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**Abstract**

We calculate Coulomb excitation of $^{208}$Pb, $^{90}$Zr and $^{96}$Zr by a $^{208}$Pb nucleus at 641 MeV/A. We go beyond the standard multiphonon picture by considering the anharmonicities coming from the mixing of two-phonon states among themselves and with one phonon states and by taking into account the terms of the external field connecting states differing by 0, 1 or 2 in the number of phonons. We show with different examples the importance of the non-linearities and anharmonicities for the excitation cross section. We find an increase ranging from 10% to 16% of the excitation cross section corresponding to the energy region of the double giant dipole resonance with respect to the "standard" calculation. We also find important effects in the low energy region. The predicted cross section in the DGDR region is found to be rather close to the experimental observation.

*talk given by E. G. Lanza

# 1 Introduction

Collective vibrations in nuclear systems have been studied for many years. They are present both at low and high energy (Giant Resonances). Double excitation of these states have been known since many years but, while the low-lying ones have been observed long time ago, only recently there have been measurements of double giant resonance state. For a review see ref. [1].

The standard approach to describe the collective vibrational motion of the nucleus is to assume a harmonic description, by means of the microscopic Random Phase Approximation (RPA), while the excitation of one nucleus in a ion-ion collision is described as due to the mean field of the other partner of the reaction. The mean field is usually assumed to be linear in the creation and annihilation operators of phonons.

Within this framework the calculated cross sections and widths of the double giant resonance are systematically smaller than the experimental ones, see ref. [1, 2] and references therein. With the help of a schematic one-dimensional model [3] of an anharmonic oscillator we have shown that the introduction of small anharmonicities (less than 10%) can change significantly the Coulomb excitation cross section.

With this result in mind we have calculated the Coulomb excitation cross section of a heavy nucleus in a relativistic ion-ion collision for which experimental data are available. To go beyond the standard approach description based on RPA, we have taken into account anharmonicities by including the residual interaction which couples one-phonon with two-phonon states and two-phonon states among themselves. Furthermore we have allowed direct transitions between two one-phonon states or from the ground state to a two-phonon state. These transitions arise from the particle-particle and hole-hole components of the external field, usually neglected in the standard multiphonon picture. These components can be expressed as quadratic forms in the phonon creation and annihilation operators. In the following we will refer to them as non-linear terms. The calculations done for the systems $^{208}$Pb + $^{208}$Pb, $^{208}$Pb + $^{90}$Zr and $^{208}$Pb + $^{96}$Zr show that the effect of the anharmonicities and non-linearities on the cross section is of the order of 10% or more.These results reduce the disagreement between the calculated and experimental cross section. A complete and detailed description of the model can be found in ref. [4].

## 2 Beyond the standard multiphonon picture

Within a semiclassical approach of heavy ion collisions one can separate the relative motion, treated classically, from the internal degrees of freedom treated quantum mechanically. This means that one has to consider grazing collisions where the densities of the two colliding nuclei have a small overlap. In this case the hamiltonian is the sum of two hamiltonians

$$H = H_A + H_B \quad ; \quad H_i = H_i^0 + W_i(t) \tag{1}$$

where $H_i^0$ is the internal hamiltonian of nucleus $i$ and $W_i(t)$ descibes the excitation of nucleus i by means of the mean field of the other nucleus. The time dependence is through the relative coordinate R(t). In the standard microscopic approach as RPA, the excited states are described as coherent sums of particle-hole and hole-particle configurations with respect to the ground state. When the RPA phonons are mapped onto bosons, the hamiltonian of the nucleus reduces to that of uncoupled harmonic oscillator, the RPA phonons $\nu$, which are then treated as independent. Within the same approximation the external field is linear in the collective boson operators.

To go beyond this approximation, in the internal hamiltonian we can introduce part of the residual interaction, namely the pppp and hhhh terms which produce a coupling between two-phonon states and the ppph and hhhp terms that mix one- and two-phonon states. The technique also allows to take into account corrections due to the Pauli principle [5]. These higher order terms give rise to anharmonicities.

By diagonalizing the residual interaction in the space of one- and two-phonons states, the eigenstates of the nucleus become a mixing of the one- and two-phonon states $|\nu_1>$ and $|\nu_1\nu_2>$, respectively

$$|\Phi_\alpha>= \sum_\nu c_\nu^\alpha |\nu> + \sum_{\nu_1\nu_2} d_{\nu_1\nu_2}^\alpha |\nu_1\nu_2> \tag{2}$$

In the external field we introduce both linear and non-linear terms, thus allowing transitions between states differing by zero, one or two in the number of phonons.

The time dependent coupling potential W(t) is obtained by performing a Fourier transform of the Winther-Alder expression [6] of the semi-classical first-order excitation amplitude in relativistic nucleus-nucleus collisions. In order to get the probability of exciting the mixed states $|\phi_\alpha>$ (Eq. 2), we have solved the time dependent Schrödinger equation for the wavefunction $|\Psi(t)>=\sum_\alpha A_\alpha(t)e^{-iE_\alpha t}|\Phi_\alpha>$ leading to the set of coupled differential equations

$$\dot{A}_\alpha(t)=-i\sum_{\alpha'}e^{i(E_\alpha-E_{\alpha'})t}<\Phi_\alpha|W(t)|\Phi_{\alpha'}>A_{\alpha'}(t) \tag{3}$$

for the amplitudes. The excitation probability of the state $\alpha$ is then given by $P_\alpha=|A_\alpha(t=+\infty)|^2$ and the associated cross section

$$\sigma_\alpha=2\pi\int_0^{+\infty}P_\alpha(b)T(b)bdb. \tag{4}$$

In the results we are going to discuss we have used a sharp cut-off transmission coefficient $T$.

# 3 Results

We have done calculations for the Coulomb excitation of $^{208}$Pb, $^{90}$Zr and $^{96}$Zr nuclei by a projectile $^{208}$Pb at 641 MeV/A. We have constructed the one-phonon basis with a RPA calculation using the Skyrme interaction SGII [7]. Only the most collective states, exhausting more than 5% of the corresponding Energy Weighted Sum Rule (EWSR) have been considered (see table 1). All possible two-phonon states built from the selected phonon basis are explicitly taken into account. Then, using the same SGII interaction, we have computed the residual interaction between one- and two-phonon states as well as among the two-phonon states. For each spin and parity the total matrix has been diagonalised in order to get the mixed states (Eq.2). Non-natural parity states can be excited in our coupled channel calculation but their cross section comes out to be small[4].

In the case of $^{208}$Pb, the coupling between one- and two-phonon states comes out to be of the order of 1/2 MeV up to 1 MeV, whereas the one between two-phonon states is, on the average, one order of magnitude smaller. Since these coupling matrix elements are quite small, the mixing coefficients in Eq.(2) are dominated by one component which can be used to label the mixed state. The other coefficients are small: in average around 0.05 and at maximum around 0.2. The energies of the mixed states are shifted, on the average, by a few hundred keV compared with the harmonic RPA-multiphonon

Table 1:

**One-phonon basis for the nuclei $^{208}Pb$, $^{90}$Zr and $^{96}$Zr.**

| $^{208}Pb$ | |
|---|---|
| Phonons | $E(MeV)$ |
| $GMR_1$ | 13.6 |
| $GMR_2$ | 15.0 |
| $GDR_1$ | 12.4 |
| $GDR_2$ | 16.7 |
| $2^+$ | 5.5 |
| $ISGQR$ | 11.6 |
| $IVGQR$ | 21.8 |
| $3^-$ | 3.5 |
| $HEOR$ | 21.3 |

| $^{90}Zr$ | |
|---|---|
| Phonons | $E(MeV)$ |
| $GDR_1$ | 15.3 |
| $GDR_2$ | 18.0 |
| $2^+$ | 5.1 |
| $ISGQR$ | 14.1 |
| $IVGQR$ | 26.5 |
| $3_1^-$ | 2.9 |
| $3_2^-$ | 5.0 |
| $3_3^-$ | 27.0 |

| $^{96}Zr$ | |
|---|---|
| Phonons | $E(MeV)$ |
| $GDR_1$ | 14.8 |
| $GDR_2$ | 17.1 |
| $2^+$ | 0.6 |
| $ISGQR_1$ | 13.9 |
| $ISGQR_2$ | 15.3 |
| $IVGQR$ | 25.2 |
| $3_1^-$ | 1.9 |
| $3_2^-$ | 7.5 |
| $3_3^-$ | 25.5 |

energies. Each multiplet appears to be splitted with a characteristic spreading equal to the global shift[4].

The Coulomb excitation cross sections for the GDR, the DGDR and the sum of all the states lying in the DGDR region obtained in the reaction $^{208}Pb$ on $^{208}Pb$ at 641 MeV per nucleon are shown in Table 2. Indeed, many multiphonon states, such as the one constructed from one dipole and one quadrupole single-phonons, are adding their cross-section in this energy region. Experimentally, they have not been separated from the DGDR excitations, therefore, only the total excitation cross-section including all possible states should be compared with experiments. From Table 2 we can see that the inclusion of all the multiphonon states considered in our calculation and lying in the vicinity of the DGDR states increases the cross section in this region from 0.22 barn to 0.28.

In table 3 we have reported separately the contribution of the various multipoles to the cross section in the DGDR region for different cases in order to disentangle the contribution of the anharmonicities and of the non-linearities. First of all we note that both the anharmonicities and non-linearities give an increase similar in magnitude and this is essentially due to the dipole states. The introduction of anharmonicities in the internal hamiltonian as well as non-linearities in the external field increases the cross section in the DGDR energy region of about 10 to 16%.

Table 2:
Comparison between our theoretical results and the experimental cross sections[8] (in barn) for the Pb + Pb reaction at 641 MeV per nucleon. The theoretical results (first line) correspond to the sum of all GDR (first column) and all DGDR (second column) cross-section. The third column contains the cross sections associated with all the states above the IVGQR (E > 22 MeV). The theoretical cross sections are obtained from the non-linear and anharmonic calculation while the numbers in parenthesis refer to the linear and harmonic limit. The experimental results are reported in the second line.

| | GDR | DGDR | DGDR energy region |
|---|---|---|---|
| $\sigma_{\mathrm{th}}$ | 3.13 (3.14) | 0.21 (0.22) | 0.31 (0.28) |
| $\sigma_{\mathrm{exp}}$ | $3.28 \pm 0.05$ | $0.38 \pm 0.04$ | |

Table 3: Coulomb excitation cross section in the DGDR region for various target A in the reaction $^{208}$Pb + A at 641 MeV/A.

| | Phonons | har. & lin. | harm. & non-lin. | anh. & lin. | anh. & non-lin. |
|---|---|---|---|---|---|
| $^{208}$Pb | L=0 | 43.4 | 43.1 | 43.4 | 43.1 |
| | L=1 | 4.7 | 10.0 | 7.6 | 14.5 |
| | L=2 | 175.7 | 173.3 | 180.8 | 180.5 |
| | L=3 | 54.8 | 65.6 | 56.0 | 68.2 |
| | total | 278.6 | 292.0 | 287.8 | 306.3 |
| $^{90}$Zr | L=0 | 6.4 | 6.4 | 6.5 | 6.6 |
| | L=1 | 0.6 | 3.0 | 2.5 | 3.9 |
| | L=2 | 30.2 | 29.5 | 31.6 | 31.3 |
| | L=3 | 10.4 | 13.1 | 10.7 | 13.4 |
| | total | 47.6 | 52.0 | 51.3 | 55.2 |
| $^{96}$Zr | L=0 | 6.0 | 6.1 | 6.5 | 6.3 |
| | L=1 | 0.5 | 1.6 | 2.4 | 2.9 |
| | L=2 | 24.9 | 24.4 | 25.4 | 25.2 |
| | L=3 | 6.3 | 8.2 | 6.7 | 8.1 |
| | total | 37.7 | 40.3 | 41.0 | 42.5 |

Table 4:

**Theoretical results on the cross sections (in millibarn) for the excitation of the dipolar $|2^+ \otimes 3^- >$ state for various target A in the Pb + A reaction at 641 MeV per nucleon. The various columns correspond to various types of calculations including different coupling terms one by one (see text).**

| | har.& lin. | $W^{11}$ | $W^{20}$ | anh. | anh. & non-lin. |
|---|---|---|---|---|---|
| $^{208}$Pb | 0.03 | 0.04 | 16.21 | 2.60 | 29.53 |
| $^{90}$Zr | 0.0008 | 0.001 | 3.66 | 0.59 | 7.02 |
| $^{96}$Zr | 0.0007 | 0.0008 | 10.33 | 5.48 | 23.07 |

The low lying part of the spectrum is strongly affected by the anharmonic and non-linear terms. A clear example is given by the two-phonon state which is built with the low-lying $|2^+ >$ and $|3^- >$ states [4]. In Table 4 we report the cross section for the excitation of the $|2^+ \otimes 3^- >$ state for various target excited by a $^{208}$Pb projectile at 641 MeV/A. We see that in all the case shown in the table, in the harmonic and linear approximation the cross section is almost zero. This is because the only possible way to excite it is by a two step excitation of multipolarities 2 and 3 which are not favoured. When we include the non-linear terms, we can excite this state with new possible ways. Because of the presence of ground state correlations the non-linearities may induce direct transitions from the ground state to the two-phonon states. Since the $|2^+ \otimes 3^- >$ multiplets can be coupled to dipole $(1^-)$ quantum numbers this is an E1 transition which is favoured in Coulomb excitation process. Finally, the presence of anharmonicities introduces a mixing between the $|2^+ \otimes 3^- >$ two-phonon state and the GDR itself. Since the GDR excitation is strong, even a small mixing may induce a strong excitation of the $|2^+ \otimes 3^- >$ state. When all these effects are taken into account at the same time they give an increase by a factor 1000 or bigger of the cross-section associated with this state. The different behaviour of the Zr isotopes depends on the different anharmonicity present in the two nuclei.

In order to show a global view on the effects of both non-linearities and anharmonicities, we have computed the complete inelastic cross section by

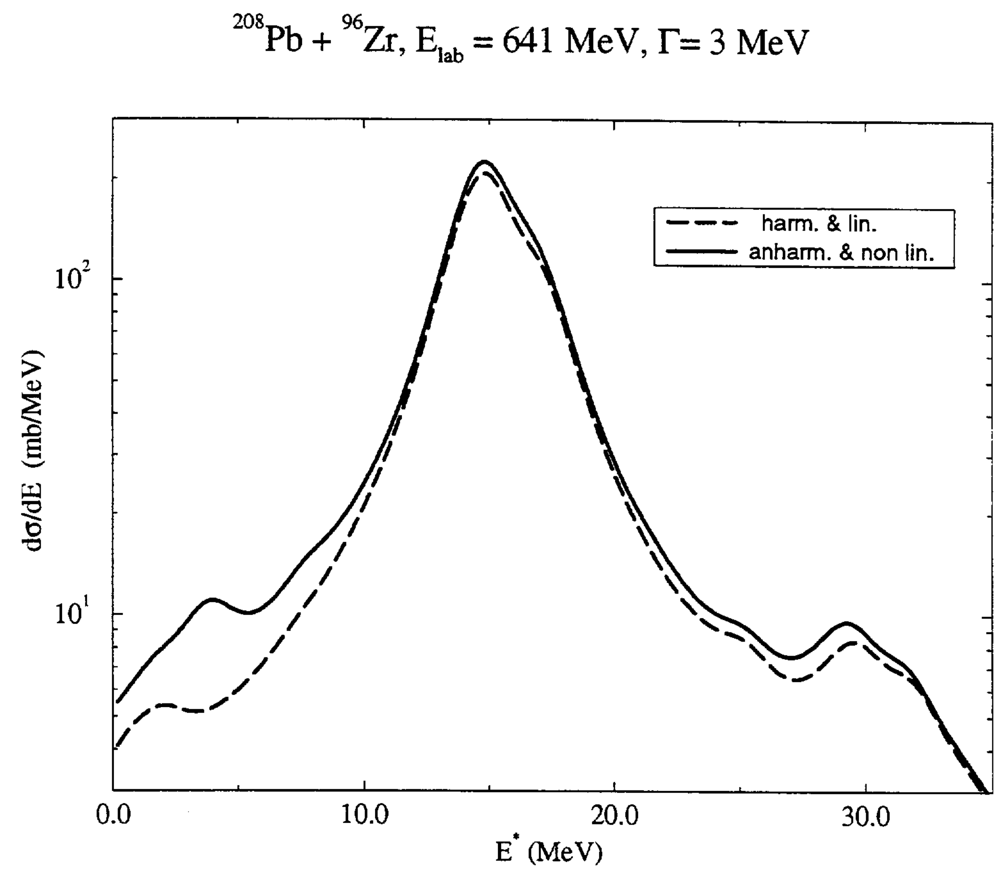


Figure 1: Relativistic Coulomb target dipole excitation cross section for the $^{208}$Pb+$^{96}$Zr system at 641 MeV/A as a function of the excitation energy. The cross section of the single state $|\Phi_\alpha>$ has been smoothed by a lorentzian with a 3 MeV width. The dashed line corresponds to the calculation done in the harmonic and linear case, while the solid line refers to the other extreme case, namely anharmonic and non-linear.

summing up all the contributions coming from the various states after a smoothing of each individual line shape by a lorentzian with a 3 MeV width. In fig. 1 we show the results of the calculations done in the two extreme cases: the harmonic and linear case (dashed line) and the anharmonic and non-linear one (solid line). The figure puts clearly in evidence the two region of interest where the effects of both non-linearities and anharmonicities are stronger.

# 4 Conclusion

In conclusion, we have seen that the Coulomb excitation cross sections are very sensitive to anharmonicities and non-linearities. In the cases analyzed here, namely $^{208}$Pb, $^{90}$Zr and $^{96}$Zr our calculations show that the increase of the cross section in the region of the DGDR ranges from 10 to 16%. In particular, in the case of $^{208}$Pb the agreement with the experimental data for the GDR is satisfactory and for the DGDR we have got a theoretical estimation far by only 18% from the experimental results. The latter increase is mainly due to the excitation of two-phonon states whose energies are in the DGDR regions and whose population is due to the presence of the anharmonicities and non-linearities.

We have also found a big effect at low energy where two-phonon states like $|2^+ \otimes 3^- >$ with a GDR component are strongly excited. Since these states are strongly mixed and since their energy is low they may be strongly excited by the nuclear part of the mean field at an incident energy lower than the one considered here. Work in this direction is in progress.

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