Meson loop effects in the Nambu-Jona-Lasionio model

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We compare two different possibilities to include meson-loop corrections in the Nambu–Jona-Lasinio model: a strict $1/N_c$-expansion in next-to-leading order and a non-perturbative scheme corresponding to a one-meson-loop approximation to the effective action. Both schemes are consistent with chiral symmetry, in particular with the Goldstone theorem and the Gell-Mann–Oakes–Renner relation. The numerical part at zero temperature focuses on the pion and the ρ-meson. We also investigate the temperature dependence of the quark condensate. Here we find consistency with chiral perturbation theory to lowest order. Similarities and differences of both schemes are discussed.

I. INTRODUCTION

During the last few years one of the principal goals in nuclear physics has been to explore the phase structure of QCD. Along with this comes the investigation of hadron properties in the vacuum as well as in hot or dense matter. In principle, all properties of strongly interacting particles should be derived from QCD. However, at least in the low-energy regime, where perturbation theory is not applicable, this is presently limited to a rather small number of observables which can be studied on the lattice, while more complex processes can either be addressed by chiral perturbation theory or within effective model calculations which try to incorporate the relevant degrees of freedom.

So far the best descriptions of hadronic spectra, decays and scattering processes are obtained within phenomenological hadronic models. For instance the pion electromagnetic form factor in the time-like region can be reproduced rather well within a simple vector dominance model with a dressed ρ-meson which is constructed by coupling a bare ρ-meson to a two-pion intermediate state [1,2]. Models of this type have been successfully extended to investigate medium modifications of vector mesons and to calculate dilepton production rates in hot and dense hadronic matter [3].

In this situation one might ask how the phenomenologically successful hadronic models emerge from the underlying quark structure and the symmetry properties of QCD. Since this question cannot be answered at present from first principles it has to be addressed within quark models. For light hadrons chiral symmetry and its spontaneous breaking in the physical vacuum through instantons plays the decisive role in describing the two-point correlators [4] with confinement being much less important. This feature is captured by the Nambu–Jona-Lasinio (NJL) model in which the four-fermion interactions can be viewed as being induced by instantons. Furthermore the model allows a study of the chiral phase transition as well as the examination of the influence of (partial) chiral symmetry restoration on the properties of light hadrons.

The study of hadrons within the NJL model has of course a long history. In fact, mesons of various quantum numbers have already been discussed in the original papers by Nambu and Jona-Lasinio [5] and by many authors thereafter (for reviews see [6–8]). In most of these works quark masses are calculated in mean-field approximation (Hartree or Hartree-Fock) while mesons are constructed as correlated quark-antiquark states (RPA). This corresponds to a leading-order approximation in $1/N_c$, the inverse number of colors. With the appropriate choice of parameters chiral symmetry, which is an (approximate) symmetry of the model Lagrangian, is spontaneously broken in the vacuum and pions emerge as (nearly) massless Goldstone bosons. While this is clearly one of the successes of the model, the description of other mesons is more problematic. One reason is the fact that the NJL model does not confine quarks. As a consequence a meson can decay into free constituent quarks if its mass is larger than twice the constituent quark mass m. Hence, for a typical value of $m \sim 300$ MeV, the ρ-meson with a mass of 770 MeV, for instance, would be unstable against decay into quarks. On the other hand the physical decay channel of the ρ-meson into two pions is not included in the standard approximation. Hence, even if a large constituent quark mass is chosen in order to suppress the unphysical decays into quarks, one obtains a poor description of the ρ-meson propagator and related observables, like the pion electromagnetic form factor.

Similar problems arise if one wants to study the phase structure of strongly interacting matter within a mean-field calculation for the NJL model, although this has been done by many authors (see e.g. [7–10]). In these calculations the thermodynamics is entirely driven by unphysical unconfined quarks even at low temperatures and densities, whereas the physical degrees of freedom, in particular the pion, are missing.

This and other reasons have motivated several authors to go beyond the standard approximation scheme and to include mesonic fluctuations. Since the most important feature of the NJL model is chiral symmetry, one should use an approximation scheme which conserves the symmetry properties, to ensure the existence of massless Goldstone bosons. Within the present article we will discuss two different approximation schemes and compare the results: A strict (perturbative) expansion in $1/N_c$ up to next-to-leading order and a non-perturbative symmetry conserving approximation scheme which has been derived in Ref. [11] using a one-meson-loop approximation (MLA) to the effective action in a bosonized NJL model. The latter scheme has before been presented, derived by completely other means, in Ref. [12]. It has been shown [12,11,13] that both schemes are consistent with the Goldstone theorem and the Gell-Mann–Oakes–Renner relation, two of the most important low energy chiral theorems.

In vacuum we focus the discussion of our results for the pion and the ρ-meson. The inclusion of meson loop effects should also improve the thermodynamics of the model considerably. A first insight on the influence of mesonic fluctuations upon the thermodynamics can be obtained via the temperature dependence of the quark condensate. The low-temperature behavior in both schemes is dominated by pionic degrees of freedom which is a considerable improvement on calculations in Hartree approximation where quarks are the only degrees of freedom. The results are consistent with the lowest-order chiral perturbation theory result [13].

This article is organized as follows. In Sec. II we begin with a brief summary of the standard approximation scheme used in the NJL model to describe quarks and mesons. Afterwards we present the two possibilities to go beyond the standard scheme. The numerical results at zero temperature will be presented in Sec. III. The temperature dependence of the quark condensate will be studied in Sec. IV. Finally, we will summarize in Sec. V.

II. THE MODEL

As discussed in the Introduction, we will employ two different schemes for describing mesons within the NJL model including meson loops: a strict (perturbative) $1/N_c$-expansion and the non-pertubative one-meson loop approximation (MLA). In this section we give a brief outline of these schemes. Details of the formalism can be found in Refs. [13,14].

We consider an NJL-type Lagrangian for two flavors and three colors with scalar-isoscalar and pseudoscalar-isovector interaction:

$$\mathcal{L} \;=\; \bar\psi(i\partial\!\!\!/ - m_0)\psi \;+\; g_s\,[(\bar\psi\psi)^2 + (\bar\psi i\gamma_5\vec\tau\psi)^2] \;-\; g_v\,[(\bar\psi\gamma^\mu\vec\tau\psi)^2 + (\bar\psi\gamma^\mu\gamma_5\vec\tau\psi)^2]\;. \tag{1}$$

Here g_s and g_v are dimensionful coupling constants of order $1/N_c$. In the limit $m_0 = 0$ the Lagrangian is symmetric under $SU(2)_L \times SU(2)_R$ transformations.

Starting point of the standard approximation scheme is to solve the gap equation for the quark propagators in Hartree approximation. This gap equation is displayed in Fig. 1. The mesons are then described in RPA, i.e., by iterated quark-antiquark loops, as illustrated in Fig. 2. The Hartree+RPA approximation scheme corresponds to the leading order in a $1/N_c$-expansion.

FIG. 1. *The gap equation for the quark propagator in Hartree approximation (solid). Dashed lines denote propagators of bare quarks.*

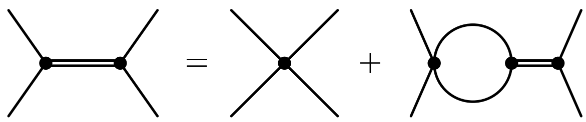

FIG. 2. *The Bethe-Salpeter equation for the meson propagators in RPA (double line). The solid lines indicate quark propagators.*

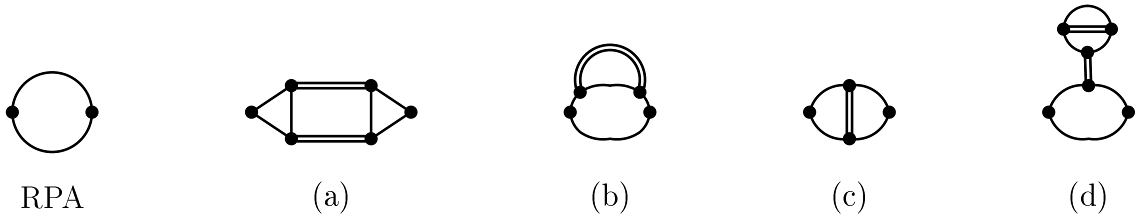

FIG. 3. *Contributions to the meson polarization functions in leading (RPA) and next-to-leading order in* $1/N_c$*.*

The $1/N_c$-correction terms to the meson polarization functions together with the leading order (RPA) quark anti-quark loop are shown in Fig. 3. Solid lines denote quark propagators in Hartree approximation, i.e. in leading order. These diagrams contain meson propagators calculated in RPA, as described above. To obtain the "improved" meson propagators, the entire polarization function, including the $1/N_c$-correction terms, is iterated in the same manner as before the RPA polarization function alone. It can be shown analytically that the pion constructed in this way is again massless in the chiral limit [13]. Note that we do not use the improved meson propagators for evaluating the correction terms. Such a selfconsistent procedure, although desirable from a phenomenological point of view, spoils the $1/N_c$ counting for the polarization functions scheme and leads to inconsistencies with chiral symmetry.

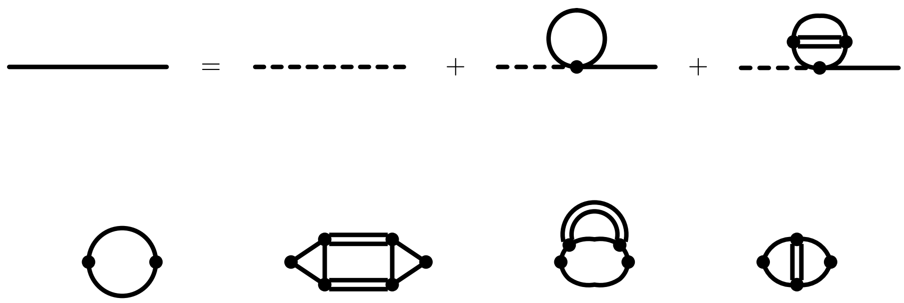

FIG. 4. *Extended gap equation (upper part) and contributions to the meson polarization functions (lower part) in the MLA.*

Within the MLA the gap equation for the quark propagators is modified selfconsistently by adding a term containing a meson loop. This is illustrated in the upper part of Fig. 4. Here the double line is constructed in the same way as a meson propagator in RPA (see Fig. 2), but starting from quarks which emerge as solutions of the modified gap equation itself. The terms contributing to the meson polarization functions are shown in the lower part of Fig. 4. Here the solid lines and the double lines have the same meaning as in the upper part of the figure. To obtain the meson propagators the polarisation functions are iterated as before. Again, it can be shown analytically that the pion constructed in this way is massless in the chiral limit [11,?,13].

III. NUMERICAL RESULTS

A non-renormalizable model is incomplete without defining how to regularize divergent loops. In addition, it is not sufficient to regularize the divergent loops in Hartree + RPA, since new divergencies and hence new cutoff parameters emerge if one includes meson loops[1]. In our case we have to deal with two types of different divergent loops: quark and meson loops. For example let us look at diagram (b) of Fig. 3. This diagram contains one intermediate meson which itself consists of quark loops. This intermediate meson has to be calculated in a first step. We are then left with two loops: one quark loop with four vertices and an integration over the four-momentum of the intermediate meson. It is quite natural to regularize the quark loop in the same way as the polarization diagrams which enter into the intermediate meson. In fact, this is necessary in order to preserve chiral symmetry. We have chosen to use Pauli-Villars regularization with a cutoff Λ_q. However, there is no stringent reason, why the remaining meson loop should also be regularized in this way. We therefore follow Refs. [12,11] and introduce an independent meson cutoff

[1] A possibility to circumvent the problem of additional cutoff parameters is to use a non-local version of the NJL model [15,16]

Λ_M. The results are, of course, strongly dependent on this parameter.

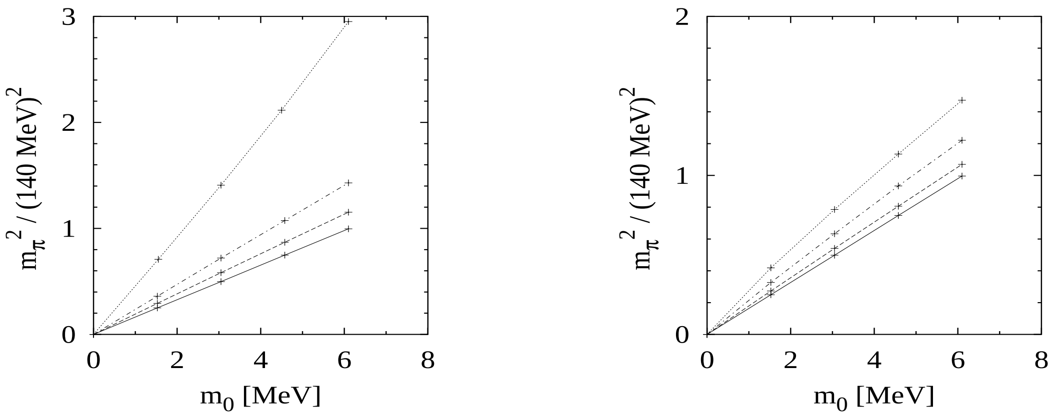

FIG. 5. *Squared pion mass as a function of the current quark mass m_0 for different meson-loop cutoffs; (left) $1/N_c$-expansion scheme: Λ_M = 0 MeV (solid), 500 MeV (dashed), 900 MeV (dashed-dotted) and 1300 MeV (dotted); (right) MLA: Λ_M = 0 MeV (solid), 300 MeV (dashed), 500 MeV (dashed-dotted) and 700 MeV (dotted). The calculated points are explicitly marked.*

Let us first study the influence of mesonic fluctuations on the NJL pion propagator and related quantities. We begin with the leading order, which corresponds to a meson cutoff $\Lambda_M = 0$. With Λ_q = 800 MeV, $g\Lambda_q^2$ = 2.9 and m_0 = 6.1 MeV we obtain a reasonable fit for the pion mass, the pion decay constant and the quark condensate: $m_\pi^{(0)}$ = 140 MeV, $f_\pi^{(0)}$ = 93.5 MeV and $\langle\bar{\psi}\psi\rangle^{(0)}$ = -2 (241.1 MeV)^3. Here and in the following the superscript (0) is used to denote quantities which are calculated in Hartree + RPA. The above parameters correspond to a relatively small constituent quark mass of 260 MeV.

Now we turn on the mesonic fluctuations by taking a non-zero meson cutoff Λ_M. Fig. 5 displays the squared pion mass as a function of the current quark mass m_0 for different values of Λ_M, on the left hand side for the $1/N_c$-expansion scheme and on the right hand side for the MLA. Obviously all points which correspond to the same meson cutoff lie almost on a straight line through the point $(m_0 = 0, m_\pi^2 = 0)$. The latter was calculated analytically whereas all other points are numerical results. This demonstrates the consistency of our scheme with chiral symmetry and the stability of the numerics.

Of course we should not stay with the model parameters determined in leading order, but perform a refit of various observables including the effect of the meson loops. Here we proceed in two steps [14,13]: For various fixed values of the meson cutoff Λ_M we first fix the current quark mass m_0, the quark-loop cutoff Λ_q, and the scalar coupling constant g_s to fit the pion mass, the pion decay constant f_π, and the quark condensate $\langle\bar{\psi}\psi\rangle$. f_π is calculated from the one-pion to vacuum axial vector matrix element, which basically corresponds to evaluating the mesonic polarization diagrams coupled to an external axial current and to a pion. The quark condensate is given by the trace of the quark propagator:

$$\langle\bar{\psi}\psi\rangle = -i \int \frac{d^4p}{(2\pi)^4} \,\mathrm{Tr}\,\mathrm{S(p)}\ . \tag{2}$$

In the $1/N_c$-expansion scheme we have to take into account not only the Hartree contribution but also the two contributions to the quark self energy in next-to-leading order which are shown in Fig. 6.

In the second step we try to determine the two remaining parameters, g_v and Λ_M, by fitting the data for the pion

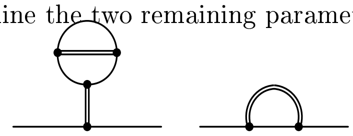

FIG. 6. *The $1/N_c$-correction terms to the quark self-energy.*

electromagnetic form factor in the time-like region. This observable is very well suited for this purpose because it is dominated by the ρ-meson which, besides being a vector state, cannot be described reasonably without including intermediate pion loops. In the $1/N_c$-expansion scheme it is given by the diagrams shown in Fig. 7. Since in this model the phenomenologically important two-pion intermediate state consists of RPA pions, we are forced to fit $m_\pi^{(0)}$ and not m_π to the empirical pion mass if we desire to get the correct threshold behavior for the ρ-meson. However, in most cases we find that the difference between m_π and $m_\pi^{(0)}$ is not very large.

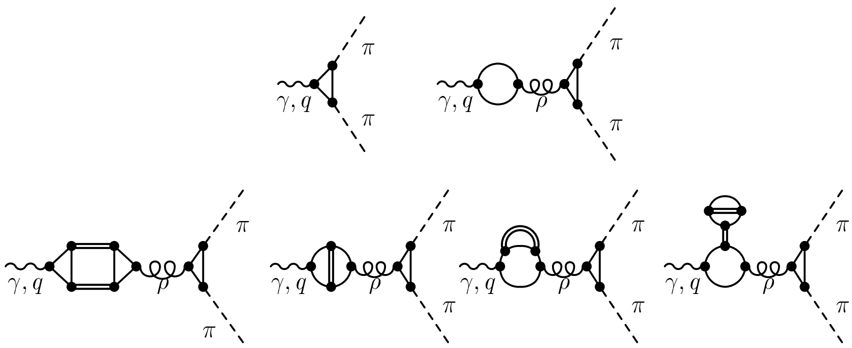

FIG. 7. *Contributions to the pion electromagnetic form factor in the $1/N_c$-expansion scheme. The propagator denoted by the curly line corresponds to the $1/N_c$-corrected rho-meson, while the double lines indicate RPA pions and sigmas.*

The parameter space is further restricted by the requirement that the unphysical $q\bar{q}$-threshold must lie well above the peak in the ρ-meson spectral function in order to obtain a realistic description of the pion electromagnetic form factor. This means, the constituent quark mass has to be larger than about 400 MeV.

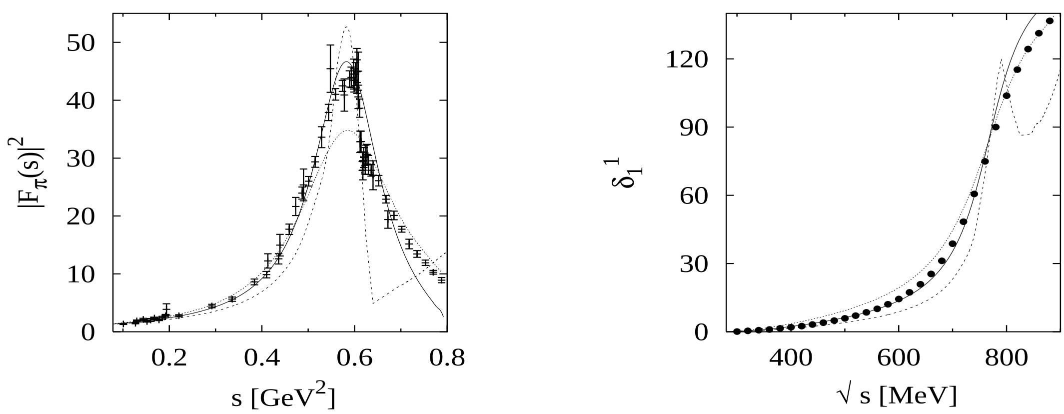

FIG. 8. *Pion electromagnetic form factor (left panel) and the $\pi\pi$-phase shifts in the vector-isovector channel (right panel) for $\Lambda_M = 500$ MeV (dashed), $\Lambda_M = 600$ MeV (solid) as well as $\Lambda_M = 700$ MeV (dotted) in the $1/N_c$-expansion scheme. The data points are taken from Refs. [17] and [18], respectively.*

It was an important result of the analysis in Ref. [14] that such a set of parameters can be found in the $1/N_c$-expansion scheme. Our results for the pion electromagnetic form factor for $\Lambda_M = 500, 600$ and 700 MeV, are displayed on the left hand side of Fig. 8. The value of g_v has been chosen such that the position of the maximum coincides with the data. It is obvious that influence of the two-pion intermediate state is underestimated for $\Lambda_M = 500$ MeV and overestimated for $\Lambda_M = 700$ MeV. Besides, the results for $\Lambda_M = 500$ Mev suffer from the fact that the constituent quark mass of 396 MeV which emerges from the fit in the pion sector is too small to keep the quark-antiquark threshold well above the peak in the form factor. This is improved for larger values of Λ_M, for $\Lambda_M = 600$ MeV we already obtain a constituent mass of 446 MeV and for $\Lambda_M = 700$ MeV we find a mass of 550 MeV. We can conclude that the overall agreement with the data for the form factor is good for $\Lambda_M = 600$ MeV. This is confirmed if we look at the phase shifts in the vector-isovector channel, which have been calculated from the dominant s-channel ρ-meson exchange. Our results for the same values of the cutoff as before are shown on the right hand side of Fig. 8.

In contrast to the $1/N_c$-expansion scheme, we did not succeed in performing a reasonable fit for the MLA. There we encountered instabilities in the ρ-meson propagator for values of the meson cutoff Λ_M which lead to a sufficiently large constituent quark mass [13].

IV. QUARK CONDENSATE AT NONZERO TEMPERATURE

It is commonly believed that chiral symmetry, which is spontaneously broken in vacuum, gets restored at high temperatures. At zero densities this is confirmed by lattice results, whereas in wide ranges of densities and temperatures this notion is based on model calculations for lack of fundamental knowledge. Only at low temperatures and low densities model independent results can be obtained by considering a gas of pions and nucleons. These are the dominant degrees of freedom in that range because pions are by far the lightest hadrons, while nucleons are the lightest particles carrying non-zero baryon number. This is in strong contrast to standard NJL-model calculations in Hartree(-Fock) approximation (see e.g. [7–10]), where the thermodynamics is entirely driven by unconfined quarks, i.e., by unphysical degrees of freedom. Hence, although the fundamental problem of lack of confinement cannot be overcome, we can hope to improve the results by introducing mesonic degrees of freedom within an approximation beyond Hartree + RPA, i.e. in the $1/N_c$-expansion scheme or the MLA. Since both schemes do not contain any nucleons, essential ingredients at nonzero densities, we will restrict our examinations to nonzero temperatures but zero density.

We first compare the low-temperature behavior of the quark condensate with model independent considerations from chiral perturbation theory. To lowest order in temperature the change of $\langle\bar\psi\psi\rangle$ with temperature in the chiral limit is given by [19]

$$\langle\bar\psi\psi\rangle_T = \langle\bar\psi\psi\rangle\Big(1 - \frac{T^2}{8f_\pi^2} + \dots\Big)\ . \tag{3}$$

Here $\langle\bar\psi\psi\rangle$ denotes the quark condensate at zero temperature. The term proportional to T^2 arises from a pure (massless) pion gas. In Hartree approximation this behavior is completely failed since excitations of the massive quarks are exponentially supressed. In the $1/N_c$-expansion scheme as well as in the MLA, however, the T^2-behavior can be reproduced. The only difference is that in these schemes the fluctuations consist of RPA pions, and therefore the coefficient corresponds to the RPA quantities instead of the full ones. The good agreement of our results with a free pion gas behavior at low temperatures can also be seen in Fig. 9, where the quark condensate is shown as a function of temperature. For comparison we also show the behavior in Hartree approximation.

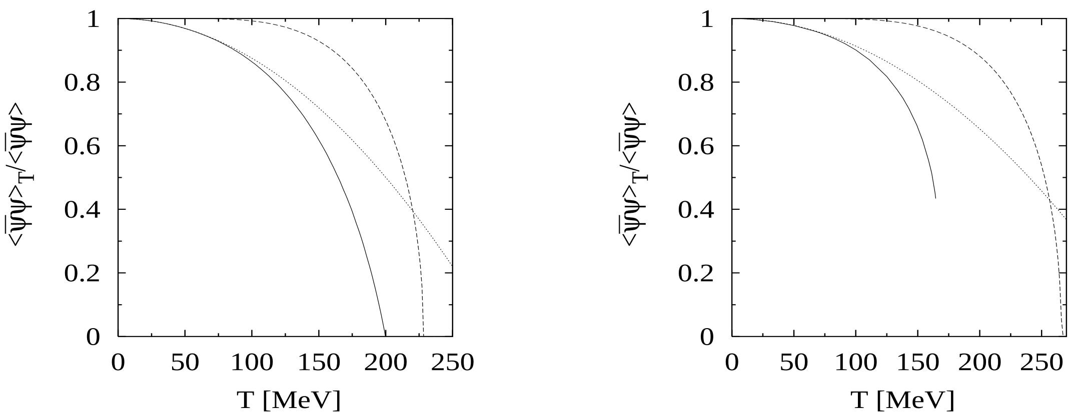

FIG. 9. *Quark condensate as a function of temperature, normalized to the vacuum value in the chiral limit. Left: $1/N_c$-expansion scheme (solid), pion gas (dotted) and Hartree approximation (dashed), right: MLA (solid), pion gas (dotted) and Hartree approximation (dashed).*

In both schemes deviations from the pure pion gas behavior become visible at $T \gtrsim 100$ MeV. These deviations arise from quark effects which could be tolerated only close to the phase transition. In contrast to the $1/N_c$-expansion scheme an examination of the phase transition is possible in the MLA. The present parameter set leads to a critical temperature of $T_c = 164.5$ MeV. Hence quark effects cease to be negligible at a temperature of about $.6T_c$, where one cannot avoid admitting that they are completely unphysical.

In accordance with the findings in Ref. [20] the phase transition is of first order whereas it is of second order in Hartree approximation. This is already indicated by the jump in the order parameter at $T = T_c$. Besides the unphysical quark degrees of freedom, this is probably another shortcoming of the approximation, because universality arguments suggest that the finite-temperature chiral phase transition in QCD with two massless quarks is of second order [21,22]. This conjecture is based on the assumption that the critical behavior is completely determined by the four (at T_c massless) bosonic degrees of freedom and that QCD therefore lies in the same universality class as the $O(4)$ model which is known to exhibit a second order phase transition. The same arguments can be applied to the NJL model, which has the same underlying symmetry as QCD with two massless flavors. However, one of the objections one might raise against the above hypothesis is that a theory with composite boson fields not necessarily belongs to the same universality class as the $O(4)$ model [23]. This objection is among others valid for the NJL model.

V. SUMMARY

We have investigated quark and meson properties within the Nambu–Jona-Lasinio model, including meson-loop corrections. These have been generated in two different ways. The first method is a systematic expansion of the self-energies in powers of $1/N_c$ up to next-to-leading order [12,24,14,13]. In the second scheme, a local correction term to the standard Hartree self-energy is self-consistently included in the gap equation [12,11]. Both schemes, the $1/N_c$-expansion scheme and the MLA, are consistent with chiral symmetry, leading to massless pions in the chiral limit.

The relative importance of the mesonic fluctuations is controlled by a parameter Λ_M, which cuts off the three-momenta of the meson loops. The value of Λ_M, has to be determined, together with the other parameters, by fitting physical observables. The ρ-meson and related quantities are very well suited for this purpose, since the meson loops are absolutely crucial in order to include the dominant $\rho \to \pi\pi$-decay channel, while the Hartree+RPA approximation contains only unphysical $q\bar{q}$-decay channels. Of course, a priori it is not clear to what extent these unphysical decay modes, which are an unavoidable consequence of the missing confinement mechanism in the NJL model, are still present in the region of the ρ-meson peak.

For the $1/N_c$-expansion scheme we obtain a reasonable fit of f_π, $\langle\bar{\psi}\psi\rangle$ and the pion electromagnetic form factor with a constituent quark mass of $m = 446$ MeV. This means, the unphysical $q\bar{q}$-decay channel opens at 892 MeV, about 120 MeV above the maximum of the ρ-meson peak. Unfortunately we did not succeed to obtain a similar fit within the MLA [13]. Since in this scheme the meson-loop effects lower the constituent quark mass as compared to the Hartree mass, it is much more difficult to evade the problem of unphysical $q\bar{q}$-decay channels in the vicinity of the ρ-meson peak.

In the last part of this article we have investigated the temperature dependence of the quark condensate. In both schemes the low-temperature behavior is consistent with lowest-order chiral perturbation theory, i.e., the temperature dependence arising from a free pion gas. This is a considerable improvement over the mean-field result, where the temperature dependence is entirely due to thermally excited quarks, i.e., unphysical degrees of freedom.

At higher temperatures, however, thermal quark effects also become visible in the two extended schemes. We argued that this could be tolerable only near the chiral phase boundary. Whereas the perturbative treatment of the mesonic fluctuations within the $1/N_c$-expansion scheme does not allow an examination of the chiral phase transition, this is possible in the MLA. For our model parameter set we found a critical temperature of 164.5 MeV. On the other hand, quark effects are visible already at a temperature of $\sim$ 100 MeV. Obviously this is still too early to be realistic.

In agreement with Ref. [20] we found a first-order phase transition in that scheme. This contradicts the general belief that the non-zero temperature chiral phase transition in a model with two light flavors should be of second order and is probably an artifact of the approximation.

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