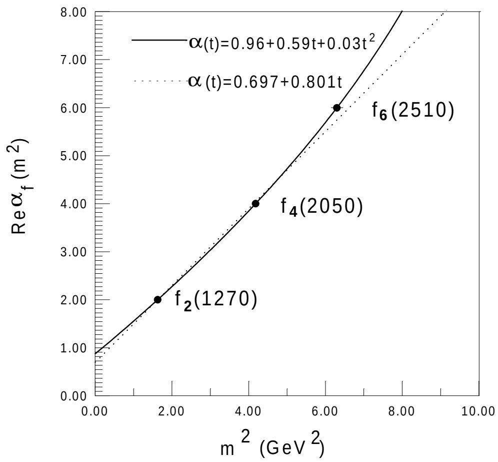

Figure 1: Real part of the f-trajectory versus m^2, squared mass of the resonance. Solid line is the parabola passed through known resonaaces. Dashed straight line is the result of a fit.

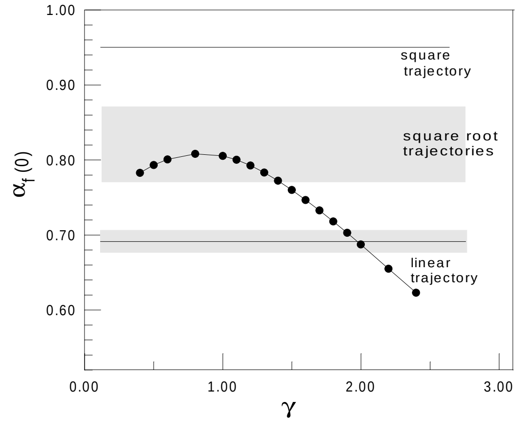

Figure 2: Intercept of the f-trajectory correlated with the power γ of $\ln s$ in $\sigma_{tot}(s)$. Intervals for an intercept of various parametrizations are shown.

3 The modified additive quark model at $t \neq 0$

The amplitudes under interest are written

$$A_{pp}^{\bar{p}p}(s,t) = A_{\mathcal{P}}(s,t) + A_f(s,t) \pm [A_{\mathcal{O}}(s,t) + A_\omega(s,t)] \ . \tag{7}$$

The normalization is

$$\sigma_{tot}(s) = 8\pi \Im m A(s, t=0), \qquad \frac{d\sigma}{dt}(s,t) = 4\pi |A(s,t)|^2 \ . \tag{8}$$

The starting points for a parameterization of all terms in (10) at $t \neq 0$ are the corresponding partial amplitudes defined at $t = 0$ in [2] and rewritten in detail in (1-6). A special discussion will be devoted to the Odderon amplitude $a_{\mathcal{O}}(s,t)$ which is out of he game at $t = 0$.

Let us recall that in accordance with the main assumption of additive quark model (as well as of our MAQM [2]) there is an interaction of two (in fact of 3 X 3) constituent quarks (or lines in terms of diagram), each of them carrying only a part of the momentum $\vec{p}$. Therefore we must define $s_q = (p_1 + p_2)^2 = s/9$ (for protons under assumption that the whole momentum is distributed equally between all quarks in each of them). As concerns the t channel, we consider for Pomeron and f Reggeon a single exchange (one line in t channel in terms of diagram); our Odderon also is supposed to behave as a one Reggeon exchange. Consequently, we have no reason to divide t in the final amplitude by any number [9].

[9] Such a division, *e.g.* by 9, occurs when three gluon or three Pomeron exchanges (three lines in t channel) are considered (as was done in [17]), implying a distribution of the momentum $\vec{q}$ along three lines, each of them being assumed to carry an averaged momentum $\vec{q}/3$.

3.1 Pomeron

The most direct generalization of the $A^{(i)}_{\mathcal{P}}(s,0)$ (in (4)) is to consider all "coupling constants" $g_1, g_2, \zeta_{\mathcal{P}}$ as functions of t and to multiply $A^{(i)}_{\mathcal{P}}(s,0)$, $(i=1,2,3)$ by the usual Regge factor $\Big(-is/s_0\Big)^{\alpha_{\mathcal{P}}(t)-1}$. Namely, we write

$$A^{(1)}_{\mathcal{P}}(s,t) = ig_1^2(t)[-\zeta_{\mathcal{P}}(t) + L(s)]\Big(-is/s_0\Big)^{\alpha_{\mathcal{P}}(t)-1}, \tag{9}$$

$$A^{(2)}_{\mathcal{P}}(s,t) = ig_1(t)g_2(t)[-\zeta_{\mathcal{P}}(t) + L(s)]\Big(-is/s_0\Big)^{\alpha_{\mathcal{P}}(t)-1}, \tag{10}$$

$$A^{(3)}_{\mathcal{P}}(s,t) = ig_2^2(t)[-\zeta_{\mathcal{P}}(t) + L(s)]\Big(-is/s_0\Big)^{\alpha_{\mathcal{P}}(t)-1}. \tag{11}$$

As the simplest variant we choose the linear Pomeron trajectory (again with an intercept equal 1)

$$\alpha_P(t) = 1 + \alpha'_P t, \tag{12}$$

however we considered also the case of a logarithmic trajectory which is discussed in details in Section 4.

Of course, more sophisticated models can be proposed but they lead to an extra number of parameters and we will not consider them.

Finally, to avoid proliferation of parameters, we will assume, as usual exponential "residue functions" $g_i(t)$, $(i=1,2)$ and $\zeta_{\mathcal{P}}(t)$:

$$g_1(t) = g_1\exp(b_1 t), \qquad g_2(t) = g_2\exp(b_2 t), \qquad \zeta_{\mathcal{P}}(t) = \zeta_{\mathcal{P}}\exp(b_{\zeta_{\mathcal{P}}} t). \tag{13}$$

Summing up, the Pomeron contribution at $t \neq 0$ will have the form

$$A_{\mathcal{P}}(s,t) = 9[A^{(1)}_{\mathcal{P}}(s/9,t) + 2A^{(2)}_{\mathcal{P}}(2s/9,t) + A^{(3)}_{\mathcal{P}}(4s/9,t)]\ , \tag{14}$$

where the partial amplitudes are defined by (9-11). Thus there are 7 ($g_1, g_2, \xi_{\mathcal{P}}, b_1, b_2, b_{\zeta_{\mathcal{P}}}$, $\alpha'_{\mathcal{P}}$) free parameters for the Pomeron term of amplitude. With this Pomeron model the Froissart-Martin bound is not violated and the total cross-section behaves as lns.

3.2 Secondary Reggeons

Generalizing [2], the f-Reggeon amplitude in the MAQM is written as

$$A_f(s,t) = (5+4\lambda_f)A_f^{(qq)}(s/9,t)\ , \tag{15}$$

where

$$A_f^{(qq)}(s,t) = ig_f^2\Big(-i\frac{s}{s_0}\Big)^{\alpha_f(t)-1}e^{b_f t}, \quad \alpha_f(t) = \alpha_f(0) + \alpha'_f t\ . \tag{16}$$

For the f-Reggeon trajectory (as well as for the ω-Reggeon, see below) we take a linear dependence on t. The only reason for such a choice is simplicity and a minimal number of parameters. The value of λ_f is unimportant if only pp and $\bar{p}p$ processes are considered (it is sufficient to redefine the coupling g_f). Nevertheless, for the present work we can put $\lambda_f = 0.439$ which was obtained from the fit at $t = 0$ (see [2]). The number of free parameters is then 4 ($g_f, \alpha_f(0)$, α'_f, b_f).

Following the previous considerations, we write for the ω-Reggeon

$$A_\omega(s,t) = (5+4\lambda_\omega)A_\omega^{(qq)}(s/9,t)\ , \tag{17}$$

where

$$A_\omega^{(qq)}(s,t) = g_\omega^2 r_\omega(s,t)\Big(-i\frac{s}{s_0}\Big)^{\alpha_\omega(t)-1}e^{b_\omega t}, \quad \alpha_\omega(t) = \alpha_\omega(0) + \alpha'_\omega t\ . \tag{18}$$

The additional factor $r_\omega(s,t)$ describes so called a "cross-over phenomenon", namely, a zero at small t in the difference of the $\bar{p}p$ and pp differential cross-sections, $d\sigma(\bar{p}p)/dt - d\sigma(pp)/dt$. A weak dependence on s of the cross-over point t_ω determined from the condition $r_\omega(s,t_\omega) = 0$ allows to reproduce (as was noted in [11]) the experimental data at $|t| \leq 0.7$ GeV^2 with a quite high quality. It is necessary to note that there was only one crossing-odd term of amplitudes in [11]. The factor $r_\omega(s,t)$ was very important under this condition. Now we extend the kinematic region under consideration up to $|t| \approx 14$ GeV^2 and also include Odderon contribution, which, however, is dumped at small $|t|$ (see below). Therefore we left the factor $r_\omega(s,t)$, and in order to avoid a possible enhancement of this factor at large $|t|$ and s we choose it in the form

$$r_\omega(s,t) = 1 + t[r_1 + r_2/\ln(-is/s_0)]\ , \tag{19}$$

instead of $r_\omega(s,t) = 1 + t[r_1 + r_2\ln(-is/s_0)]$ used in [11].

Again, repeating the arguments given above for the f-Reggeon, we put $\lambda_\omega = 1$. If $r_\omega(s,t) \neq 1$ there are 6 free parameters, $g_\omega, \alpha_\omega(0), \alpha'_\omega, b_\omega, r_1, r_2$. In opposite case we have 4 parameters.

3.3 Odderon

This additional crossing-odd contribution (with respect to the ω-term) is a quite delicate and ambiguous point lacking sufficiently precise data. A widespread consensus [12, 13, 15], however, is that an Odderon contribution while insignificant at $t = 0$ is very relevant in the large-$|t|$ domain.

As compared with the previous contributions, it is important to notice that it is impossible to apply to the Odderon any additive quark model rule. Odderon, in contrast with Pomeron and secondary Reggeons, in fact, interacts with the whole proton rather than with separate quarks since three-gluons (or the Odderon by Donnachie and Landshoff [19]) couple simultaneously with three quarks in each p or $\bar{p}$.

As repeatedly mentioned, in this paper we stick to the Born approximation and rescattering corrections are not taken into account. This is known to be inadequate from the conceptual point of view on many counts and not just for practical reasons of restoring unitarity in the cases in which the Born approximation would yield its violation. The point is particularly delicate concerning the Odderon which, by universal consensus, should be important at large-$|t|$. For this reason, in this paper we parameterize the Odderon, somewhat artificially, as the sum of two contributions which we denote as a "soft" and a "hard" term

$$A_{\mathcal{O}}(s,t) = A_{\mathcal{O}}^{(s)}(s,t) + A_{\mathcal{O}}^{(h)}(s,t)\ . \tag{20}$$

As it is known, the contribution of Odderon at $t = 0$ is negligible. We take into account this fact multiplying its both components by a factor vanishing at $t = 0$. For the soft

Odderon let assume like for the Pomeron a dipole form [10]

$$A_{\mathcal{O}}^{(s)}(s,t) = g_{\mathcal{O}s}(t)\Big[(1-e^{\beta_s t})\ln(-is/s_0)\Big]^{\mu}\Big[-\zeta_{\mathcal{O}s}(t)+\ln(-is/s_0)\Big]\Big(-i\frac{s}{s_0}\Big)^{\alpha_{\mathcal{O}}(t)-1}, \quad (21)$$

while for the hard one, we choose

$$A_{\mathcal{O}}^{(h)}(s,t) = g_{\mathcal{O}h}(1-e^{\beta_h t})[\ln(-is/s_0)]^{\nu}\frac{1}{(1-t/t_{\mathcal{O}h})^4}\ . \quad (22)$$

We should give here a few comments concerning the choice of the Odderon amplitude defined by the above equations.

1) A contribution of the soft Odderon to $\sigma_{elastic}$ is dominated by the region where $|t|$ is small, namely, $|t| \lesssim \frac{const}{\alpha'_O \ln(s/s_0)}$. In this domain the factor $(1-e^{\beta_s t})\ln(-is/s_0)$ is constant. It means that the soft Odderon (21) does not violate the evident inequality $\sigma_{elastic} \le \sigma_{tot}$ at any value of μ.

2) At the same time an amplitude should not have a singularity at $t=0$, therefore μ must be an integer. In the fit we have considered $\mu = 1$ and $\mu = 2$.

3) The hard Odderon (22) has not an exponential decrease with $|t|$, therefore it does not violate restriction $\sigma_{elastic} \le \sigma_{tot}$ at any s only if $\nu \le 1/2$.

Again, as for the Pomeron, we consider the linear Odderon trajectory

$$\alpha_{\mathcal{O}}(t) = 1 + \alpha'_{\mathcal{O}}t \quad (23)$$

and the soft residue functions in an ordinary exponential form

$$g_{\mathcal{O}s}(t) = g_{\mathcal{O}s}e^{b_{\mathcal{O}}t}, \qquad \zeta_{\mathcal{O}s}(t) = \zeta_{\mathcal{O}}e^{b_{\zeta_{\mathcal{O}}}t}\ . \quad (24)$$

Thus the Odderon contribution is controlled by 9 additional parameters : $g_{\mathcal{O}s}, \zeta_{\mathcal{O}}, \beta_s, b_{\mathcal{O}}, b_{\zeta_{\mathcal{O}}}, \alpha'_{\mathcal{O}}, g_{\mathcal{O}h}, t_{\mathcal{O}h}, \beta_h$.

The grand total number of free parameters is 26, however, this number may be reduced by fixing some of parameters by virtue of some special arguments. For example, this paper being devoted to pp and $\bar{p}p$ angular distributions, all coupling constants and intercepts are fixed from the fit to σ_{tot} and ρ at $t=0$ that is reported in [2]; furthermore, the slopes of trajectories could be fixed at their "world" values and so on (not excluding simplifications in the chosen form for the Odderon).

3.4 Comparison with the standard additive quark model

In the SAQM, the Pomeron contribution at $t=0$ reads

$$A_{\mathcal{P}}(s,0) = 9[A_{\mathcal{P}}^{(qq)}(s/9,0)]\ , \quad (25)$$

with

$$A_{\mathcal{P}}^{(qq)}(s,0) = A_{\mathcal{P}}^{(1)}(s,0)\ . \quad (26)$$

The generalization to $t \neq 0$ is straightforward

$$A_{\mathcal{P}}(s,t) = 9[A_{\mathcal{P}}^{(qq)}(s/9,t)]\ , \quad (27)$$

[10] Strictly speaking it has dipole form if $\mu = 0$.

with the quark-quark amplitude

$$A_{\mathcal{P}}^{(qq)}(s,t) = A_{\mathcal{P}}^{(1)}(s,t) \ , \tag{28}$$

defined above in Eq.(9). We can, similarly, generalize the contributions of the Reggeons

$$A_f(s,t) = 9[A_f^{(qq)}(s/9,t)] \tag{29}$$

and

$$A_\omega(s,t) = 9[A_\omega^{(qq)}(s/9,t)] \ , \tag{30}$$

where the quark amplitudes are given by (16) and (18). To make a meaningful comparison (where only the counting rules are changed by the generalization SAQM → MAQM) one must also enter in the amplitude the Odderon which is taken strictly the same in the two models (given in (20-24)).

Consequently, the difference between the two models is due mainly to the Pomeron contribution.

4 Results and discussion

4.1 Previous results at $t = 0$ [2]

As a first step, we refer to the results found in [2], where 217 $t = 0$ data (for pp and $\bar{p}p$) with 4 GeV$\leq \sqrt{s} \leq$1800. GeV ([20]) have been taken into account, within combined fits of pp , $\bar{p}p$ $\pi^- p$, $\pi^+ p$, $\gamma\gamma$, γp total cross-sections and ρ ratios. The selected results under interest here are given in the Table 1 which exhibits the improvement brought to the old AQM by the modifications called in the revisited AQM.

Observable	σ_{tot}^{pp}	$\sigma_{tot}^{\bar{p}p}$	ρ^{pp}	$\rho^{\bar{p}p}$
Number of points	85	51	64	17
χ^2 SAQM	220	240	157	17
χ^2 MAQM	53	59	147	18

Table 1. The partial χ^2-s obtained by fitting at $t = 0$ [2] in the SAQM (old) and the MAQM (modified).

The $\chi^2/d.o.f.$ (for the all processes) was 3.04 in the SAQM and 1.78 in the MAQM. The recalculated $\chi^2/d.o.f.$ (specifically for pp, $\bar{p}p$ processes) is 3.00 in the SAQM and 1.32 in the MAQM. The corresponding behavior of $\sigma_{tot}(s)$ and of $\rho(s) = \Re e A(s,0)/\Im m A(s,0)$ are plotted in [2]. To be complete, we give in Table 2, for the two models, the parameters issued from the combined fits in [2] and used here for the pp and $\bar{p}p$ process.

Parameter	g_1(GeV^{-1})	g_2(GeV^{-1})	ζ_P	g_f(GeV^{-1})	$\alpha_f(0)$	g_ω(GeV^{-1})	$\alpha_\omega(0)$
Value in SAQM	0.247	–	– 0.418	0.770	0.741	0.374	0.515
Value in MAQM	0.316	– 0.0239	3.396	1.112	0.810	0.395	0.422

Table 2. Values of the parameters controlling the SAQM and MAQM amplitudes at $t = 0$ [2]. Recall the Pomeron intercept equals 1. and the values quoted for g_f and g_ω are coupled to $\lambda_f = 0.439$ and $\lambda_\omega = 1.$ respectively.

4.2 Comparison between the SAQM and MAQM at $t \neq 0$

The previous 7 parameters are kept fixed to the values determined by the $t = 0$ combined fits: only the remaining parameters are fitted from the angular distributions. Of course, the final χ^2 could be slightly improved by refitting the complete set of parameters for the pp and $\bar{p}p$ processes alone but we decide against doing so.

In the second step, we account for 959 $t \neq 0$ data [20] with 0.1< $|t| \leq 14.2$ GeV^2; 19.4≤ $\sqrt{s}$ ≤630 GeV. The selection $|t| > 0.1$ GeV^2 is used in order to exclude the Coulomb-nuclear interference region. It could be included as a refinement.

After preliminary trials, we select the following conditions for the current parameterization

(i) the "cross-over" factor $r_\omega(s,t)$ must be used in the Reggeon amplitude (*i.e.* r_1 and r_2 parameters must be fitted) to obtain a better quality result

(ii) $\mu = 2$ is chosen here because it gives a slightly better χ^2, however, taking $\mu = 1$ does not influence significantly the quality of the fit: once more the data do not seem precise enough to select a specific form of the Odderon

(iii) $\nu = 1/2$: for this (maximal) value, the Odderon does not violate unitarity, the results at high energy (LHC) do not seem abnormal and the Odderon can be considered as an effective phenomenological contribution

(iv) linear trajectories are used throughout, which represents the more economic (if not the more efficient) solution

(v) furthermore, as unitarity requires [21], we constrained $\alpha'_{od} \leq \alpha'_p$. We found the results distributed according to Table 3.

Observable	$(d\sigma/dt)^{pp}$	$(d\sigma/dt)^{\bar{p}p}$
Number of points	758	201
χ^2 SAQM	2205	673
χ^2 MAQM	1435	540

Table 3. Resulting partial χ^2-s obtained by fitting the angular distributions in the SAQM and MAQM with the parameters in Table 4.

The $\chi^2/d.o.f.$ (for $t \neq 0$) is 2.85 in the SAQM and 2.11 in the MAQM. The corresponding $\chi^2/d.o.f.$, recalculated together with the $t = 0$ and $t \neq 0$ data (with the fitted couplings and intercepts given in Table 2) is 2.88 in the SAQM and 1.97 in the MAQM.

We found within the MAQM a pretty good reproduction of the data, exhibited in Figs. 3-4 (including the dip and the high $|t|$ regions). The MAQM (again, as in the case $t = 0$, above) is better than the SAQM (a significant improvement reveals essentially in the dip region at the lowest energies, below the ISR). The only disagreement of the MAQM curiously concerns the $\bar{p}p$ angular distribution at 53 GeV.

As already stated, we have no arguments to fix the slopes of the trajectories at any definite values (even at the so called "world" values: $\alpha'_{\mathcal{P}} = 0.25$, $\alpha'_f = 0.84$, $\alpha'_\omega = 0.93$ GeV^{-2} [4, 5]). These parameters were free in our best fit. However, one constat that the slopes obtained in fitting with the MAQM do not differ strongly from this slopes, but they lead to a noticeable improvement of agreement with data. Nevertheless, we have performed the fit with the slopes fixed at the above quoted values : we obtain $\chi^2/d.o.f. \simeq 2.83$ with a rather good reproduction of the data.

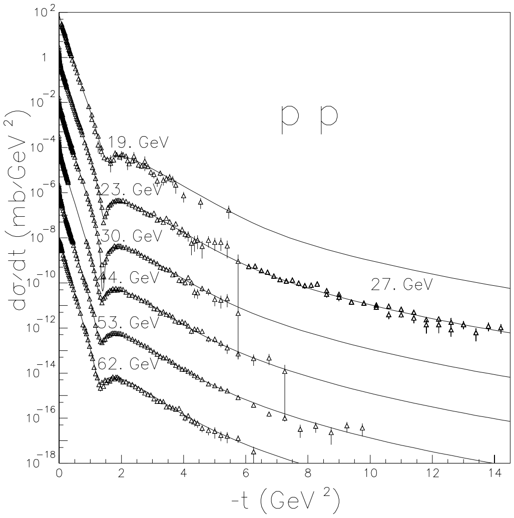

Figure 3: Fit of the differential cross-sections of pp interaction, calculated in MAQM. A factor 10^{-2} between each successive energy is omitted.

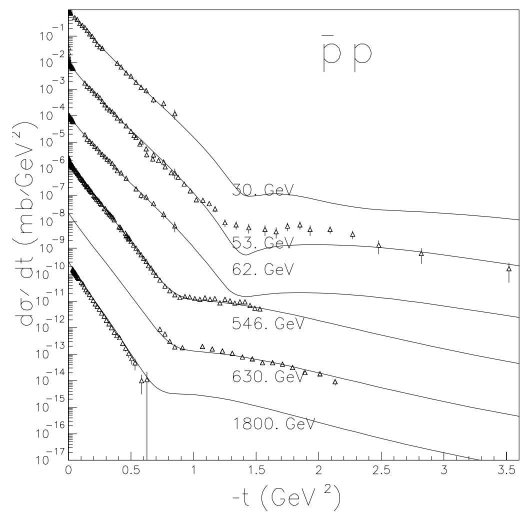

Figure 4: The same as in Fig. 3, but for $\bar{p}p$. The Tevatron data are not fitted.

Generally, one can see, comparing the results for MAQM and SAQM (mainly χ^2-s, because the plots of the angular distributions would necessitate a large scaling), that the description of data is essentially better in MAQM than in SAQM. Moreover, some of the parameters in SAQM (*e.g.* such as slopes and $b_{\zeta_{\mathcal{P}}}$) tend, if not limited, taking unreasonable values (see Table 4).

	Parameter	MAQM	SAQM
Pomeron	b_1, GeV^{-2}	.2282E+01	.1848E+01
	b_2, GeV^{-2}	-.1628E+00	—
	$b_{\zeta_{\mathcal{P}}}$, GeV^{-2}	.1060E+01	.10000E+03
	$\alpha'_{\mathcal{P}}$, GeV^{-2}	.2976E+00	.3475E+00
f-Reggeon	b_f, GeV^{-2}	.2905E+01	.1216E+01
	α'_f, GeV^{-2}	.8839E+00	.1294E+01
ω-Reggeon	b_ω, GeV^{-2}	.1936E+01	.1300E+01
	α'_ω, GeV^{-2}	.8252E+00	.6000E+00
	r_1, GeV^{-2}	− .7831E+01	.8261E+01
	r_2, GeV^{-2}	− .6944E+02	− .2077E+02
Odderon (soft)	$g_{\mathcal{O}s}$, GeV^{-2}	− .1366E-02	− .3445E-02
	$\zeta_{\mathcal{O}}$	.2100E+02	.7727E+01
	β_s, GeV^{-2}	.5078E+01	.1489E+01
	$b_{\mathcal{O}}$, GeV^{-2}	.1196E+00	− .2159E+00
	$b_{\zeta_{\mathcal{O}}}$, GeV^{-2}	.1508E+01	.3992E+00
	$\alpha'_{\mathcal{O}}$, GeV^{-2}	.2976E+00	.3475E+00
Odderon (hard)	$g_{\mathcal{O}h}$, GeV^{-2}	.4507E+01	.3010E+01
	$t_{\mathcal{O}h}$, GeV^2	.3475E+00	.3948E+00
	β_h, GeV^{-2}	.1306E+01	.1686E+01

Table 4. Parameters of the MAQM and SAQM obtained in fitting the angular distributions.

4.3 Unitarity, Odderon and logarithmic trajectories.

-1) As a by-product of the present study, we propose to consider the scattering amplitude we obtained in our MAQM from the point of view of unitarity and analyticity. To be specific we want to insure that this amplitude not only respects the Froissart-Martin bound but also exhibits a zero at low $|t|$ in $\Re e A^+(s,t)$, real part of its even (with respect to the C-parity) contribution as required by a high energy theorem recently stated by A. Martin [16].

The situation of the first zero of $\Re e A^+(s,t)$ and of $\Re e A_{\mathcal{P}}(s,t)$ is shown in Table 5 for some selected energies. A confirmation of the theorem is obtained. We also verify the well known dominance of the Pomeron at high energies (when the f-Reggeon contribution is negligible, say above the ISR energy range). Thus we agree with the results quoted in [16].

Energy (GeV)	zero of A^+ $-t$ (GeV^2)	zero of $A_{\mathcal{P}}$ $-t$ (GeV^2)
50	0.12	1.15
500	0.35	0.43
1800	0.30	0.33
14000	0.23	0.24

Table 5. First values of $-t$ cancelling $\Re e A^+(s,t)$ and $\Re e A_{\mathcal{P}}(s,t)$, versus the energy, obtained in the MAQM fit.

-2) $\Im m A^+(s,t)$, imaginary part of the even component of the amplitude, has an oscillatory behavior for large $|t|$, in spite of lacking any rescatterings corrections in the model. This is a consequence of the negative sign of the new MAQM (comparing with the old SAQM) coupling constant g_2 (see Table 2). It is interesting to note that a zero reveals at $|t| \sim 1.1$ GeV^2 in both real and imaginary parts. Oscillations, or something appearing as diffraction-like secondary structures are hiden in the differential cross-sections because of the Odderon contribution dominating in this domain. Of course our present MAQM model can not produce any oscillations at large $|t|$, because it is a model at the Born approximation level, in which the Odderon dominates at very high energy and in the high $|t|$-region with a smooth behavior. As a next step, it would be interesting to eikonalize the model to see in particular if oscillations in $d\sigma/dt$ appear. We believe that rescattering corrections (we plan to calculate them in a near future) may be important at such high energies to be investigated for instance in the TOTEM and PP2PP projects [22] and as a consequence any extrapolation (in particular those of the angular distributions) may be doubtful.

-3) As mentioned already above, the "hard" component of the Odderon with a power behaviour at large $|t|$ looks affectedly from the point of view of Regge approach (note here that for the whole set of data, the ratio $|t|/s$ is small, we are in a domain of small angle scattering, that, we believe, should be described by Regge theory while the hard Odderon component can be important at larger $|t|$).

Taking into account the above argument we have considered the model without its "hard" Odderon component, but for Pomeron and (soft) Odderon we tested the nonlinear trajectories with a logarithmic behavior

$$\alpha_i(t) = 1 + \gamma_i[1 - (1 - t_i/t)\ln(1 - t/t_i)], \qquad i = \mathcal{P},\ \mathcal{O}. \tag{31}$$

The minimal value of the threshold $t_{\mathcal{P}}$ (there are many thresholds in t-channel for pp and $\bar{p}p$) should be given by the t-channel physical state with the minimal mass (in our case a two-pion state, so $t_{\mathcal{P}}^{min} = 4m_\pi^2$). Nevertheless, in order to take into account the influence of other thresholds we consider $t_{\mathcal{P}}$ as a free parameter. A similar argument can be repeated for the Odderon trajectory.
Logarithmic trajectories mimic a power decreasing amplitude with $|t| \to \infty$. However, in order to give a sense to such a possibility, it is necessary to replace the exponential residue functions by power ones. It leads to an extra number of free parameters. Therefore we used another, probably oversimplified, method. All exponents, $\exp(b_i t)$ in the Pomeron and in the soft Odderon terms are replaced by $\exp[(\alpha_{\mathcal{P}(\mathcal{O})}(t) - 1)b_i]$. We would like to note that, making use of logarithmic trajectories, we do not aim to obtain the best fit (see our previous remark about residue functions), rather we want only to check our belief (and to demonstrate for a reader) that it is then possible to reproduce large $|t|$ data.
Resulting $\chi^2/d.o.f = 3.46$ as well as agreement with data is not so bad. The calculated angular distributions are deviated slightly from the data points mainly around of dips and for pp but, as we expected, are very well reproduced for the large-$|t|$ domain.

Summarizing, we should emphasize that the obtained results confirm and reinforce the conclusion of [2]: account of the corrections to a Pomeron-quark interaction in the standard additive quark model and new counting rules for the secondary Reggeons, in other words the modified additive quark model, leads to an essential better agreement of the model with available experimental data not only at $t = 0$, but also at $t \neq 0$. Besides we found that data on the elastic pp and $\bar{p}p$ scattering at high energies can be reproduced with very high quality in the model with the Dipole Pomeron, which has intercept equaled one, $\alpha_{\mathcal{P}}(0) = 1$ leading to an intermediate growth of the total cross-sections, $\sigma_{tot}(s) \propto \ln s$.

Acknowledgements. We are indebted to E. Predazzi for his support and discussions. Two of us (MG and EM) would like to thank the Theoretical Physics Department of the University of Torino for kind hospitality. EM thanks also the Institut de Physique Nucléaire de Lyon for the hospitality. Financial support by the INFN and the MURST of Italy and from the IN2P3 of France is gratefully acknowledged.

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