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Laboratoire de Physique Nucléaire et de Hautes Energies

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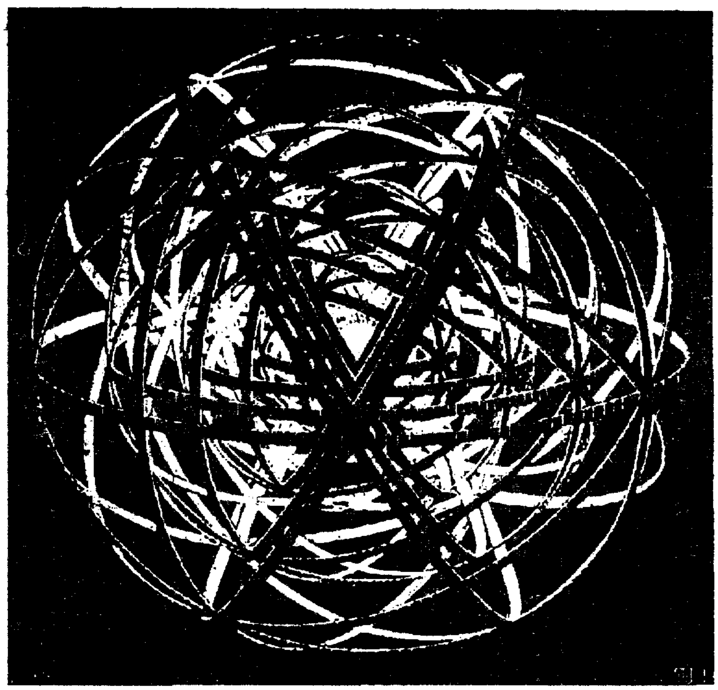

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A Search for Anomalous Contribution in $e^+e^- \longrightarrow \pi^0/\eta \quad \gamma$ Annihilations

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Abstract : Data on $e^+e^- \rightarrow \eta/\pi^0 \quad \gamma$ collected by the Neutral Detector at Novosibirsk in a c.m. energy range up to 1.04 GeV have been studied with a purpose to single out of the scattering amplitude the triangle anomaly contribution predicted by QCD, PCAC and Chiral Theories. We show that the cross section for $e^+e^- \rightarrow \pi^0\gamma$ exhibits no sensitivity to the π^0 triangle anomaly ; instead, the cross section for $e^+e^- \rightarrow \eta\gamma$ shows a clear sensitivity to the value of the η triangle anomaly and the best fit corresponds to a value for the η width in good correspondence with the world average value. It is also shown that the value of the $\rho \rightarrow \eta\gamma$ branching fraction is noticeably influenced by the existence of the anomaly. Finally, we give for the first time the absolute value for the hadronic phases of the vector mesons for the decay modes $\eta\gamma$ and $\pi^0\gamma$.

Free keywords : triangle anomaly, meson radiative decays.

1 Introduction

The triangle anomaly, expected to exist within QCD, has been discovered long ago[1, 2] from PCAC and explains rather well the $\gamma\gamma$ decay of the pseudoscalar mesons (π^0, η, η'). Together with higher order anomalies (like the box anomaly) it is now incorporated into the Wess–Zumino lagrangian[3, 4]. Several recent attempts have been done in order to incorporate within a single effective lagrangian a number of different phenomena such as the anomalous sector as defined by the Wess–Zumino lagrangian, the interactions of photons, pseudoscalar and vector mesons as defined for instance in refs. [5, 6] and the radiative decays of light flavor mesons (see e.g. ref.[7]). Other models following the same line have also been considered (see for instance refs.[8, 9, 11, 10, 12]). They mainly prove that anomalies and interactions of pseudoscalar and vector mesons can be considered as independent processes. In other words, there is no obvious double counting when superimposing contributions involving anomalies and standard VDM contributions.

It is indeed known for a long time that a consistent study of anomalies requires that one takes into account that pure anomalous contributions appear frequently mixed with resonance contributions. For instance, in order to extract properly the box anomaly contribution to $\gamma \to \pi^+\pi^-\pi^0$ from data[13, 14], one should take into account the ρ contribution at small transfers ; this was done following various procedures[15, 16]. In order to extract from experimental data on $\eta/\eta' \to \pi^+\pi^-\gamma$ the box anomalies for the η/η' mesons, the dominant ρ contribution should and can be properly accounted for as shown in ref.[17].

Recently it has also been shown [18] that the triangle anomaly contribution can be extracted from the $\eta/\eta' \to \mu^+\mu^-\gamma$ decays and the numerical values obtained in this way are in good agreement with the two-photon decay widths of the η and η' mesons. In the last study it was even shown that, taking for the triangle anomaly the values corresponding to $\eta/\eta' \to \gamma\gamma$ and for the η/η'–ρ–γ coupling constants the values extracted from the $\eta/\eta' \to \pi^+\pi^-\gamma$ decays (after removal of the box anomaly contribution), one can achieve a perfect description of the η/η' electromagnetic form factor[19] without any free parameter.

The success of these approaches tends to indicate that resonance and anomalous contributions can be considered as independent at the level of accuracy permitted by the existing data. Indeed, the anomalous contribution (measured in this way) is numerically consistent with what can be expected from the known values of f_π or the π^0 decay width[13, 14] or with the two–photon decay widths[18] of the η/η' mesons.

The anomalous contribution in decay or scattering processes appears as a broad phase space contribution. Narrow resonances like the ω or ϕ mesons cannot shadow it ; however broad structures like the ρ^0 meson should be well under control, otherwise a phase space contribution can be absorbed at an arbitrary level or arbitrarily enhanced. This is more important in processes involving η or η' with extended regions of invariant mass than for π^0 where only the zero tranfer limit of the ρ contribution has been used up to now. The results on the box and triangle anomalies obtained in refs. [17, 18] tend to show that the ρ parametrizations proposed there are acceptable. Indeed, the numerical values following from fits are in pretty good agreement with the measured two–photon decay widths and with predicted values of the box anomaly following from theory by means of the Chanowitz[20, 21] and AFN[22, 23] equations. As these measurements and predictions are intrinsically independent of the ρ^0 meson, they tend to prove that the M_1 and M_2 ρ^0–models proposed in ref.[17, 18] carry the correct information even if leaving open some ambiguity.

As the anomaly phenomenon is expected to contribute anywhere it is allowed to, this is a motivation to look for a possible triangle anomaly contribution in the annihilation processes $e^+e^- \to (\eta/\pi^0)\gamma$. If the corresponding data exhibit some sensitivity to the (possible) anomalous contribution, it becomes also interesting to compare the values of physical parameters extracted in this way with those already extracted from the same data without assuming the anomalous contribution[24, 25]. It is, for instance, the case for the single measurements of the $\rho^0 \to (\eta/\pi^0)\gamma$ branching fractions. Moreover, since both magnitude and phase of the triangle anomaly are known, one can get for the first time the *absolute* hadronic phase which affects each vector meson contribution in a final state state dependent way [26] rather than resonance phases relative to ρ^0 [24].

In section **2**, we briefly describe data samples. Section **3** outlines the model used to describe the $e^+e^- \to \eta/\pi^0\ \gamma$ cross sections and remind some formulae of relevance for the analysis of branching fractions. The interplay of triangle anomalies is formulated here. Sections **4** and **5** are devoted to a detailed analysis of the $e^+e^- \to \eta\ \gamma$ and $e^+e^- \to \pi^0\ \gamma$ cross sections and to the derivation of the branching fractions and hadronic phases associated with the ρ^0, ω, ϕ in radiative decays. In section **6**, we give a summary of our main results ; we also discuss the relevance of the $\eta'\gamma$ final state. Finally, section **7** is devoted to conclusions.

2 Data Samples

Although a lot of efforts were devoted to studies of the radiative decays of the ρ^0, ω, and ϕ mesons (see, [25] and a recent precise measurement of the decay $\phi \to \eta\gamma$ [27]), the consistent measurement of both $\pi^0\gamma$ and $\eta\gamma$ modes in a wide energy range has been performed with the Neutral Detector (ND) only [28, 29, 24]. ND has collected data at the Novosibirsk e^+e^- collider VEPP–2M in the c.m. energy range $0.5 \div 1.4$ GeV. The main part of the detector was an electromagnetic calorimeter with an energy resolution $\sigma/E = 4\%/\sqrt{(E[\text{GeV}])}$. Full details on the apparatus can be found in ref. [24]. Careful analysis of all data on radiative decays of the ρ^0, ω and ϕ mesons accumulated by ND has been performed in this work allowing to decrease significantly systematic errors.

The ND detector has collected a total integrated luminosity of 13 pb^{-1} corresponding to 14 millions of ρ^0, ω and ϕ decays. During the experiment, an energy range studied was repeatedly scanned with a minimum step of 0.5 MeV. The stability of the c.m. beam energy was 0.07 MeV. The absolute energy scale of the experiment has been determined using the world average mass values [25] for the ω and ϕ meson masses.

Most of the collected data are concentrated in the ω and ϕ peak regions while the number of observed events at the tails of the ρ meson is small. This is somehow unfortunate since the sensitivity to anomalies is the largest in precisely the regions outside the ω and ϕ peaks. Therefore, for the fine study of ρ and anomalies, we have chosen our binning in such a way that the number of events is significant in each bin. Corrections for non-linearity in the cross sections inside bins and energy instabilities have been taken into account in error calculations.

The ND data represent a systematic measurement of the cross section of $e^+e^- \to (\eta/\pi^0)\gamma$ from 540 MeV up to 1040 MeV. Events of the $\pi^0\gamma$ reaction were selected by the decay $\pi^0 \to \gamma\gamma$ while those of the reaction $e^+e^- \to \eta\gamma$ used the decays of η mesons to $\gamma\gamma$ or to 3 π^0. This corresponds to 9975 events for the $\pi^0\gamma$ and 3525 events for the $\eta\gamma$ final state.

The background to 3–γ final states is mainly due to 3–γ annihilations and has been subtracted, backgrounds to 6 and 7 γ have been found negligible [24]. Radiative corrections have been taken into account using the results of ref. [30]. Systematic errors due to uncertainties in normalization and simulation of detection efficiencies are estimated to be 4%. No systematic energy depending effects have been found.

3 The Model and its Consequences

The amplitude for any annihilation process $e^+e^- \to X\gamma$, where X stands for a pseudoscalar meson (π^0, η, η'), is given by diagrams of Fig.1. In (a), the contribution due to the anomaly is shown, whereas in (b) we sketch the more standard VDM contribution expected from ρ^0, ω, and ϕ mesons. The corresponding cross section writes :

$$\sigma(m^2) = \frac{\alpha}{24m^6} \; (m^2 - m_X^2)^3 \; \left| B_X + ie \sum_{V=\rho,\omega,\phi} e^{i\Phi_V} g_{V\gamma} \frac{F_V(X)}{D_V(m^2)} \right|^2 \quad . \tag{1}$$

In this expression α ($= e^2/4\pi$) is the fine structure constant, m_X is the mass of the outgoing pseudoscalar meson X and m is the e^+e^- invariant mass. B_X is the triangle anomaly, which will be approximated by its value at the chiral limit. Its expected value is thus related to the two–photon decay of the meson X (π^0, η, η') :

$$\Gamma(X \to \gamma\gamma) = \frac{m_X^3}{64\pi} |B_X|^2 \quad , \quad X = \pi^0, \; \eta, \; \eta' \quad ; \tag{2}$$

we shall not go beyond the constant approximation for B_X despite recent work[31], as the available data do not exhibit any sensitivity to such a refinement[1]. The constants B_X are real[1, 4, 20] ; moreover, with the convention we use, they carry a minus sign.

The sum in rel.(1) is the standard VDM contribution, the single one usually taken into account [24]. $g_{V\gamma}$ describes the transition from photon to vector mesons ; within the common understanding[33] of VDM, they are constant and related to the decay width of the vector mesons V to e^+e^- :

$$\Gamma_V(e^+e^-) = \frac{4\pi\alpha^2}{3m_V^3} \; g_{V\gamma}^2, \quad V = \rho, \;\; \omega, \;\; \phi \quad . \tag{3}$$

These constants will be fixed (not fitted) to the corresponding values. The $F_V(X)$s are the coupling constants at vertices $VX\gamma$ defined as in ref.[18] in order to allow numerical comparison with the corresponding quantities found in other processes. Φ_V is the hadronic phase for the vector meson V corresponding to the given final state $X\gamma$. From first principles [26], one does not expect that they are necessarily identical for different final states of the same vector meson.

Finally, the quantitites $D_V(m^2)$ are the inverse propagators associated with the vector mesons V :

[1]It should be noted nevertheless, that $1/m^2$ behaviour is asymptotically required otherwise the cross section would tend to a finite value when $m^2 \to \infty$. However, matching with low energy predictions of chiral theories is not necessarily as smooth as suggested in ref. [31], as illustrated by ref. [32]. Apparently, for photon virtualities below 1 GeV, the constant approximation is still acceptable[12, 18].

$$D_V(m^2) = m_V^2 - m^2 - i m_V \Gamma_V(m^2) \quad , \quad V = \rho,\ \omega,\ \phi \quad , \tag{4}$$

where m_V is the meson mass and $\Gamma_V(m^2)$ its varying width. In the case of the ρ meson these mass and width are taken from models M_1 and M_2 in refs.[17, 18]. These models have been defined in order to fit all the available data[34] on $e^+e^- \to \pi^+\pi^-$ below 1 GeV with a width parametrized as :

$$\Gamma_\rho(m^2) = \Gamma_\rho \left[\frac{q_\pi(m)}{q_\pi(m_\rho)}\right]^3 \left[\frac{m_\rho}{m}\right]^\lambda \tag{5}$$

where $q_\pi(m)$ is the pion momentum in the ρ^0 decay and λ a fitted parameter describing the fall–off of the ρ mass distribution. In model M_2, the ρ^0 parameters are :

$$m_\rho = 780.8^{+0.5}_{-0.4} \ \text{MeV}, \quad \Gamma_\rho = 153. \pm 2. \ \text{MeV}, \quad \lambda = 0.66 \pm 0.05 \quad , \tag{6}$$

whereas in model M_1, they are :

$$m_\rho = 769.9 \pm 0.5 \ \text{MeV}, \quad \Gamma_\rho = 142.8^{+1.5}_{-2.4} \ \text{MeV}, \quad \lambda = 1.75 \pm 0.08 \quad ; \tag{7}$$

where here the ρ mass has been chosen at its accepted value[25]. Such a low value for the ρ mass is mainly due to pure hadronic processes[25], whereas e^+e^- annihilations tend to provide larger values [34, 17]. We refer the reader to ref.[17, 18] for full discussion, mentioning only that both parametrizations provide perfect description of the data. Let us, however, remind that model M_2 is in agreement[33] with the most common understanding of VDM while model M_1 introduces a non–resonant $\gamma\pi^+\pi^-$ coupling in order to fit the cross section for $e^+e^- \to \pi^+\pi^-$, following in this way recent trends in effective lagrangian theory[5, 33].

In the case of ω and ϕ mesons, their small width could allow to choose a constant Γ_V. However, for sake of completeness we have preferred taking mass dependent widths and we have taken into account all decay modes above the 1% level ; the two–body decay channels are treated as usual (*i.e.* like in rel. (5) above with $\lambda = 1$), while the three–body decay channels have been treated as proposed in ref.[35], assuming moreover that the three pion final state is dominated by $\rho\pi$. Except for the coupling constants of ω and ϕ mesons to $\pi^0\gamma$ and $\eta\gamma$, which will be fitted, all parameters of these resonances have been chosen at their world average values[25].

The coupling constant $F_V(X)$ is related to the radiative decay width $V \to X\gamma$ by :

$$\Gamma(V \to X\gamma) = \frac{1}{96\pi} \left[\frac{m_V^2 - m_X^2}{m_V}\right]^3 |F_V(X)|^2 \quad . \tag{8}$$

In the following study, our fitting parameters will be the coupling constants $F_V(X)$ and the hadronic phases Φ_V, for each annihilation process. As the anomalous term has a definite phase, one can in principle determine absolute hadronic phases Φ_V. We shall moreover take advantage of the fact that in the quark model Φ_ρ and Φ_ω are expected to be close to each other [36]. Having obtained the coupling $F_V(X)$ from fit to the data by means of rel. (1), we derive the

the corresponding radiative width using rel. (8) above and the branching fraction by dividing this partial width by the full width taken from PDG [25].

It is customary to relate the coupling constant describing a vertex $VX\gamma$ to the coupling constants describing vertices $VV'X$ (two vector and one pseudoscalar particles) by means of VDM. Up to possible phase factors, we have (see ref.[7, 12] for instance) :

$$g_{VX\gamma} = \sum_{V'} \frac{g_{VV'X} g_{V'\gamma}}{m_{V'}^2} \quad , \tag{9}$$

using obvious notations. Let us notice [33] that $m_{V'}^2/g_{V'\gamma}$ is also denoted by $g_{V'}$. Correspondingly, VDM allows[6, 24] to relate the coupling constant at a $X\gamma\gamma$ vertex to the coupling constants $VX\gamma$; in this case we have :

$$g_{X\gamma\gamma} = \sum_{V} e^{i\Phi_V} \frac{g_{VX\gamma} g_{V\gamma}}{m_V^2} \quad , \tag{10}$$

by introducing the expected hadronic phases. If the anomaly picture and the consequences of VDM on effective lagrangian are indeed dual, one could expect that :

$$|g_{X\gamma\gamma}| = |B_X| \tag{11}$$

With the available data samples on e^+e^- annihilations, we are in position to test this assumption for two different final states ($\eta\gamma$, $\pi^0\gamma$).

Using rel. (2) above and the accepted values for the two–photon decay widths of the π^0 (7 eV), η (0.46 keV) and η' (4.26 keV), one can determine the value of each B_X : $B_{\pi^0} = [-25.1 \pm 1.0]\ 10^{-3}$ GeV^{-1}, $B_\eta = [-23.7 \pm 1.0]\ 10^{-3}$ GeV^{-1}, $B_{\eta'} = [-31.2 \pm 0.7]\ 10^{-3}$ GeV^{-1}.

On the other hand, the output from fits we are going to describe, are the coupling constants $F_V(X)$. These are nothing but the $g_{VX\gamma}$ in rel. (10) for ρ, ω and ϕ. Therefore, using equation (10), we can check to what extent rel. (11) is fulfilled.

One also expects that F_ρ for the $\eta\gamma$ final state is close to the coupling constant named F_η (for the corresponding ρ model) extracted from the analysis [17, 18] of the data [37] on the $\eta \to \pi^+\pi^-\gamma$ decay spectrum. Strictly speaking, this is surely true if the vertex function for η–ρ^0–γ is constant. In this case the branching fraction for $\rho^0 \to \eta\gamma$ appears to be considerably smaller[18] than given by a previous analysis[24] of the same data where the contributions depicted in Fig. (1a) have been discarded.

On the other hand, if one assumes that SU(3) is not too hardly broken[8, 38], the analysis of anomalies[18] predicts also a value for the branching fraction $\rho^0 \to \pi^0\gamma$ comparable to the branching fraction [25] $\rho^\pm \to \pi^\pm\gamma$ and considerably smaller than the previous result obtained using the present data set [24].

4 Study of $e^+e^- \to \eta\gamma$

The original data sample contains 3525 events collected at 114 measured energy points. We have first performed fits in order to test the sensitivity of the data to the magnitude of the anomalous contribution (the first term in brackets in rel. (1)). From this point of view, we have found no change in the quality of fits when going from M_1 to M_2 models for the ρ^0 (relations (6) and (7) above). For a very large spectrum of values for B_η, the χ^2/N_{df} remains highly favorable (of the order 0.7), showing that 3 free phases and 3 free branching fractions allow too much freedom in fitting data where most of the information is carried by two sharp peaks. In such a situation, the existence of a minimum χ^2 (for some given parameter values) is more meaningful that its numerical value. Fig. 2 shows the minimum χ^2 reached for fixed values of the anomaly as a function of the corresponding two–photon decay width Γ_η of the η meson ; M_1 and M_2 lead to practically the same curve. The curve in Fig. 2 clearly shows a sharp peak at $\Gamma_\eta = 0$ and illustrates that an anomalous contribution corresponding to a value of Γ_η ranging between 400 eV and about 7 keV is preferred. To be more precise, introducing an anomalous contribution at the value corresponding to $\Gamma_\eta = 0.46$ keV allows to improve the χ^2 by about 10 units (without any change in the number of free parameters). We consider that this curve shows evidence for the triangle anomaly contribution expected to exist in $e^+e^- \to \eta\gamma$, and that data exhibit a clear preference for an anomallly magnitude close to the measured value of Γ_η [25].

no anomaly	Dolinsky (ref [24])	Model M_1	Model M_2
$\rho^0 \to \eta\gamma$	$(4.0 \pm 1.1)\ 10^{-4}$	$(3.46 \pm 0.67)\ 10^{-4}$	$(3.92 \pm 0.87)\ 10^{-4}$
$\omega \to \eta\gamma$	$(7.3 \pm 2.9)\ 10^{-4}$	$(6.12 \pm 2.30)\ 10^{-4}$	$(6.27 \pm 2.37)\ 10^{-4}$
$\phi \to \eta\gamma$	$(1.30 \pm 0.07)\ 10^{-2}$	$(1.15 \pm 0.05)\ 10^{-2}$	$(1.16 \pm 0.05)\ 10^{-2}$
χ^2/dof		44/64	43/64
$\Gamma_\eta = 0.46$ keV			
$\rho^0 \to \eta\gamma$		$(2.42 \pm 1.21)\ 10^{-4}$	$(2.68^{+1.44}_{-1.12})\ 10^{-4}$
$\omega \to \eta\gamma$		$(8.23 \pm 2.72)\ 10^{-4}$	$(8.45 \pm 2.82)\ 10^{-4}$
$\phi \to \eta\gamma$		$(1.24 \pm 0.10)\ 10^{-2}$	$(1.24 \pm 0.12)\ 10^{-2}$
χ^2/dof		35/63	35/63
$\rho^0 \to \eta\gamma$ predicted (ref. [18])		$(2.18 \pm 0.53)\ 10^{-4}$	$(1.00 \pm 0.22)\ 10^{-4}$

Table 1 : Branching fractions obtained from fit to the $e^+e^- \to \eta\gamma$ cross section using two ρ^0 models. In the upper part of the the table results obtained assuming no anomaly are given ; the lower part presents results obtained assuming an anomaly corresponding to the world average value [25] for the two–photon decay width Γ_η. The last line reminds the prediction for the ρ branching fraction deduced[18] from the analysis of the $\eta \to \pi^+\pi^-\gamma$ decay.

As the triangle anomaly is expected from first principles, it seems natural to pursue the study of this spectrum with B_η set at its expected value $B_\eta = -23.7\ 10^{-3}\ \mathrm{GeV}^{-1}$ rather than zero. To be more precise, if there is any significant difference in the fit values of the parameters, then results obtained with $B_\eta = -23.7\ 10^{-3}\ \mathrm{GeV}^{-1}$ are surely more reliable than those [24] following from using $B_\eta = 0$. In order to decrease the influence of statistical fluctuations on the parameter values, we use from now on the sample described above, after rebinning to 69 points. In Fig. 3 we give for illustrative purposes the data points and one of the best fits obtained using model M_1 ; there is no visible difference in all fits using M_1 or M_2, which always give a $\chi^2/N_{df} \simeq 35/64 \div 41/64$, depending on the additional constraints.

We have first performed our fits assuming that no anomaly was at work in the process ($\Gamma_\eta = 0$). This allows to see the effect of various ρ models by comparing our results with those of Dolinsky[24]. Our results are given in the upper part of table 1 and they show good consistence with the results in ref. [24] where a conventional Breit–Wigner shape has been used in order to take into account the ρ. When performing these fits, one phase should be fixed ; indeed, rel. (1) shows that one phase is arbitrary if one sets $B_\eta = 0$. We have discarded solutions providing a negative ρ^0–ω interference [24] as this is disfavored by the quark model.

After this cross–check, we have performed our fits assuming that the anomaly is constant and is given by the world average value for the two–photon decay width of the η meson (see rel. (2)). As can be seen from table 1, the central value for the $\rho \to \eta\gamma$ branching fraction becomes smaller than when assuming no anomaly and we get results in good correspondence with the prediction from the decay process $\eta \to \pi^+\pi^-\gamma$, for both models M_1 and M_2. Variations for the ω and ϕ branching fractions are much less noticeable. Moreover, the branching fraction for ω is consistent with the model independent measurement of GAMS [39] in all fits.

The hadronic phases obtained in our fits are :

$$\left\{ \begin{array}{lll} \Phi_\rho = & -90^{+64}_{-53} & \text{degrees} \\ \Phi_\omega = & -37^{+60}_{-53} & \text{degrees} \\ \Phi_\phi = & +87^{+45}_{-47} & \text{degrees} \end{array} \right. \tag{12}$$

when using model M_2 ; model M_1 essentially leads to the same values. This represents the first measurement of absolute hadronic phases ; it is allowed because of the anomaly contribution for which theory predicts both magnitude and phase.

This result shows that the ρ and ω hadronic phases associated with the $\eta\gamma$ final state are indeed consistent with being equal within one standard deviation. It is therefore meaningful to introduce in our fitting procedure the condition $\Phi_\rho = \Phi_\omega$ in order to make the influence of correlations on branching fractions smaller. Our results in this case are summarized in table 2.

These results show that the branching fraction for $\rho \to \eta\gamma$ becomes close to predictions [18] from $\eta \to \pi^+\pi^-\gamma$. It should be noted that the branching fraction for $\rho \to \eta\gamma$ of Dolinsky[24] was found by far too large to be consistent with data [19] on $\eta \to \mu^+\mu^-\gamma$; comparing the results in table 1 (upper part) and in table 2, clearly shows that this claim was correct and that the disagreement was indeed produced by a merging of the anomaly and ρ contributions.

The comparison of coupling constant values extracted from our fits with predictions[18] does not exhibit discrepancies. Therefore, if the couplings are mass dependent, this dependence is close to be flat[2].

$\Gamma_\eta = 0.46$ keV	Model M_1	Model M_2
$\rho^0 \rightarrow \eta\gamma$	$(1.67 \pm 0.56)\ 10^{-4}$	$(1.89 \pm 0.61)\ 10^{-4}$
$\omega \rightarrow \eta\gamma$	$(6.42 \pm 2.41)\ 10^{-4}$	$(6.56 \pm 2.46)\ 10^{-4}$
$\phi \rightarrow \eta\gamma$	$(1.21 \pm 0.07)\ 10^{-2}$	$(1.21 \pm 0.07)\ 10^{-2}$
$F_\rho(\eta)$ [GeV^{-1}]	0.36 ± 0.06	0.37 ± 0.06
$F_\omega(\eta)$ [GeV^{-1}]	0.16 ± 0.03	0.16 ± 0.03
$F_\phi(\eta)$ [GeV^{-1}]	0.21 ± 0.01	0.21 ± 0.01
χ^2/dof	40/64	39/64
predictions (ref. [18])		
$\rho^0 \rightarrow \eta\gamma$	$(2.18 \pm 0.53)\ 10^{-4}$	$(1.00 \pm 0.22)\ 10^{-4}$
$F_\rho(\eta)$ [GeV^{-1}]	0.41 ± 0.05	0.27 ± 0.03

Table 2 : Branching fractions and coupling constants obtained from fit to the $e^+e^- \rightarrow \eta\gamma$ cross section using the two ρ^0 models, assuming $\Phi_\rho = \Phi_\omega$.

Moreover, the branching fraction for ω remains consistent with the GAMS result [39] and becomes closer to the corresponding result obtained when the anomaly is neglected (see upper part of table 1) ; this gives us some confidence about the relevance of the condition $\Phi_\rho = \Phi_\omega$ expected from the quark model. Indeed, one can expect that anomaly and ρ contributions could be somehow mixed as the ρ is broad and of small intensity and thus can mimic phase space. However, we do not expect a correlation between an anomalous contribution and an object as narrow as the ω meson. Therefore, the numerical (statistically insignificant) difference between the lower part of table 1 and table 2 should be considered as an artefact caused by too much freedom in fits. Indeed, one does not expect the anomaly could influence the branching fraction for ω that much. Finally, the phases become :

$$\begin{cases} \Phi_\rho = \Phi_\omega = & -39^{+33}_{-118} \quad \text{degrees} \\ \Phi_\phi = & 115^{+28}_{-78} \quad \text{degrees} \end{cases} \tag{13}$$

and are close to each other for both ρ models ; note that the errors become asymmetric. As can be seen, the phase for ω is nearly unaffected by the constraint and the main change is produced in the ρ sector by the interplay of ρ and anomaly. The values we get are consistent with 0° for $\Phi_\rho - \Phi_\omega$ and with 180° for $\Phi_\phi - \Phi_\rho$, as expected from the quark model.

Comparing the results with and without anomaly, one sees a clear difference, pointing to the fact that the anomalous contribution cannot be discarded in the interpretation of the reaction $e^+e^- \rightarrow \eta\gamma$ without introducing important systematic errors. Instead, it is also interesting to

[2]Indeed, if couplings are mass dependent, the prediction of ref. [18] is kind of a mean value of F_ρ over a mass range from the two–pion threshold up to the η mass ; in the present data averaging goes up to the ϕ mass and includes the ρ peak location.

remark that our data do not exhibit significant sensitivity to the model used to describe the ρ lineshape ; therefore systematic errors due to modelling are negligible.

Finally, using the hadronic phases in relations (10) and the coupling constants given in table 2, we can compute the sum defined by rel. (10) :

$$|g_{\eta\gamma\gamma}| = (21.5 \pm 4.2)\ 10^{-3}\ \ \mathrm{GeV}^{-1} \quad ; \tag{14}$$

this is in good correspondence with the triangle anomaly value for B_η ($[-23.7 \pm 1.0]\ \ 10^{-3}$ GeV^{-1}). It should be noted that rel. (14) is actually saturated by the ρ contribution only and therefore the precise values of the phases do not affect the estimate of $|g_{\eta\gamma\gamma}|$. Indeed, keeping only the ρ term in rel.(10) would give $|g_{\eta\gamma\gamma}| = (21.5 \pm 3.6)\ 10^{-3}\ \mathrm{GeV}^{-1}$. As a matter of fact, the ω and ϕ contributions are small and of comparable magnitude, but opposite in phase ; they mainly contribute by increasing the errors.

Using $g_{\eta\gamma\gamma}$ as an estimate for B_η in rel. (2), one can get an estimate for the two-photon decay width of the η meson and then its branching fraction :

$$\mathrm{Br}(\eta \rightarrow \gamma\gamma) = (31 \pm 12)\% \quad . \tag{15}$$

Let us emphasize that this value is dominated by the ρ contribution, and remind that the world average value is [25] $(38.8 \pm 0.5)\%$. Instead, if one uses the coupling constants coming from the fits performed with a zero anomalous contribution, the result would be :

$$\mathrm{Br}(\eta \rightarrow \gamma\gamma) = (64.5 \pm 12.5)\% \tag{16}$$

which is twice as big as claimed in ref. [18] from analyzing the data from ref. [19]. This clearly illustrates that neglection of anomalous contributions can sum up in large systematic effects.

Therefore, introducing in fits an anomalous contribution corresponding to the accepted value of $\Gamma(\eta \rightarrow \gamma\gamma)$, improves consistency of the information given by the $e^+e^- \rightarrow \eta\gamma$ cross section with all related data and/or predictions[18, 25, 39].

All this leads us to conclude that, **i/** there is no way to distinguish here the correct model for ρ lineshape among M_1 and M_2, **ii/** the ρ contribution is reasonably enough estimated and systematic errors due to uncertainties in the treatment of the ρ are negligible, **iii/** the anomaly and the ρ contributions are well disentangled and the anomaly contribution cannot be discarded, **iv/** the duality between VDM and anomalies expressed through rel. (10) is fulfilled.

Therefore, we consider the results displayed in table (2) as satisfactory and final.

5 Study of $e^+e^- \rightarrow \pi^0\gamma$

Similarly to the $\eta\gamma$ final state, we have first studied the sensitivity of $e^+e^- \rightarrow \pi^0\gamma$ data to the magnitude of the anomaly. Whatever is the ρ^0 model used, we have not found a significant change of the χ^2 by allowing an anomalous contribution (the χ^2 never improves by more than 2 units). Therefore, for the $\pi^0\gamma$ final state we cannot claim any evidence for the anomalous contribution. One can likely infer that the accuracy of the data does not allow in this case to feel a (small) anomalous contribution as it has been for the $\eta\gamma$ final state. Indeed, the χ^2/N_{df} is found even more favorable ($\simeq 0.5$) than it was for $\eta\gamma$ ($\simeq 0.7$) for a large spectrum of cases. Therefore this study has to wait for future data with smaller error bars and for more measurements outside the ω and ϕ peaks in order to increase the sensitivity to an additional contribution.

However, we have pursued the study of $e^+e^- \to \pi^0\gamma$ by fixing the anomalous (constant) contribution to the value expected from the π^0 decay. Indeed, as the anomalous contribution is expected to exist from first principles and because we have already detected it in the parent process $e^+e^- \to \eta\gamma$, consistency implies that any analysis of measurements should take into account the triangle anomaly.

As it has been illustrated with the $\eta\gamma$ channel, the broad phase space contribution provided by the anomaly arises practically mixed with the ρ contribution if not taken into account explicitly, and as a result the accepted [25] branching fraction $\rho \to \pi^0\ \gamma$ can be noticeably affected by systematic errors.

no anomaly	Dolinsky (ref [24])	Model M_1	Model M_2
$\rho^0 \to \pi^0\gamma$	$(7.9 \pm 2.0)\ 10^{-4}$	$(8.04 \pm 1.31)\ 10^{-4}$	$(7.84 \pm 1.36)\ 10^{-4}$
$\omega \to \pi^0\gamma$	$(8.88 \pm 0.62)\ 10^{-2}$	$(8.49 \pm 0.24)\ 10^{-2}$	$(8.50 \pm 0.24)\ 10^{-4}$
$\phi \to \pi^0\gamma$	$(1.23 \pm 0.14)\ 10^{-3}$	$(1.14 \pm 0.14)\ 10^{-3}$	$(1.15 \pm 0.15)\ 10^{-3}$
Φ_ρ [degrees]	**0**	**0**	**0**
Φ_ω [degrees]	**0**	2 ± 5	3 ± 5
Φ_ϕ [degrees]	**155**	164^{+14}_{-7}	160^{+15}_{-7}
χ^2/dof		29/57	29/57
$\Gamma_{\pi^0} = 7.7$ eV			
$\rho^0 \to \pi^0\gamma$		$(11.43 \pm 2.0)\ 10^{-4}$	$(12.05 \pm 1.20)\ 10^{-4}$
$\omega \to \pi^0\gamma$		$(8.63 \pm 0.29)\ 10^{-2}$	$(8.56 \pm 0.26)\ 10^{-4}$
$\phi \to \pi^0\gamma$		$(1.16 \pm 0.16)\ 10^{-3}$	$(1.15 \pm 0.14)\ 10^{-3}$
Φ_ρ [degrees]		-93 ± 8	-93 ± 8
Φ_ω [degrees]		-92 ± 11	-100 ± 10
Φ_ϕ [degrees]		115 ± 15	108 ± 16
χ^2/dof		29/56	27/56

Table 3 : Branching fractions obtained from fit to the $e^+e^- \to \pi^0\gamma$ cross section using the two ρ^0 models. Numbers given in boldface are fixed. In the upper part of the table, results obtained assuming no anomaly are given ; the lower part presents the results obtained with an anomalous contribution corresponding to the accepted two–photon decay width Γ_{π^0} [25].

In order to have a clear idea of systematics produced by the modelling of the ρ shape, we have performed our fits assuming first that no anomaly was at work ; moreover, in order to match the fits of ref. [24] we have fixed[3] $\Phi_\rho \equiv 0°$. The results are gathered in the upper part

[3]In any case, if there is no anomalous contribution, one of the three hadronic phases should be fixed.

of table 3. Figure 4 shows a typical fit of $e^+e^- \to \pi^0\gamma$ data.

Models M_1 and M_2 give results in agreement with each other. Differences with the fit of ref. [24], can be attributed to the fact that we allow for more freedom (Φ_ω and Φ_ϕ are left free while they are fixed in ref. [24]). These results actually show that the phase difference between the ρ and ω contributions is consistent with zero, while the phase found for ϕ relative to ρ is close to the value found in fits to $e^+e^- \to \pi^+\pi^-\pi^0$ cross section (155°) [24].

In the lower part of table 3, we display the results obtained assuming the anomaly at the value inferred from the π^0 decay width. Here again, both ρ models give results in agreement with each other and support expectations from the quark model ($\Phi_\omega - \Phi_\rho \simeq 0°$ and $\Phi_\phi - \Phi_\rho \simeq 180°$). Note, however, that there is a jump of about 50° between the two sets of fits gathered in table 3.

As already mentioned, there is no significant difference in the fit quality when working with or without anomalous contribution. The absolute hadronic phases are found with good accuracy. Branching fractions for the ω and ϕ mesons are not affected by the additional contribution and are in agreement with their accepted values [25, 24]. However, the branching fraction for the ρ^0 meson is noticeably changed.

We have repeated our fits assuming $\Phi_\omega = \Phi_\rho$, as it is supported by our data. We have found two local minima, one with $\chi^2/N_{df} \simeq 0.5$ (solution A) and another one with $\chi^2/N_{df} \simeq 0.7$ (solution B). The results are gathered in table 4.

$\Gamma_{\pi^0} = 7.7$ eV	Model M_1		Model M_2	
	sol. A	sol. B	sol. A	sol. B
$\rho^0 \to \pi^0\gamma$ (units 10^{-4})	(11.51 ± 2.00)	(6.17 ± 1.57)	(11.67 ± 2.00)	(6.77 ± 1.72)
$\omega \to \pi^0\gamma$ (units 10^{-2})	(8.62 ± 0.26)	(8.39 ± 0.24)	(8.64 ± 0.27)	(8.39 ± 0.24)
$\phi \to \pi^0\gamma$ (units 10^{-3})	(1.15 ± 0.13)	(1.20 ± 0.16)	(1.21 ± 0.16)	(1.26 ± 0.17)
$F_\rho(\pi^0)$ [GeV^{-1}]	0.346 ± 0.030	0.236 ± 0.033	0.350 ± 0.030	0.270 ± 0.034
$F_\omega(\pi^0)$ [GeV^{-1}]	0.709 ± 0.011	0.699 ± 0.010	0.709 ± 0.011	0.701 ± 0.010
$F_\phi(\pi^0)$ [GeV^{-1}]	0.038 ± 0.002	0.039 ± 0.003	0.039 ± 0.03	0.040 ± 0.003
$\Phi_\rho = \Phi_\omega$ [degrees]	-94 ± 8	124 ± 9	-90 ± 7	125 ± 11
Φ_ϕ [degrees]	114 ± 14	248 ± 9	119^{+11}_{-18}	248^{+18}_{-10}
χ^2/dof	29/57	38/57	29/57	36/57
$\rho^\pm \to \pi^\pm\gamma$ (ref. [25])	$(4.50 \pm 0.50)\ 10^{-4}$			
predictions (ref. [18]) $\rho^0 \to \pi^0\gamma$	$(2.44 \pm 0.87)\ 10^{-4}$		$(5.23 \pm 0.79)\ 10^{-4}$	

Table 4 : Branching fractions, phases and coupling constants obtained from fit to the $e^+e^- \to \pi^0\gamma$ cross section using two ρ^0 models and assuming $\Phi_\rho = \Phi_\omega$. The last line gives predictions subject to SU(3) symmetry and associated with model M_1 or M_2.

A priori, solution A is the best, however the fit quality of solution B is also acceptable. Let us compare somehow these two solutions.

In solution A, the phase difference $\Phi_\phi - \Phi_\rho$ is found around 210°, close to expectations from the quark model (180°) ; however, the ρ^0 branching fraction is a factor of 2 larger than expected from the charged mode [25] or from SU(3) symmetry [18]. Such a deviation can be explained theoretically [7, 38] and is not larger than the observed deviation of $\phi \to \eta\gamma$ from exact SU(3) symmetry[10, 12, 18], or even that of $K^{*0}/K^{*\pm}$ radiative decays.

On the other hand, for solution B, we find $\Phi_\phi - \Phi_\rho$ at about 120°, relatively far from quark model expectations, but in this case the ρ^0 branching fraction is found in better agreement with the charged mode data [25] and with expectations from SU(3) symmetry [18].

As a further check of consistency, we have computed the decay width for π^0, which can be estimated using the fitted values for $F_V(\pi^0)$ together with rels. (10), (11) and (2). The results are given in table 5.

	Solution A	Solution B
All contributions	78.50 ± 12.50	72.90 ± 15.0
ρ contribution alone	68.60 ± 11.50	32.70 ± 8.20

Table 5 : π^0 decay width in percents of 7.79 eV for solutions A and B. The expected value is 98.8%.

Thus, solutions A and B predict quite comparable – and acceptable – π^0 width ; one should remark that the ρ contribution only does not saturate rel. (11), as it was the case with the $\eta\gamma$ final state.

Therefore, the main property of the solution A is to provide a phase difference $\Phi_\phi - \Phi_\rho$ in agreement with quark model expectations, while it leads to a branching fraction for $\rho^0 \to \pi^0\gamma$ of twice its expected value. Accordingly, the solution B gives a branching fraction for $\rho^0 \to \pi^0\gamma$ in good agreement with expectations from the charged mode and SU(3) symmetry, while the phase difference $\Phi_\phi - \Phi_\rho$ is found somehow smaller than expected from the quark model.

However, one should notice that, even if it can be explained, symmetry breaking in the ρ system is *a priori* less favored[4] than a deviation from the quark model predictions for the angles for OZI–suppressed decays (radiative ϕ meson decay into π^0). Therefore, the solution B seems more likely. Moreover, looking at the expectations from the charged mode and at the predictions from SU(3) symmetry (last lines in table 4), the results for M_2 look more reliable.

6 Summary and Perspective

From the previous analysis, we have been able to exhibit the contribution of the triangle anomaly in the $\eta\gamma$ final state. On the other hand, the triangle anomaly contribution for the $\pi^0\gamma$ final state remains presently undetected ; the accuracy of the existing data and the lack of measurements outside the ω and ϕ peaks are likely to be the reason. Our final results for the branching fractions and hadronic phases are displayed in table 6.

[4]Actually, it is also a large SU(2) symmetry breaking in the non–strange sector.

$\Gamma_\eta = 0.46$ keV	$\eta\gamma$ final state	$\pi^0\gamma$ final state
$\mathrm{Br}(\rho^0)$	$(1.89^{+0.61}_{-0.78})\ 10^{-4}$	$(6.77 \pm 1.72)\ 10^{-4}$
$\mathrm{Br}(\omega)$	$(6.56^{+2.41}_{-2.55})\ 10^{-4}$	$(8.39 \pm 0.24)\ 10^{-2}$
$\mathrm{Br}(\phi)$	$(1.21 \pm 0.07)\ 10^{-2}$	$(1.26 \pm 0.17)\ 10^{-3}$
$\Phi_\rho = \Phi_\omega$ [degrees]	-39^{+33}_{-118}	125 ± 11
Φ_ϕ [degrees]	115^{+28}_{-78}	248^{+18}_{-10}

Table 6 : Branching fractions and hadronic phases obtained from fit to $e^+e^- \to \eta/\pi^0\gamma$ cross sections, assuming a contribution from triangle anomaly at the level expected from two–photon decays.

These results take into account the anomalous contribution at the level expected from π^0 and η decays as discussed in section **3**.

For the $\eta\gamma$ final state, the present analysis leads to the following :

– the branching fraction for ϕ is consistent at the 1σ level with the previous analysis [24] of the same data and in pretty good agreement with the recent result [27] of CMD–2 ($[1.18 \pm 0.11]\%$),

– the branching fraction for ω is found to be slightly smaller than previous values [24, 39] but all remain consistent at better than the 1 σ level ; moreover our central value is in accordance with a recent result from CRYSTAL BARREL[40] ($[6.8 \pm 1.7]\ 10^{-4}$),

– the branching fraction for ρ^0 is found to be significantly smaller than the value obtained by neglecting the anomaly [24], but now in good agreement with the value expected [18] from the analysis of $\eta \to \mu^+\mu^-\gamma$ and $\eta \to \pi^+\pi^-\gamma$ data.

For the $\pi^0\gamma$ final state, the conclusions from the present analysis are as follows :

– the branching fractions for ω and ϕ are in good agreement with previous analysis and other data [24]

– the value for the ρ branching fraction is in better agreement with expectations from SU(3) symmetry. In this case, however, another solution exists with analogous results for ω and ϕ mesons but with a large SU(3) symmetry breaking effect in the ρ^0 radiative decay.

In all cases, we have obtained the absolute phases (*i.e.* resonance phases relative to the anomalous contribution). They confirm that $\Phi_\omega = \Phi_\rho$ is fulfilled with a good accuracy.

The model developed in section **3** is also valid *mutatis mutandis* for $e^+e^- \to \eta'\gamma$. This reaction has not yet been observed. The best upper limit comes from a recent measurement of CMD–2 : $\mathrm{Br}(\phi \to \eta'\gamma) < 2.4\ 10^{-4}$ (at the 90% C.L.) [41] ; this corresponds to an upper limit for the cross section of about 1 nb[5]. Improved results about this decay are expected to come soon from experiments at VEPP–2M [41, 42].

This process is important as it permits a test of QCD. Indeed, all models in agreement with QCD [7, 10, 12, 18] predict a branching fraction of the order $1 \div 2\ 10^{-4}$; however violation of QCD is predicted [18] if this branching fraction is of the order 10^{-6}, assuming SU(3) symmetry

[5]This result superseds the bound from ND [24], which corresponds to a cross section bound of 1.7 nb.

is not broken. One can consider as likely a symmetry breaking effect of a factor $2 \div 4$. This leaves some room for the test, as these two kinds of predictions should still be different by a factor of about 20 for likely magnitudes of symmetry breaking effects. If this branching ratio is found below $5.\ 10^{-5}$, it would be of importance [18].

However, this measurement could be shadowed by the anomalous η' contribution and the ρ contribution which is governed by a large known coupling constant [17, 18] $F_\rho(\eta') \simeq 0.4$ GeV^{-1}. In terms of cross section (see rel. (1)), this corresponds to a function which strongly varies with the (unknown) ρ hadronic phase and ranges between 0.01 pb to 0.70 pb, depending on this phase. In other words, anomaly and ρ contributions do not represent a significant background to the ϕ decay to $\eta'\gamma$, as the predictions for this decay correspond to cross sections ranging between a few pb and 400 pb.

7 Conclusions

We have performed a new analysis of the data collected by the Neutral Detector on $e^+e^- \rightarrow \eta/\pi^0\gamma$ annihilations in order to detect the anomalous triangle contribution expected to exist in these decays. For the $\eta\gamma$ final state, we have found that the anomalous contribution was indeed present at a level consistent with the η decay width to two photons. The accuracy of the data for the $\pi^0\gamma$ final state and the small number of measured points in the appropriate region prevent from such a firm conclusion.

As the ρ contribution is both broad and small, we have shown that it is indeed mixed with the anomalous (broad) contribution and we have illustrated the systematic effects this mixing is responsible for. Thus, the anomalous contribution cannot be avoided.

Introducing the anomaly in the cross sections, we have performed a new analysis of these data. All results are found to be in good agreement with expectations coming from parent processes concerning the ρ meson, thus confirming the conclusions reached by the analysis of several other processes affected by anomalies. On the other hand, the branching fractions for ω and ϕ have been refined and found to be in fair agreement with recent measurements.

Finally, taking advantage of the fact that the anomaly contribution has a definite phase, we have been able to extract for the first time the absolute magnitude of hadronic phases.

The effects due to the modelling of the ρ^0 lineshape have been proved to provide negligible systematic effects.

We have also argued on the importance of a measurement of the branching fraction $\phi \rightarrow \eta'\gamma$ as a test of QCD and estimated the physics background which affects it, because of the η' triangle anomaly and the relatively strong coupling of the ρ meson to $\eta'\gamma$. The contamination affecting a possible ϕ signal in this channel has been found to depend sharply on the ρ hadronic phase, but it never exceeds 1 pb.

One could hope that the results expected to come out from VEPP–2M and DAPHNE will allow to estimate more accurately the effects of anomalies in annihilation processes and in radiative decays of vector mesons.

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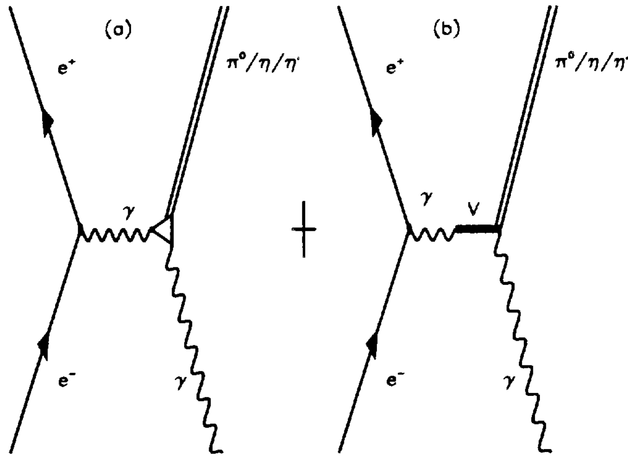

Figure 1: Contributions to $e^+e^- \rightarrow P\gamma$, for $P = \pi^0,\ \eta,\ \eta'$. In (a) the diagram associated with the triangle anomaly ; in (b), the diagram associated with intermediate vector mesons.

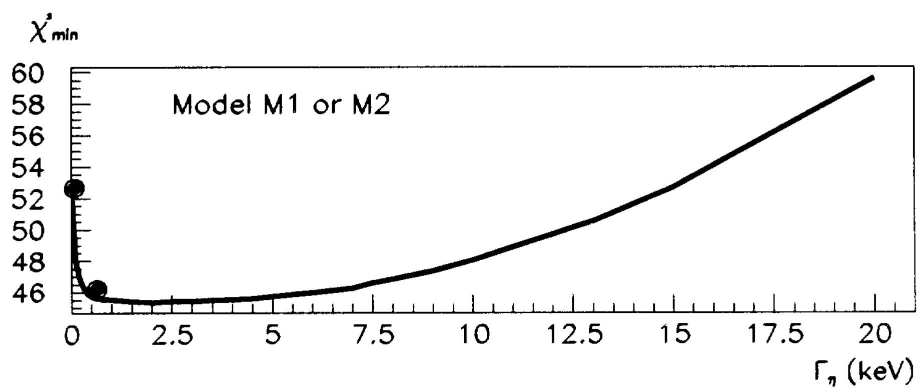

Figure 2: Minimum χ^2 in the fit of $e^+e^- \rightarrow \eta\gamma$ cross section as a function of the triangle anomaly constant. The horizontal axis is expressed in terms of the corresponding width $\eta \rightarrow \gamma\gamma$. The two full dots indicate the usual VDM (no anomaly) and the value corresponding to the width $\eta \rightarrow \gamma\gamma$ (0.46 keV).

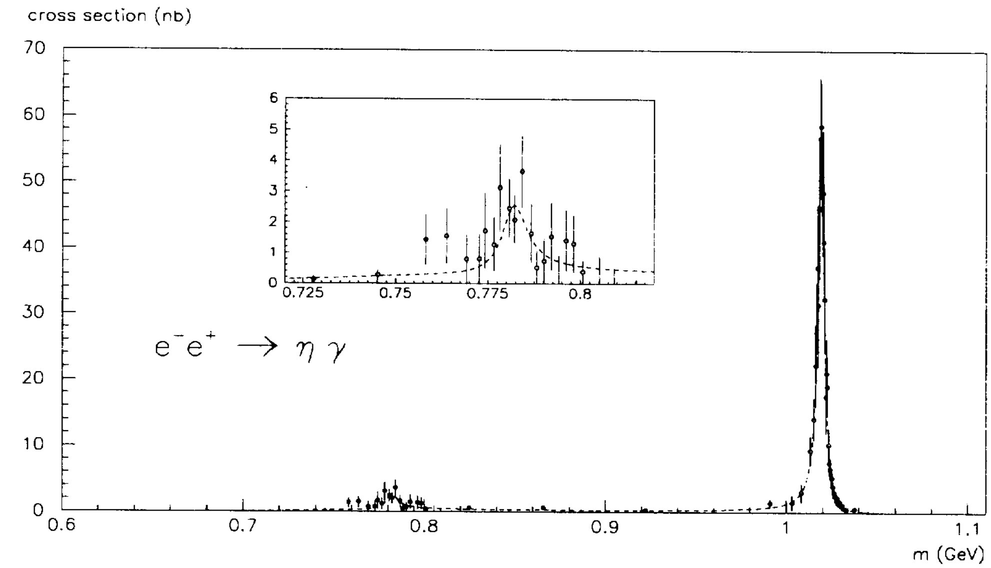

Figure 3: Cross section for $e^+e^- \to \eta\gamma$. The fit represented has been done using model M_1 for the ρ^0 meson ; the fit with model M_2 is identical. Inset shows the ω region.

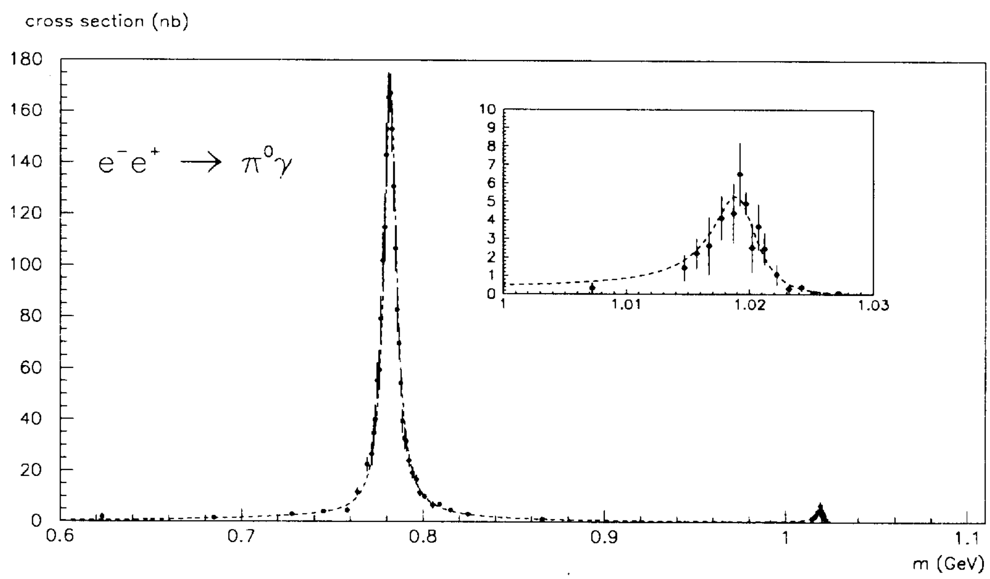

Figure 4: Cross section for $e^+e^- \to \pi^0\gamma$. The fit represented has been done using model M_1 for the ρ^0 meson ; the fit with model M_2 is identical. Inset shows the ϕ region.