Fringe field minimisation of solenoids

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It is well known that the fringe fields of solenoids can disturb the cooling and operation of the cavities. One way to limit the fringe fields consists in putting at each extremity of the main coil a corrective coil, as suggested for example in P. Ostroumov [1]. This study presents a method to optimise the characteristics of these correction coils.

Let's consider a main coil of length « L », of radius « a », with a magnetic field on axis « B_z » and defined in such a way that the centre of the coil corresponds to $z = 0$.

According to E. Durand [2], the response along the axis z can be written :

$$(1) \qquad B_z(z) = B_z(0)\frac{\sqrt{\frac{L^2}{4}+a^2}}{L}\left(\frac{z+\frac{L}{2}}{\sqrt{(z+\frac{L}{2})^2+a^2}}-\frac{z-\frac{L}{2}}{\sqrt{(z-\frac{L}{2})^2+a^2}}\right)$$

We will develop this formula to the infinity, using *L/z*. For ε small we get :

$$(2) \qquad (1+\varepsilon)^{-1/2} = 1-\frac{1}{2}\varepsilon+\frac{3}{8}\varepsilon^2-\frac{5}{16}\varepsilon^3+\ldots$$

which gives :

$$(3) \qquad \frac{z+\frac{L}{2}}{\sqrt{(z+\frac{L}{2})^2+a^2}} = sign(z)\left(1+\frac{L}{2z}\right)\left(1+\frac{L}{z}+\frac{1}{4}\frac{L^2+4a^2}{z^2}\right)^{-1/2}$$

$$= sign(z)\left(1+\frac{L}{2z}\right)\left[1-\frac{L}{2z}-\frac{1}{8}\frac{L^2+4a^2}{z^2}+\frac{3}{8}\left(\frac{L^2}{z^2}+\frac{L(L^2+4a^2)}{2z^3}+\ldots\right)-\frac{5}{16}\frac{L^3}{z_3}+\ldots\right]$$

$$\approx sign(z)\left(1-\frac{a^2}{2z^2}+\frac{1}{2}\frac{a^2L}{z^3}\right)$$

In the same way :

$$(4)\qquad \frac{z-\frac{L}{2}}{\sqrt{(z-\frac{L}{2})^2+a^2}} \approx sign(z)\left(1-\frac{a^2}{2z^2}-\frac{1}{2}\frac{a^2L}{z^3}\right)$$

from which we deduce :

$$(5)\qquad B_z(z) \approx B_z(0)\, a^2\sqrt{\frac{L^2}{4}+a^2}\ \frac{sign(z)}{z^3}$$

This shows us that the fringe field vanishes in $1/z^3$.

The idea is now to choose the two correction coils in such a way that ***the global coefficient of 3rd order contribution is zero.***

We notice that we have in fact several degrees of freedom : the corrective coil length, its radius, its position and its relative magnetic field. This means that we could in fact develop the relation (1) to superior orders, namely 5 or even 7. We just consider here the 3rd order.

Let's consider a corrective coil of length « L_c », of radius « a_c », with a magnetic field on axis «B_z^c » and defined in such a way that the centre of the coil corresponds to $z = z_c$. If we don't want the main and corrective coils to overlap, we must satisfy :

$$(6)\qquad z_c \geq \frac{L+L_c}{2}$$

We can writte :

$$(7)\qquad B_z^c(z) = B_z^c(0)\frac{\sqrt{\frac{L_c^2}{4}+a_c^2}}{L_c}\left(\frac{z-z_c+\frac{L_c}{2}}{\sqrt{(z-z_c+\frac{L_c}{2})^2+a_c^2}}-\frac{z-z_c-\frac{L_c}{2}}{\sqrt{(z-z_c-\frac{L_c}{2})^2+a_c^2}}\right)$$

$$\approx B_z^c(0)\, a_c^2\sqrt{\frac{L_c^2}{4}+a_c^2}\ \frac{sign(z-z_c)}{(z-z_c)^3} \approx B_z^c(0)\, a_c^2\sqrt{\frac{L_c^2}{4}+a_c^2}\ \frac{sign(z)}{(z)^3}$$

The other coil development gives :

$$(8)\qquad B_z^c(z) \approx B_z^c(z_c)\, a_c^2\sqrt{\frac{L_c^2}{4}+a_c^2}\ \frac{sign(z-z_c)}{(z-z_c)^3} \approx B_z^c(z_c)\, a_c^2\sqrt{\frac{L_c^2}{4}+a_c^2}\ \frac{sign(z)}{(z)^3}$$

The mutual cancellisation of the 3 developments gives the equation :

$$(9)\qquad B_z(0)\, a^2\sqrt{\frac{L^2}{4}+a^2} = 2B_z^c(z_c)\, a_c^2\sqrt{\frac{L_c^2}{4}+a_c^2}$$

We introduce the notations :

$$(11) \quad \mu = \frac{L_c}{L} \quad ; \quad \lambda = \frac{B_z^c(z_c)}{B_z(0)} \quad ; \quad x = a_c$$

so that we obtain the polynomial equation of degree 6, satisfied by the radius a_c :

$$(12) \quad 4\lambda^2 x^6 + \lambda^2\mu^2 L^2 x^4 - \frac{L^2 a^4}{4} - a^6 = 0$$

The radius a_c can be obtained graphically or by using the iterative Newton method.

Example

We consider a solenoid with main coil length and radius of 20 cm and 3 cm and with a nominal field of 9 Tesla.
We chose correction coils with a length equal to 2 cm and a magnetic field 20% of the main one with no gap between the coils. So we have :

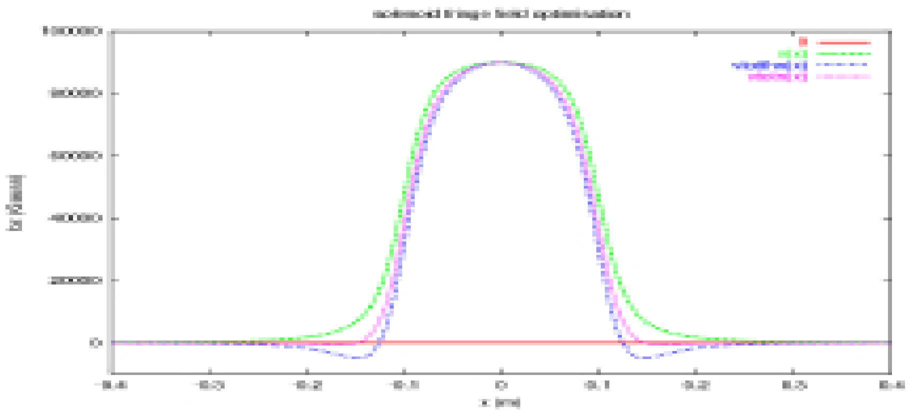
$$
(13) \quad
\begin{array}{llll}
L & = & 0.2 & m \\
a & = & 0.03 & m \\
B_z(0) & = & 9. & T \\
L_c & = & 0.02 & m \\
B_z^c & = & 0.18 & T \\
z_c & = & 0.11 & m \\
\mu & = & 0.1 & \\
\lambda & = & 0.20 &
\end{array}
$$

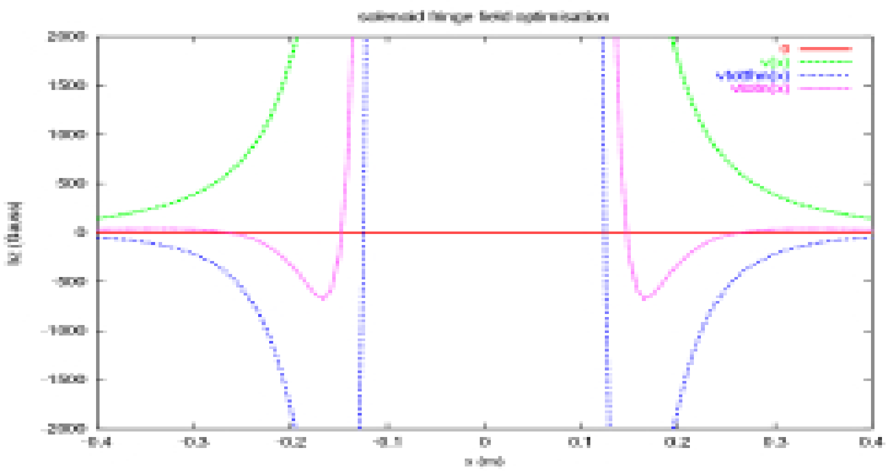
Using 12, we obtain :

$$(14) \quad a_c \approx 0.061435 \quad m$$

We remember that in the equation (9), the factor 2 in the right side is due to the contribution of both corrective coils for z very large. As shown in the table below, we can adjust this coefficient in order to avoid the magnetic undershoot appearing in the neighbourhood of the solenoid. Notice also that we have scaled all the field contributions in order to keep 9T in the middle of the solenoid, the corrective coils giving a slight negative field at the centre.

Z (m)	main coil field (Gauss)	with corrective coils coef = 2 (gauss)	With corrective coils coef = 1.2 (gauss)
0.00	90000.000	90000.000	90000.000
0.05	86355.233	82786.919	84255.142
0.10	46461.593	30258.764	36925.601
0.15	6360.522	-5114.224	-392.809
0.20	1748.219	-1848.146	-368.381
0.25	740.707	-585.864	-40.031
0.30	388.205	-222.253	28.926
0.35	230.599	-98.292	37.034
0.40	148.820	-48.789	32.519
0.45	101.905	-26.436	26.371
0.50	72.963	-15.336	20.995
1.00	8.616	-0.451	3.279
2.00	1.062	-0.013	0.428
9.00	0.011	-8.332e-06	0.004





in green : the main coil without correction coils
in blue : result after correction with coefficient equal 2.0
in pink : result after correction with coefficient equal 1.2

References

[1] P.N. Ostroumov, K.W. Shepard, S.H. Kim, E.S. Lessner, R. Laxdal, R. Wheatley, "A New Generation of Superconducting Solenoids for Heavy-Ion Linac Application", LINAC2003, Gyeongiu.

[2] E. Durand, Magnétostatique, Masson et Cie, p.98, 1968 .