Quarks and gluons : tests of QCD in e+e- annihilations

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abstract

This article gives an overview of the main experimental tests of perturbative QCD performed at LEP. It covers the following topics: determination of α_s from event shapes, tests of flavour independence of α_s, studies of heavy quark mass effects, differences between quark and gluon jet fragmentation and study of the triple gluon vertex.

keywords: QCD, e^+e^-, LEP, jets

résumé

Cet article offre une revue des résultats importants obtenus au LEP pour tester la validité de la théorie de la chromodynamique quantique (QCD). Les sujets abordés concernent la mesure de α_s à partir de la topologie des événements hadroniques, les tests précis de l'indépendence de saveur de α_s, des études QCD dans le secteur des quarks lourds, l'étude des différences de fragmentation des jets de quarks et de gluons et l'étude du vertex à trois gluons.

mots clés: QCD, e^+e^-, LEP , jets

1 Introduction

Quantum Chromodynamics (QCD) [1] is the gauge theory proposed for the strong interaction. It describes the interactions between the quarks (spin 1/2 charged particles), and the vector gauge bosons mediating the strong interactions, the gluons. Quarks and gluons carry a quantum number, called colour, which allows the existence of a coupling between gluons. This gluon self interaction leads to a fundamental property of QCD, called 'asymptotic freedom', predicting the decrease of the strong coupling constant, α_s, with energy scale. In the last 25 years the study of hadronic events produced in e^+e^- annihilation has made a major contribution to establishing QCD as the theory of the strong interaction. This is largely due to the fact that e^+e^- interactions offer a very clean environment to study basic QCD processes. The latter occur only in the final state; there is no contamination from beam remnants, and, apart from initial and final state electromagnetic radiation, the hadronic center of mass energy is well defined. The observed hadronic event structure is directly related to the gluon radiation pattern produced in the parton (quark and gluon) QCD processes. The LEP experiments have been very active since 1989 in performing quantitative tests of QCD. Due to its large hadron branching ratio, negligible background from other processes, and a strong suppression of initial state radiation, the Z resonance has offered unique conditions for detailed QCD studies. In addition, the precise microvertex detectors of the tracking systems of the LEP experiments have allowed flavour-dependent QCD studies to be performed with the high statistics ($\simeq$ 4M hadronic events per experiment) Z-peak data. The higher centre-of-mass energies obtained during LEP2 running have allowed studies of the energy-scale dependence of QCD predictions.

The QCD results which are reviewed in this chapter include: precise determinations of α_s from event shapes, studies of the triple gluon vertex and a test of the non-abelian gauge structure of QCD, gluon splitting into heavy quark pairs, flavour independence of α_s, running m_b and differences in quark and gluon jet fragmentation.

Other QCD results obtained at LEP, which are not covered here, concern low energy phenomena like studies of hadronisation mechanisms and particle production and tests of soft gluon coherence and particle correlations.

2 Jets in e^+-e^- annihilation

The process of hadron production from a quark-antiquark pair in e^+e^- annihilations may be viewed as composed of two different stages in time, governed by the strong interaction, and referred to as perturbative and non-perturbative phases. In the first, the quarks of the quark-antiquark pair radiate gluons (parton cascade) while they are moving apart. Most of this radiation is at low energy and collinear to the parent quark, thus conserving information about the initial quark direction. In some cases it may happen that one of the radiated gluons is sufficiently energetic that it can detach itself from the remaining partons and start its own cascade. Such an event, illustrated in Figure 1, is called a three-jet event. Since the probability of emission of a hard gluon is proportional to α_s, this phenomenon can be used to measure the latter. In the second stage the emitted partons combine, when at an energy scale of 1-2 GeV, to

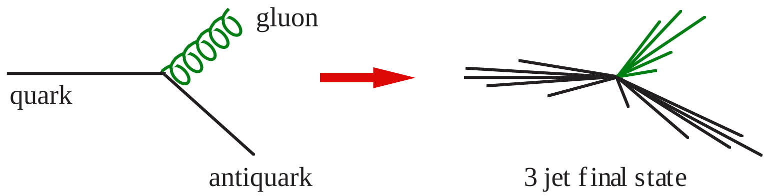

Figure 1: 3 jet structure of a $q\bar{q}g$ event

form the observed hadronic particles. This non-perturbative stage is described by heuristic models such as string or cluster hadronisation. The observable end-result of the two stages is the appearance of collimated beams of particles called "jets", which still carry information about the initial hard partons. From the preceding it is clear that we can also have four-jet events, five-jet events, and so on, each time with an overall probability diminished by a factor α_s.

In practice, jets are constructed using so-called "jet-finding algorithms" which allow each observed particle of an event to be assigned to a particular jet. An important feature of such algorithms is a resolution parameter, generally called y_{cut}. This parameter defines a "distance" between two particles, often based on their relative transverse momentum p_t. If two particles have a "distance" which is smaller than a chosen value of the y_{cut} parameter, they are considered to belong to the same jet. This results in the number of jets being a function of the chosen y_{cut}, the smaller it is the more jets one finds in an event. Figure 2a shows the production rate of 2,3,4 and 5 jet events as a function of y_{cut} measured by ALEPH [2] with the Durham algorithm and compared to standard QCD Monte Carlo models. These

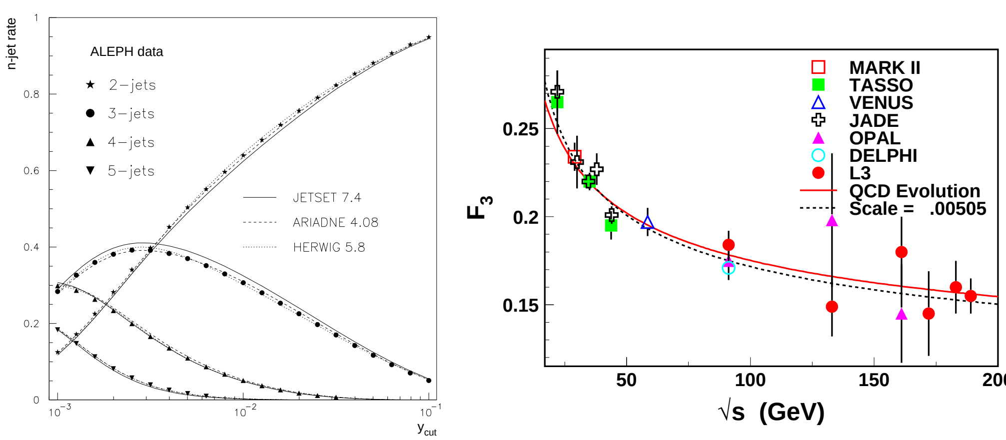

Figure 2: a) production rate of multijet events as a function of y_{cut} with the Durham algorithm. b) 3-jet rate measured with the Jade algorithm (y_{cut}=0.08) as a function of e^+e^- centre-of-mass energy

models incorporate the basic QCD principles and have been found to agree quite well with experiment (see Figure 2a). They can therefore be used, with confidence, to correct measured hadronic distributions for detector effects, and to estimate systematic errors due to poor knowledge of the non-perturbative hadronisation process. A basic QCD test is the measurement of the 3-jet rate which is, at leading order, proportional to α_s. Figure 2b shows measurements of this quantity for a fixed value of y_{cut} as a function of e^+e^- centre-of-mass energy obtained at different colliders. The results are in agreement with the running of α_s with energy scale as expected in QCD.

3 Determination of α_s from event shapes

The most important parameter of QCD is the strong coupling constant α_s. The theory does not predict a fixed value for α_s but only its energy scale behaviour. Event shape variables, built from linear sums of measured particle momenta, are sensitive to the amount of hard gluon radiation and offer one of the most direct ways to measure α_s in e^+e^- annihilation. They are insensitive to soft and collinear radiation ('infra-red safe') and so can be reliably calculated in perturbative QCD. At LEP the most commonly used variables for which fixed order perturbative predictions exist are the thrust, the heavy jet mass, the C-parameter and the 3-jet resolution parameter, y_{23}[1]. The differential distribution of any of these variables y, calculated to $\mathcal{O}(\alpha_s^2)$ at parton level, is given by [3]:

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{dy}} = \frac{\alpha_\mathrm{s}(\mu)}{2\pi}\mathrm{A(y)}+\left(\frac{\alpha_\mathrm{s}(\mu)}{2\pi}\right)^2\left[\mathrm{B(y)}+2\pi\beta_\circ\ln(\mu^2/\mathrm{s})\mathrm{A(y)}\right]$$

where μ is the renormalisation scale at which α_s is evaluated. At first order, these event shape variables are proportional to α_s, giving a strong sensitivity to α_s. The coefficients A(y) and B(y) have been calculated by integrating the second order matrix elements [4]. To compare the analytical calculations with the experimental distributions, the effects of hadronisation and decays are corrected using Monte Carlo models. However the main limitation coming from the fixed order calculation is the renormalisation scale and renormalisation scheme dependence due to the power series truncation. This μ dependence should disappear when the expression is computed to all orders. The usual recipe is to vary μ in a range (typically from 0.5 to 2.0 times) around $\sqrt{\mathrm{s}}=m_Z$ and to associate the resulting change in the value of α_s to the uncertainty due to missing higher order terms. The problem of scale definition has recently been differently addressed by DELPHI [5]. $\mathcal{O}(\alpha_s^2)$ calculations, including event orientation, have been fitted for 18 event shape variables measured with 1.4 M hadronic events. The renormalisation scale ambiguity has been treated by doing a combined fit of α_s and the μ scale for each variable. The resulting 18 values of α_s are shown in Figure 3. As can be seen, consistent values of α_s are obtained, although

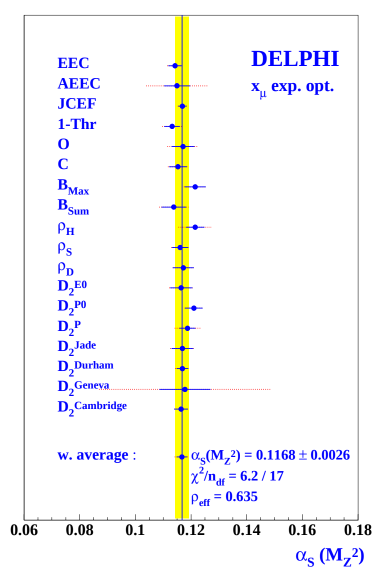

Figure 3: Results of the fits of the 18 event shape distributions to $\mathcal{O}(\alpha_s^2)$ calculations together with their weighted average.

[1]This is the value of y_{cut} a given event changes from a 2-jet to a 3-jet topology.

μ varies from $0.055\sqrt{s}$ to $2.66\sqrt{s}$ for different observables. The weighted average taking into account correlations between the observables yields the value: $\alpha_s(m_Z) = 0.1168 \pm 0.0026$ with a quite small error. A concensus is still awaited on the validity of such a procedure. This method provides the best measurement of $\alpha_s(m_Z)$ by DELPHI.

The use of resummed calculations, available since 1992, in which leading and next-to-leading logarithmic terms are summed to all orders in α_s, provides a partial solution to the μ scale ambiguity. These calculations are performed at parton level for a few observables [6]. When matched properly to fixed order calculations, they give quite reliable QCD predictions (O(α_s^2)+NLLA) over a wider range of values of the event shape variables. An example is given in Figure 4a where the differential 2-jet rate distribution measured by OPAL is shown as a function of y_{cut}. The fixed order predictions (dotted and dash-dotted lines) cannot fit the low value region, corresponding to 2-jet topologies, while the resummed calculation, represented by the solid line, agrees much better with the measurements. The possibility to extend the fit

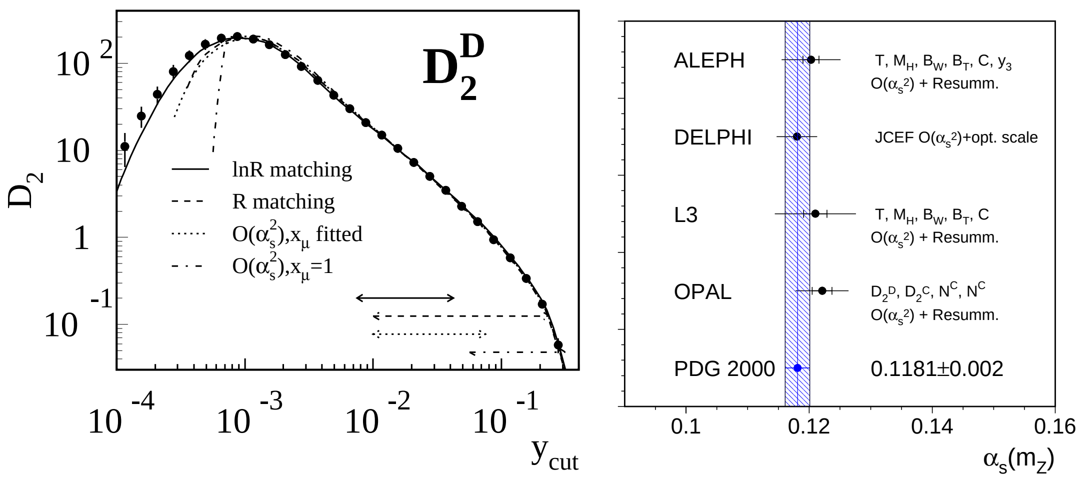

Figure 4: a) OPAL measurement of the differential 2-jet rate at m_Z compared to various QCD predictions. b) Summary of recent measurements of $\alpha_s(m_Z)$ in comparison with the world average (vertical cross-hatched band).

range is an advantage when the data samples are small as at LEP2 energies. The resummed calculations are found to give consistent results, for different observables, with μ close to $\sqrt{s}$ instead of the wide range of different values preferred by the O(α_s^2) analyses.

The most recent ALEPH [2], L3 [7] and OPAL [8] measurements of α_s at m_Z use these resummed calculations. Results are obtained from combined fits to different variables. The different experiments use different variables, different fit ranges and different averaging methods for the variables to obtain their values. Despite this, the values found are very consistent with each other and agree with the α_s world average [9] (See Figure 4b).

4 Running of α_s

Measurements of the strong coupling constant at energies above the Z pole, ranging from $\sqrt{s} = 130$ GeV to 208 GeV, have been performed by ALEPH [10], DELPHI [11], L3 [7] and OPAL [8, 12]. However the event statistics above m_Z is relatively small (a few hundred events to a few thousand events) and large backgrounds from initial state radiation and hadronic W-pair decays have to be considered. Each experiment has derived α_s at the different energies using the same experimental technique and the same calculation, allowing them to study the energy dependence of α_s, which is subject only to energy-uncorrelated errors. All results are compatible with the energy evolution predicted by QCD. L3 has exploited a wider centre-of-mass energy range by the use of radiative Z pole events to determine six α_s values between 30 GeV and 88 GeV [13]. These measurements are shown in Figure 5 together with the higher energy values obtained up to 208 GeV. The errors shown are experimental only . A fit to the QCD evolution equation gives [7] α_s $(m_Z) = 0.1216 \pm 0.0017$ (exp) $\pm$ 0.0058 (th). The measurements from the

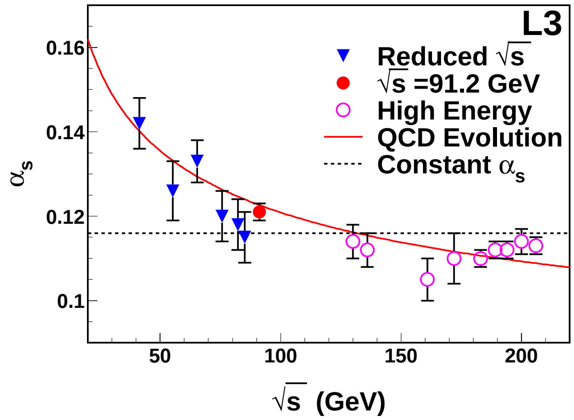

Figure 5: Measured values of α_s by L3 from event shape distributions as a function of the CM energy.

four LEP experiments have been combined by the LEP QCD working group [14]. A fit to the combined samples at the different energies results in α_s $(m_Z) = 0.1195 \pm 0.0004$ (exp) $\pm$ 0.0036 (th) where the error is mainly due to the theoretical uncertainty associated to the choice of renormalisation scale. This value is found to be in very good agreement with that (determined essentially by the hadronic width of the Z, and with very small theoretical uncertainty) obtained in the most recent fit, to all electroweak observables, by the LEP EW working group [15]: α_s $(m_Z) = 0.1183 \pm 0.0027$. It also agrees with the independent determination from τ decays, α_s $(m_Z) = 0.1202 \pm 0.0027$, as discussed in the contribution by Davier and Höcker [16].

5 Flavour independence of α_s and heavy quark mass effects

An important prediction of QCD is that the strong coupling constant at the $q\overline{q}g$ vertex is independent of the quark mass. This property has been tested by performing α_s measurements, using quark-flavour tagged samples of hadronic Z-decays, both at LEP and SLD. For such tests it is important to take into account phase space suppression due to the heavy quark mass. As shown by OPAL [17], performing an α_s measurement using global event shape variables, on a b-tagged sample, but assuming massless quarks in the theory, results in a value of α_s about 7% lower than for a light quark sample (see Figure 8b of Ref. [17]) in agreement with a simple theoretical estimate [18] of the phase space suppression factor. Because the experimental precision on α_s obtainable at LEP is $\simeq 1\%$ it is also possible, on the assumption of flavour independence, to measure the b-quark mass. This is not possible for the c-quark since the phase space suppression is only 0.7% with $M_c = 1.35$ GeV.

The early LEP measurements of the flavour independence of α_s, based on relatively small data samples used lepton (e, μ) and D* tagging of heavy quarks, while s-quarks and light (u,d) quarks were tagged using, respectively, K_S mesons and leading pions. Heavy quark phase space effects were taken into account using either a parton shower generator [19] or a Monte Carlo generator [20] based on NLO tree-level matrix elements for heavy quark production. These measurememts all agreed with flavour independence, obtaining a precision of $\simeq 4-5\%$ on the ratio[2]: $\alpha_s^b/\alpha_s^{udsc}$.

The second generation of experiments [21, 22, 17, 23] used secondary vertex tagging, made possible by silicon microstrip detectors, much improved statistics (in the case of the LEP detectors) and in some cases [17, 23] complete NLO heavy quark calculations [24]. The most accurate results obtained by each experiment are presented in Table 1. OPAL [17] and SLD [23] also tested flavour independence for c-quarks, but the statistical precision ($\simeq$ 4%) is worse in this case. It can be seen in Table 1 that the estimates of both theoretical and systematic errors vary widely from experiment to experiment. However all the mesurements agree with flavour independence within the quoted statistical errors, indicating that some experimental systematic and theoretical errors may be somewhat over-estimated.

[2] Here and in the following, $\alpha_s^i \equiv \alpha_s^i(m_Z)$ where $\alpha_s^i(m_Z)$ corresponds to a pure sample of quarks of flavour i. The label l refers to light (u,d,s) quarks. If multiple flavours are specified, it is understood that their ratios correspond to those in hadronic Z-decays

Expt.	Ref.	Observables	Theory	Results
ALEPH	[22]	T,C,D_2	LO(α_s^2)	$\frac{\alpha_s^b}{\alpha_s^{udsc}} = 1.002 \pm 0.009 \pm 0.005 \pm 0.021$
DELPHI	[25]	R_3^{bl}	NLO(α_s^2)	$\frac{\alpha_s^b}{\alpha_s^{uds}} = 1.007 \pm 0.005 \pm 0.007 \pm 0.005$
L3	[19]	R_3^{bl}	JETSET7.2	$\frac{\alpha_s^b}{\alpha_s^{udsc}} = 1.00 \pm 0.05 \pm 0.06$
OPAL	[17]	1-T,$M_H/\sqrt{s}$,B_W,y_{23},C	NLO(α_s^2)	$\frac{\alpha_s^b}{\alpha_s^{uds}} = 0.993 \pm 0.008 \pm 0.006 \pm 0.011$
SLD	[23]	R_3^{bl}	NLO(α_s^2)	$\frac{\alpha_s^b}{\alpha_s^{uds}} = 1.004 \pm 0.018^{+0.026}_{-0.031}{}^{+0.018}_{-0.029}$

Table 1: Tests of quark flavour independence of α_s at LEP and SLD. The observables are: Thrust(T), C-parameter(C), Differential 2-jet rate(D_2), Ratio of 3-jet fractions for b and light quarks (R_3^{bl}), Scaled Heavy Jet Mass ($M_H/\sqrt{s}$), Wide jet broadening (B_W) and the y_{cut} value for $2 \to 3$ jet transition (y_{23}). The errors are, in order, statistical, systematical and theoretical, except for DELPHI, where the second error corresponds to hadronisation. For L3, the systematical and theoretical errors are combined.

Measurements of the b-quark mass, assuming flavour independence of α_s, have been performed by DELPHI [25],ALEPH [26], OPAL [27] and by Brandenburg *et al.* [28] using SLD data. All of these experiments are interpreted using the complete O(α_s^2) NLO calculation for heavy quark production. Although these calculations actually use, and are completely specified by, the pole mass of the heavy quark, it is possible, using the freedom of choice of the renormalisation scheme in the NLO calculation, to replace the pole mass by the running $\overline{\rm MS}$ mass at the scale of the Z mass: $\overline{m}_b(m_Z)$. For this the one-loop QCD evolution equation: $M_b^2 = \overline{m}_b(\mu)^2\{1 + (2\alpha_s(\mu)/\pi)[4/3 - \ln(\overline{m}_b(\mu)^2/\mu^2)]\}$ is used, where the same renormalisation scale, μ, is assigned to α_s and the running quark mass. Although the parameterisations of observables in terms of M_b or $\overline{m}_b(m_Z)$ give identical results at NLO, it has become customary, following DELPHI [25], to present results for the b-quark mass in terms of the value of $\overline{m}_b(m_Z)$ (Table 2). The

Expt.	Ref.	Observables	$\overline{m}_b(m_Z)$ [GeV]
ALEPH	[26]	$< y_{23} >$ (D)	$3.27 \pm 0.22 \pm 0.22 \pm 0.38 \pm 0.16$
DELPHI	[25]	R_3^{bl} (D)	$2.67 \pm 0.25 \pm 0.34 \pm 0.27$
OPAL	[27]	$R_3^{b(dusc)}$ (J,E,E0,P,P0,D,G)	$2.67 \pm 0.03^{+0.29}_{-0.37} \pm 0.19$
SLD	[28]	$R_3^{b(dusc)}$ (E,E0,P,P0,D,G)	$2.56 \pm 0.27^{+0.28}_{-0.38}{}^{+0.49}_{-1.48}$

Table 2: Measurements of $\overline{m}_b(m_Z)$ at LEP and SLD. $< y_{23} >$ is the first moment of y_{23}. D and G denote the DURHAM and GENEVA jet finding algorithms, repectively. J,E,E0,P,P0 refer the different jet recombination schemes (see Ref. [27]) of the JADE algorithm. For ALEPH, the errors are, in order, statistical, systematical, hadronisation and theoretical. For the other experiments, the order is the same, but no hadronisation error is quoted, except for DELPHI, where the second error corresponds to this source.

typical sensitivity of the measurement of the b-quark mass is illustrated, by the ALEPH measurement using R_3^{bl}, in Figure 6. All the measurements in Table 2, when evolved down, using QCD, to the pole mass scale, are consistent with the world average value: $M_b = 4.2 \pm 0.2$ GeV [9]. ALEPH made a direct measurement of the pole mass using the observable $< y_{23} >$, obtaining the result: $M_b = 4.73 \pm 0.29({\rm stat}) \pm 0.29({\rm syst}) \pm 0.49({\rm had}) \pm 0.18({\rm th})$ GeV, also in good agreement with the world average just quoted.

6 Test of the non-abelian Gauge Structure from 4-Jet Events

In second-order QCD perturbation theory four-jet events arise from the production of four hard partons and their subsequent fragmentation. The cross-section for four-parton production by electron-positron annihilation then contains contributions from the three processes described in Figure 7. The constants C_A,T_R and C_F are called the QCD colour factors or Casimir operators of the corresponding non-abelian

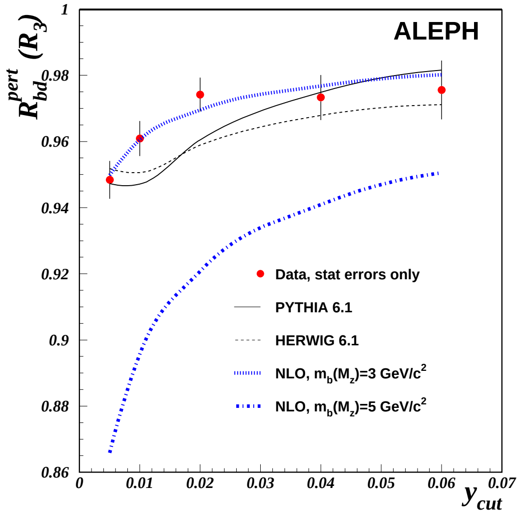

Figure 6: Results from ALEPH, from the 3-jet double ratio, on the running b-quark mass.

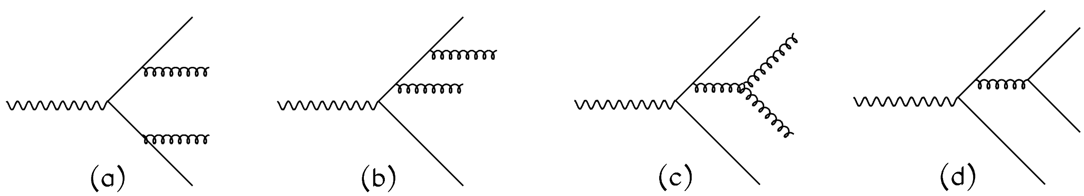

Figure 7: Elementary partonic processes in four-jet physics:
1.) Radiation of two gluons (a and b), proportional to C_F (4/3 in QCD).
2.) Radiation of one gluon splitting into two gluons (c), proportional to C_A, equal to the number of colours (3 in QCD). In an abelian QCD model this process (Triple Gluon Vertex) would be absent.
3.) Radiation of one gluon splitting into two quarks (d), proportional to $T_R = N_F\ T_F$ (=$N_F/2$ in QCD, with N_F =5 at LEP energies).

group SU(3), and the full second order expression of the four-jet cross-section is given by:

$$\frac{\mathrm{d}\sigma(x)_{\mathrm{four-parton}}}{\mathrm{d}x} = K[C_F A(x) + (C_F - \frac{C_A}{2})B(x) + C_A C(x) + T_R D(x) + (C_F - \frac{C_A}{2})E(x)]$$

which contains also interference terms between the three above-mentioned processes. The kinematical distributions A(x), B(x), C(x), D(x) and E(x) do not depend on any QCD constants, “x” can stand for any variable or angular correlation defining the relative positions of the four jets, and “K” is a constant overall factor. These functions can be obtained by integrating differential second order matrix elements [4]. They differ from each other due to the polarisation of the radiated intermediate (virtual) gluons, which can split either into two gluons (Figure 7c) or two quarks (Figure 7d). Conservation of angular momentum then requires the angular dependence of the two quarks to be different from that of the two gluons. The measurement of the colour factors consists of adjusting the relative proportions of the calculated distributions A, B, C, D and E to optimize the agreement between data and theory. Restriction to the colour factor ratios C_A/C_F and T_R/C_F only allows to avoid measurements of absolute cross-sections. Early LEP measurements all found colour factors values consistent with QCD.

More recently there have been two refinements:
1.) The ALEPH experiment [29] has combined the measurement of the colour factors, via angular correlations as described above, with the measurement of the event rate D_2 as a function of the three-jet resolution parameter, y_{23}. This introduces into the measurement the strong coupling constant α_s, which also has some sensitivity to T_R because its value depends on the number of flavours N_F.
2.) The OPAL experiment [30] has calculated the relevant quantities in third order, and include also three- and four-jet rates. To this order the angular correlations are no longer independent of α_s. The results of

this latest measurement are shown in Figure 8 together with the results of other LEP experiments; they all agree with QCD predictions.

Process No.3 of Figure 7 provides the possibility of detection of a light gluino, since its existence would increase N_F, from its QCD value of 5, to 8 [31]. However, since the number of four-quark events is only of the order of 6% of the total number of four-jet events, such an increase would be very difficult to detect by measuring the total cross-section only. The standard measurement of the colour factors uses their influence on four-jet angular correlations instead. Figure 8 shows a point representing the expected value of T_R/C_F for a light gluino, which puts the existence of a light gluino outside the 95% confidence limit of both the ALEPH and OPAL results.

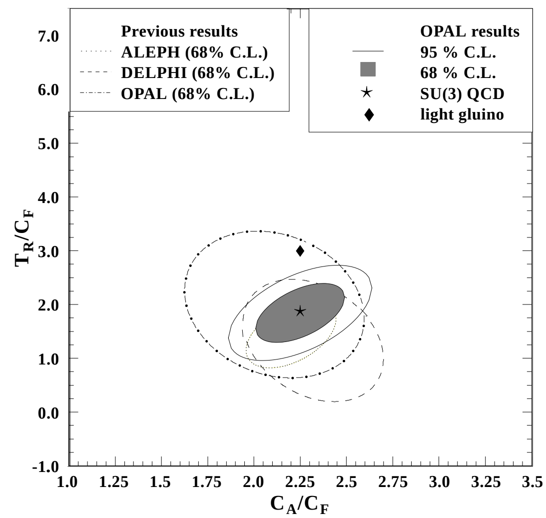

Figure 8: Two dimensional plot of the results of measurements of the colour factor ratios C_A/C_F and T_R/C_F of the LEP experiments. The black diamond represents the value of T_R/C_F expected for a light gluino, while the star shows the expectation for standard QCD.

7 Gluon splitting into heavy quarks

The process in which a gluon, radiated in the final state of hadronic Z-decay, splits into a heavy quark pair (Figure 9) has been extensively studied at LEP and SLD. It is of practical interest, since the uncertainty

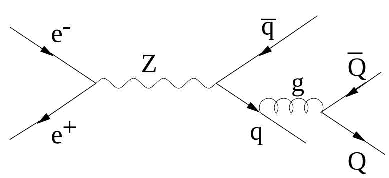

Figure 9: The process of gluon splitting into heavy quark ($Q = c, b$) pairs.

in the rate of such events is the largest source of systematic error for the measurement of the important electroweak parameter $R_b \equiv \Gamma_b(Z)/\Gamma_{had}(Z)$, and of theoretical interest, since the heavy quark mass scale should make possible reliable perturbative QCD (pQCD) predictions. The quantity $g_{Q\overline{Q}}$ ($Q = c, b$) is the fraction of all hadronic Z-decays in which the process of Figure 9 occurs. The quantity g_{4b} is similarly defined for events with gluon splitting to $b\overline{b}$ containing also a primary b quark pair.

The analyses are typically performed by selecting 3 or 4-jet events, and then flavour tagging the jets with the lowest energies and/or the smallest angular separation. The tagging methods used, and the results obtained for $g_{c\overline{c}}$ and $g_{b\overline{b}}$, are presented in Table 3.

Theoretical predictions at leading order [41] lie a factor of four below the measurements for c quarks, and a factor of two below for b quarks. The latest resummed calculation [42] is much more consistent with the data ($g_{c\bar{c}}$=2.01% and $g_{b\bar{b}}$=0.175%), but still seems somewhat low (by 2.5σ for c quarks). Heavy quark production in excess of pQCD predictions has also been recently observed in p$\overline{\text{p}}$, γp and $\gamma\gamma$ interactions. It has been observed that the ARIADNE event generator, based on the colour dipole formalism, describes the measured rates better than other QCD models like HERWIG or JETSET [43]. The quantity g_{4b} has also been measured by DELPHI [35] and OPAL [38] and found to be compatible with the predicted rate.

Expt.	g→ c$\bar{c}$ Analysis	$g_{c\bar{c}}$ (%)	g→ b$\bar{b}$ Analysis	$g_{b\bar{b}}$(%)
A [32, 33]	D* Tag	$3.23 \pm 0.48 \pm 0.53$	Vertex Tag, Evt Shape	$0.277 \pm 0.042 \pm 0.057$
D [34]			Vertex Tag, Evt Shape	$0.21 \pm 0.11 \pm 0.09$
D [35]			Vertex Jet Tag	$0.33 \pm 0.10 \pm 0.08$
L [36]	e, μ Tag, Evt Shape	$2.45 \pm 0.29 \pm 0.53$		
O [37, 38]	e, μ, D* Tag	$3.20 \pm 0.21 \pm 0.38$	Vertex Jet Tag	$0.307 \pm 0.053 \pm 0.097$
SLD [39]			Vertex-Mass Jet Tag	$0.244 \pm 0.059 \pm 0.034$
Average [40]		2.96 ± 0.38		0.254 ± 0.051

Table 3: Measurements of gluon splitting rates at LEP (A,D,L,O) and SLD. When two errors are quoted, the first is statistical, the second systematic. These are combined in quadrature, taking into account correlations, to obtain the uncertainties on the average values.

8 Fragmentation differences between quark and gluon jets

Differences between quark and gluon jets are expected due to their different colour charges (C_A=3 for gluon and C_F=4/3 for quark). As a consequence the gluon jets should have a larger multiplicity, should be broader and have a softer fragmentation function compared to quark jets. Naively, the average charged particle multiplicity for the gluon should be larger than for the quark jet by a factor equal to the ratio of the colour charges ($9/4 = 2.25$). From the early LEP analyses this ratio was measured to be between 1.10 and 1.25. Experimental progress, directed towards an understanding of this apparent discrepancy, has been made in the recent years. It is important to recall that the QCD predictions are based on quark-antiquark and gluon-gluon colour singlet systems. Experimentally the first one is well defined while the second is not. The first studies of the differences between quark and gluon jets were done by selecting symmetric 3-jet events but the results were dependent on the jet algorithm used and the selected topologies were such that the average gluon and quark jet energy was around 24 GeV. More refined studies have been performed since then improving our understanding of the observed differences. For example, OPAL has studied the properties of 3-jet events where a gluon recoils against 2 b quarks [44] in order to have an experimental gluon jet definition corresponding to the theoretical one. Figure 10 shows the rapidity distributions of the particles in quark and gluon jets measured by OPAL with respect to the sphericity axis [3]. A factor of nearly two is observed at small rapidities between the rates for gluon and quark jets. For $|y| < 1$ the multiplicity ratio is $r_{ch}(|y| < 1) = 1.919 \pm 0.047$(stat)$\pm 0.095$(syst) consistent with QCD calculation of the same quantity. Doing the measurement in a restricted rapidity interval has the advantage that energy-momentum is not locally conserved. The remaining difference with the QCD prediction of 2.25 comes from the finite energy available for the jet production process. The measurement of the ratio of the multiplicity between gluon and quark jet at 40 GeV over the full phase space gives: $r_{ch} = 1.514 \pm 0.019$(stat)± 0.034(syst) in agreement with analytical calculations including partial energy-momentum conservation [44]. DELPHI has investigated the energy scale dependence of this multiplicity ratio [45]. The ratio has been measured as a function of the jet "hardness" in 3-jet events. The "hardness" of a jet is defined as $\kappa = E_{jet} sin(\frac{\theta}{2})$ where E_{jet} is the jet energy and θ is the angle between the jet in question and closest neighbouring one. The average charged particle multiplicity for gluon and quark jets are shown in Figure 11a. A stronger increase of the multiplicity with scale is observed in gluon jets compared to quark jets. This is due to the stronger gluon radiation expected from the different colour charges. The ratio r_{ch} is shown in the Figure 11b as well as the ratio of the slopes, which is about 2.0,

[3] The rapidity, y, is defined as $y \equiv (1/2)\ln[(E + p_{\parallel})/(E - p_{\parallel})]$. It is approximately equal to $-\ln\tan(\theta/2)$, where the angle θ is that between the direction of the particle momentum and the jet axis. Regions of small θ correspond, therefore, to regions of large y.

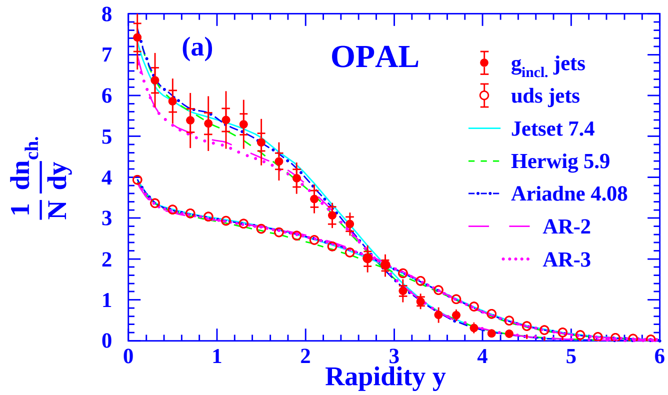

Figure 10: Charged particle rapidity distribution for 40 GeV gluon jets and 45 GeV quark jets.

corresponding to $C_A/C_F = 2.12 \pm 0.10$. This result can be interpreted as a direct evidence for the triple gluon coupling based on soft gluon radiation. It is therefore complementary to the measurement of the triple gluon coupling with 4-jet events.

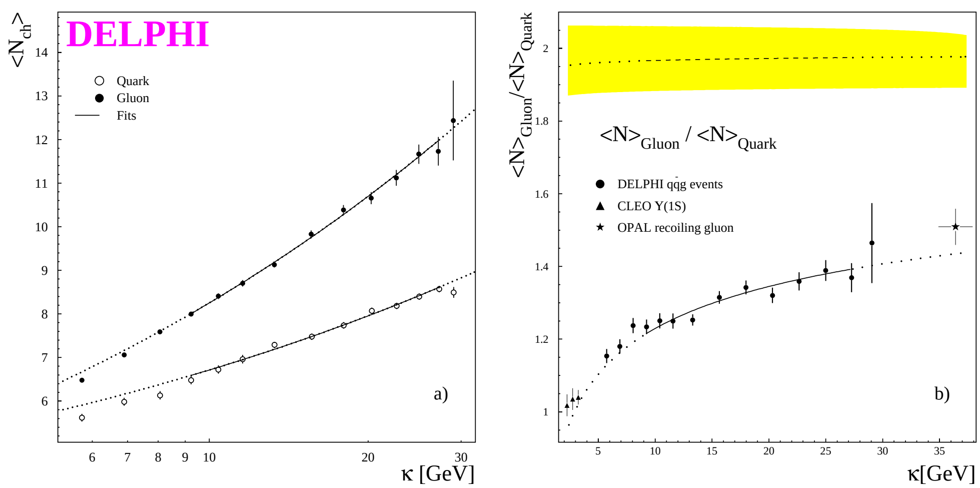

Figure 11: a) Average charged particle multiplicity for quark and gluon jets as a function of the κ scale. b) Ratio of gluon to quark multiplicity and ratio of the slopes (dashed line) as a function of scale.

9 Conclusion

The QCD studies in e^+e^- annihilation performed at LEP and the SLC include several precise tests of the theory of the strong interaction. The large amount of data collected at the Z pole has allowed very accurate determination of the strong coupling constant, α_s, as well as demonstration of the flavour independence of α_s at a few percent level. The uncertainty on the determination of α_s from event shape distributions is completely dominated by theoretical errors, but its value has been found to be very consistent with other LEP determinations using more inclusive variables, like the hadronic width of the Z and some properties of hadronic tau decays, for which more precise, $\mathcal{O}(\alpha_s^3)$, theoretical predictions exist. The high energies available at LEP2 have allowed determinations of α_s over a wide energy range, clearly confirming the expected energy scale variation predicted by QCD.

The experiments have developed efficient analysis techniques for detailed flavour dependent QCD studies. The processes of gluon splitting into $c\bar{c}$ and $b\bar{b}$ have been measured, thus reducing significantly the theoretical error of R_b measurements, and the b quark mass has been measured. The existence of the gluon self coupling, characteristic of the non abelian nature of QCD, has been demonstrated and the measured colour factors are found to be consistent with QCD. Detailed studies of quark and gluon jet fragmentation properties confirm the differences predicted by QCD.

All these results lead to our final conclusion that QCD accurately describes most aspects of jet dynamics and properties at high energies in e^+e^- annihilation.

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