

Non perturbative renormalization in covariant light front dynamics

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Non perturbative renormalization in covariant light front dynamics

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- State vector in a scalar model
- Covariant light-front dynamics
- Physical problem
- 2-particle problem
- 3-particle problem
- Generalization
- Including spin. . .

State vector in a scalar model

Two scalar fields interacting as $g\psi^2\varphi$:

→ “Nucleon” field ψ of mass m

→ “Pion” field φ of mass μ

A nucleon state vector obeys:

$$\hat{P}^2|p\rangle = M^2|p\rangle$$

in which the nucleon is **fully dressed** with pions.

Can $|p\rangle$ be computed ?

Covariant light-front dynamics

J. Carbonell, B. Desplanques, V.A. Karmanov, J.-F. Mathiot,
Phys. Rep, **300** (1998) 215

- Why covariant?

$$t^+ = t + z \quad \Rightarrow \quad \sigma = \omega_\mu \cdot x^\mu, \quad \omega^2 = 0$$

Dependance on orientation of LF plane
parametrized by 4-vector ω^μ

- Light-front vacuum:

$$P_0 > 0 \quad \Leftrightarrow \quad P_+, P_- > 0$$

\Rightarrow light-front vacuum is **trivial**

\Rightarrow diagrams are not spoiled by vacuum bubbles

\Rightarrow **Fock space** analysis

- Fock decomposition:

$$|p\rangle = \phi_1|N\rangle + \phi_2|N\pi\rangle + \phi_3|N\pi\pi\rangle + \dots$$

The ϕ_i are the **wave functions**.

- Particles:

⇒ σ -ordered diagrams

⇒ all particles are always **on-mass shell**

⇒ the triviality of the vacuum removes many diagrams

Physical problem

$$\hat{P}^2 |p\rangle = M^2 |p\rangle, \quad \hat{P}_\mu = \hat{P}_\mu^0 + \hat{P}_\mu^{\text{int}}$$

$$\hat{P}_\mu^0 = \sum_i \int d^3\vec{k} \, d_i^\dagger(\vec{k}) d_i(\vec{k})$$

$$\hat{P}_\mu^{\text{int}} = \omega_\mu \int \frac{d\tau}{2\pi} \mathcal{H}_{\text{int}}(\tau\omega)$$

With $[\omega \cdot \hat{P}_0, \mathcal{H}_{\text{int}}(\tau\omega)] = 0$ we get:

$$\left(\hat{P}_0^2 + 2(\omega \cdot p) \int \frac{d\tau}{2\pi} \mathcal{H}_{\text{int}}(\tau\omega) \right) |p\rangle = M^2 |p\rangle$$

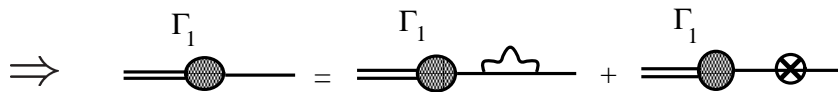
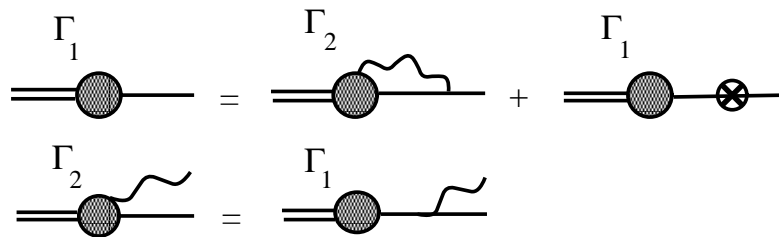
$$\frac{1}{2\pi} \int \frac{d\tau}{\tau} \mathcal{H}_{\text{int}}(\tau\omega) | \mathcal{G}(p) \rangle = - | \mathcal{G}(p) \rangle$$

Fock components of $| \mathcal{G}(p) \rangle$:

$$\Gamma_n = (s - m^2) \phi_n = \begin{array}{c} \xrightarrow{p} \text{---} \bullet \text{---} \xrightarrow{\Gamma_n} \text{wavy lines} \end{array}^{n-1}$$

2-particle problem

$$|p\rangle = \phi_1 |N\rangle + \phi_2 |N\pi\rangle$$



Trivially yields $\delta m^2 = -g^2 \Sigma(p^2 = m^2)$,
as identical to the perturbative case.

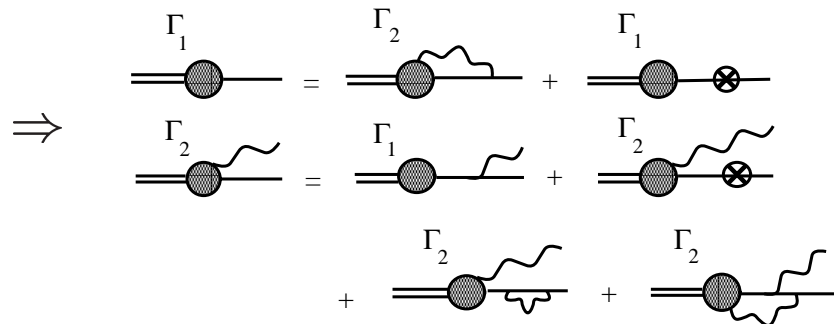
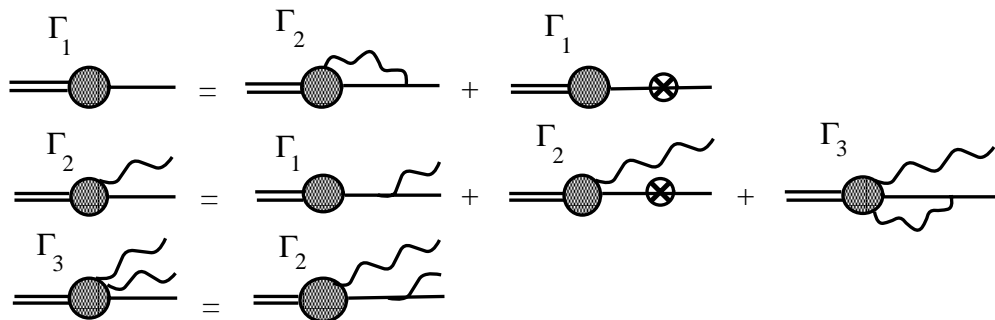
Iterating the relation:



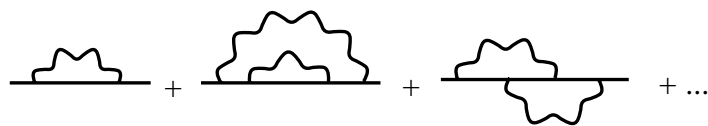
\Rightarrow only one irreducible diagram

3-particle problem

$$|p\rangle = \phi_1 |N\rangle + \phi_2 |N\pi\rangle + \phi_3 |N\pi\pi\rangle$$



Iterating the relation:



⇒ infinite number of irreducible diagrams

Equation giving Γ_1 :

$$\begin{aligned}
 & \frac{\delta m^2 + g^2 \Sigma(s_1)}{(s - m^2)x_1} \Gamma_1(\vec{q}, \vec{n}) \\
 & - \frac{g^2}{\delta m^2} \int \frac{d^3 \vec{q}'}{(2\pi)^3} \Sigma_i(\vec{q}', m^2) \Gamma_1(\vec{q}', \vec{n}) \\
 & + g^2 \int \frac{d^3 \vec{q}'}{(2\pi)^3} \Sigma_i(\vec{q}', m^2) \Pi(\vec{q}', \vec{q}, \vec{n}, m^2) \Gamma_1(\vec{q}', \vec{n}) \\
 & = \Gamma_1(\vec{q}, \vec{n}) \quad \equiv \lambda(\delta m^2) \Gamma_1(\vec{q}, \vec{n})
 \end{aligned}$$

- δm^2 is determined by condition $\lambda(\delta m^2) = 1$
- δm^2 differs from its perturbative value δm_0^2

Numerical solution

D. Bernard, T. Cousin, V.A. Karmanov, J.-F. Mathiot, Phys. Rev. D65, p. 025016 (2002)

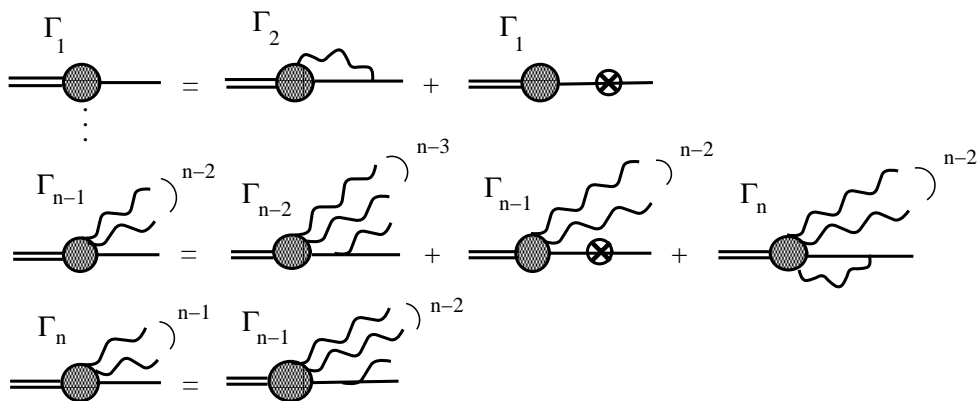
- $m = 0.94, \mu = 0.14$
- $\alpha = \frac{g^2}{16\pi m^2}$
- Regularization: $\|\vec{k}\| < L$

$L(\alpha = 3)$	5	10	50	100	200
$\frac{\delta m^2}{\delta m_0^2}$	1.052	1.040	1.025	1.022	1.020
$\langle q \rangle$	0.538	0.565	0.588	0.591	0.593

$\alpha(L = 200)$	1	10	100	1000
$\frac{\delta m^2}{\delta m_0^2}$	1.011	1.029	1.040	1.043

Generalization

Difficulty *in principle* does not grow with n .



$\Rightarrow n \rightarrow n - 1$ is always trivial

\Rightarrow iteration $i \rightarrow i - 1$ can be made systematically

\Rightarrow going from n to $n + 1$ requires to restart computation

Including spin...

Two spinor fields exchanging a scalar: $g\bar{\psi}\varphi\psi$

⇒ Same structure for the 3-body system with **3 additional contact terms**:

$$\begin{aligned}
 g - g & : \quad \text{---} \text{---} \times \text{---} \text{---} = g^2 \left(-\frac{\varphi}{2\omega \cdot p} \right) \\
 g - \delta m & : \quad \text{---} \text{---} \times \text{---} \otimes \text{---} = g \delta m \left(-\frac{\varphi}{2\omega \cdot p} \right) \\
 \delta m - \delta m & : \quad \text{---} \otimes \text{---} \times \text{---} \otimes \text{---} = \delta m^2 \left(-\frac{\varphi}{2\omega \cdot p} \right)
 \end{aligned}$$

⇒ Same difficulty *in principle* as scalar case, with **7 wavefunctions** instead of 3

Conclusion

- heavy use of **covariant** LF special features
- “satisfactory” for 2- and 3-body cases
- **iterative method** should allow for generalization to n -body case
- computational complexity stays under control, i.e. does not explode for high n
- extended to more physical models... ?