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GANIL P 97 02

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(January 13, 1997)

Abstract

A connection is established between an interacting boson model that includes $T = 1$ bosons (IBM-3) and one that includes $T = 0$ as well as $T = 1$ bosons (IBM-4). The connection relies on an IBM-4 classification that breaks Wigner's SU(4) symmetry. The resulting generalised IBM-4 is relevant for studying the competition between $T = 0$ and $T = 1$ pairing in $N \sim Z$ nuclei. Consequences for the pair structure in self-conjugate nuclei are discussed.

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One of the most active areas of current research in nuclear physics centers on the structure of heavier nuclei with roughly equal numbers of neutrons and protons and, specifically, nuclei in the $pfg_{9/2}$ shell with $20 \leq N \sim Z \leq 50$. Heavier $N = Z$ nuclei rapidly become unbound by virtue of the large excess of protons relative to stable nuclei. Experiments to elucidate the structure of such nuclei come up against the limits of current experimental sensitivity [1] and represent a notable challenge for the new generation of radioactive beam facilities currently under construction [2]. The combination of two features renders these nuclei of special interest: they have neutrons and protons in the same valence shell and the 28-50 shell is large enough for the nuclei to exhibit all aspects of nuclear collective behaviour. These characteristics imply that the neutron-proton exchange symmetry must be taken into account in describing the collective structure in this region.

A particularly topical problem in this respect concerns the competition between $T = 0$ and $T = 1$ pairing, both modes being present if neutrons and protons fill the same valence shell. This problem has previously been investigated for lighter nuclei in a Hartree-Fock-Bogoliubov (HFB) framework that includes both pairing modes [3], the summary of the analysis being that $T = 0$ is the dominant mode in $N = Z$ nuclei but quickly becomes unimportant for nuclei more than two neutrons away from the $N = Z$ line. However, recent experimental results [4] suggest that the ground states of the heavier odd-odd nuclei in fact have $T = 1$, while a recent treatment [5] of the pairing of neutrons and protons in a single shell points to problems associated with the lack of isospin invariance in the HFB approach. It is the purpose of this Letter, therefore, to study the competition between $T = 0$ and $T = 1$ pairing in a collective model which explicitly incorporates both pairing modes and is isospin invariant.

The interacting boson model (IBM) of Arima and Iachello [6] is capable of giving a simple yet realistic description of nuclear collective features in terms of s and d bosons. The model exists in different versions according to whether or not a distinction is made between neutron and proton bosons: in the original version (IBM-1) no such distinction was made [7] while it proved to be necessary in IBM-2 to establish a connection between the boson model and the shell model [8]. Furthermore, if neutrons and protons occupy the same valence shell, isospin symmetry considerations become crucial and lead to two more versions of the IBM. In IBM-3 [9] the complete isospin $T = 1$ triplet of bosons is included, comprising a neutron-proton (np) $T_z = 0$ pair in addition to the nn and pp $T_z = \pm 1$ pairs of IBM-2. With this complete isospin triplet an isospin invariant boson model can be constructed. In IBM-4 [10] the $T = 1$ bosons are assigned an intrinsic spin $S = 0$ and complemented with a set of $T = 0$, $S = 1$ bosons. The rationale behind this choice is that the two-particle isospin-spin combinations $(TS) = (10)$ and (01) are the ones favoured by Wigner's SU(4) classification [11] which is known to have physical significance in light nuclei. In heavier nuclei SU(4) symmetry is increasingly broken [12] but IBM-4 may remain a valid approximation (as it does in *sd*-shell nuclei [13,14]) if the boson L and S are equated to the *pseudo* orbital and spin angular momentum quantum numbers of two fermions [15].

The IBM offers thus two extreme possibilities in its different formulations: IBM-3 which does not contain $T = 0$ bosons and IBM-4 where these are included on an equal footing with $T = 1$ bosons. (We note that $T = 0$ *fermion correlations* can be represented in IBM-3 through boson-boson interactions; this may be a valid approximation for even-even but it is not for odd-odd nuclei.) In this Letter we establish a connection between these two limiting situations. We do so by constructing a classification in IBM-4 that does not conserve SU(4) symmetry and which has a subset of states in direct correspondence with IBM-3 states. By establishing this link between IBM-3 and IBM-4 we find at our disposal an ideal tool to study the competition between $T = 0$ and $T = 1$ pairing.

The dynamical algebra of IBM-3 is U(18) arising from the six orbital $(s + d)$ degrees of freedom times the three isospin degrees of freedom. The IBM-3 classification that conserves angular momentum L and isospin T is [9]

$$\begin{array}{ccccccccccc}
\mathrm{U}(18) & \supset & \big(\mathrm{U}_L(6) & \supset \cdots \supset & \mathrm{O}_L(3)\big) & \otimes & \big(\mathrm{U}_T(3) & \supset & \mathrm{SU}_T(3) & \supset & \mathrm{SU}_T(2)\big) \\
\downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
[N] & & [N_1N_2N_3] & & L & & [N_1N_2N_3] & & (\lambda_T\mu_T) & & T
\end{array}, \qquad (1)$$

where Elliott's SU(3) labels are used, $\lambda_T = N_1 - N_2$ and $\mu_T = N_2 - N_3$ [16]. All states are contained in a symmetric representation $[N]$ of U(18) where N is the number of bosons (i.e., half the number of valence nucleons). Due to overall symmetry in the bosons, the orbital algebra $\mathrm{U}_L(6)$ and the isospin algebra $\mathrm{U}_T(3)$ are characterised by the same representation $[N_1N_2N_3]$. The orbital reduction from $\mathrm{U}_L(6)$ to $\mathrm{O}_L(3)$ is not specified in (1) but may correspond to one of the three limits of IBM-1 [6] or any mixture thereof. Any nuclear hamiltonian must conserve angular momentum, and, to a good approximation, also isospin. This justifies the use of a classification in which L and T are good quantum numbers. Note, however, that the conservation of the $\mathrm{SU}_T(3)$ charge labels $(\lambda_T\mu_T)$ (or, equivalently, of $[N_1N_2N_3]$) is less fundamental as it depends on the details of the residual interaction.

The dynamical algebra of IBM-4 is U(36) since it involves the same orbital degrees of freedom coupled with an additional six isospin-spin $[(TS) = (01)$ and $(10)]$ degrees of freedom. The usual IBM-4 classification, as proposed by Elliott and Evans [10], is

$$
\begin{array}{ccccc}
\mathrm{U}(36) \supset & \big(\mathrm{U}_L(6) & \supset \cdots \supset & \mathrm{O}_L(3)\big) & \\
\downarrow & \downarrow & & \downarrow & \\
[N] & [N_1\ldots N_6] & & L & \\
\end{array}
$$

$$
\begin{array}{ccccc}
\otimes \; \big(\mathrm{U}_{TS}(6) & \supset \mathrm{SU}_{TS}(4) & \supset \mathrm{SU}_T(2) & \otimes & \mathrm{SU}_S(2)\big) \\
\downarrow & \downarrow & \downarrow & & \downarrow \\
[N_1\ldots N_6] & (\lambda\mu\nu) & T & & S
\end{array}
\quad , \qquad (2)
$$

where the orbital angular momentum L must be coupled with spin S to total angular momentum J (not shown). The novel element with respect to IBM-3 is the appearance of spin algebras, notably of Wigner's $\mathrm{SU}_{TS}(4)$. It can be shown [15] that, if L and S of the bosons correspond to the orbital angular momentum and spin (which in general is not the case), Wigner's supermultiplet algebra, composed of $\sum_i \vec{\tau}_i$, $\sum_i \vec{\sigma}_i$, and $\sum_i \vec{\tau}_i\vec{\sigma}_i$, maps exactly onto $\mathrm{SU}_{TS}(4)$ of (2). This then justifies the choice of basis (2) as a reasonable *ansatz*.

The states of IBM-3 are a subset of those in IBM-4. The precise relationship cannot be established via the classification (2) but rather via one in which the isospin-spin reduction is replaced by

$$\begin{array}{ccccccccc} \mathrm{U}_{TS}(6) & \supset & \mathrm{SU}_T(3) & \otimes & \mathrm{SU}_S(3) & \supset & \mathrm{SU}_T(2) & \otimes & \mathrm{SU}_S(2) \\ \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\ [N_1 \dots N_6] & & (\lambda_T\mu_T) & & (\lambda_S\mu_S) & & T & & S \end{array} . \qquad (3)$$

In Table I the IBM-3 isospin classification (1) is shown for one and two bosons as well as the corresponding IBM-4 isospin-spin classifications for the $\mathrm{SU}_{TS}(4)$ [10] and the $\mathrm{SU}_T(3) \otimes \mathrm{SU}_S(3)$ reductions of (2) and (3) respectively. The essential point is that only for $\mathrm{SU}_T(3) \otimes \mathrm{SU}_S(3)$ can a direct correspondence between IBM-3 and IBM-4 be established: IBM-4 states indicated with an arrow are scalar in $\mathrm{SU}_S(3)$ $[(\lambda_S\mu_S) = (00)]$ and their $(\lambda_T\mu_T)T$ values coincide with those found in IBM-3. An example might clarify the situation further. In Wigner's classification the favoured two-boson state with $T = S = 0$ has $(\lambda\mu\nu) = (000)$. This state can be expanded in the $\mathrm{SU}_T(3) \otimes \mathrm{SU}_S(3)$ basis $|[N](\lambda_T\mu_T)T \times (\lambda_S\mu_S)S\rangle$ as

$$|[2](000)00\rangle = \sqrt{\frac{1}{2}}|[2](00)0 \times (20)0\rangle - \sqrt{\frac{1}{2}}|[2](20)0 \times (00)0\rangle. \qquad (4)$$

Only the second component on the rhs is IBM-3 like. This illustrates that the $\mathrm{SU}_{TS}(4)$ state cannot be mapped entirely onto IBM-3 but only its fraction which is scalar in $\mathrm{SU}_S(3)$.

In general, states which have an analogue in IBM-3 have $(\lambda_S\mu_S) = (00)$. The converse is not necessarily true since some $\mathrm{SU}_S(3)$ scalar states have no counterpart in IBM-3. These, however, belong to highly non-symmetric $\mathrm{U}_{TS}(6)$ [or $\mathrm{U}_L(6)$] representations and occur at much higher energy.

The implication of this analysis is that adding an interaction which removes all non-scalar $\mathrm{SU}_S(3)$ states from the low-energy spectrum will essentially reduce IBM-4 to IBM-3. The simplest example of such an interaction is the quadratic Casimir operator of $\mathrm{SU}_S(3)$, $C_2[\mathrm{SU}_S(3)]$. The physical relevance of an interaction of this type is that it determines the relative energy of the $T = 0$ and $T = 1$ collective pairs.

The effect of the $C_2[\mathrm{SU}_S(3)]$ operator can best be illustrated in odd-odd self-conjugate nuclei where the balance between $T = 0$ and $T = 1$ pairing is thought to be most crucial. The lowest $\mathrm{U}_{TS}(6)$ representation in such nuclei is symmetric, $[N_1 \dots N_6] = [N]$, and contains the favoured $\mathrm{SU}_{TS}(4)$ representation (010). Ignoring diagonal terms in T^2 and S^2, states with $(TS) = (01)$ are degenerate with those with $(TS) = (10)$. The $C_2[\mathrm{SU}_S(3)]$ operator lifts this degeneracy and its effect can be calculated by considering the hamiltonian

$$\hat{H} = a\, C_2[\mathrm{SU}_{TS}(4)] + b\, C_2[\mathrm{SU}_S(3)]. \tag{5}$$

Its eigenvalues can be obtained by diagonalisation in the basis (3) with $[N_1 \dots N_6] = [N]$, $\lambda_T + \lambda_S = N$, and $\mu_T = \mu_S = 0$. In this basis $C_2[\mathrm{SU}_S(3)]$ is diagonal with eigenvalues $\lambda_S(\lambda_S + 3)$ and the matrix elements of $C_2[\mathrm{SU}_{TS}(4)]$,

$$V^{NTS}_{\lambda_T\lambda_S\lambda'_T\lambda'_S} \equiv \langle [N](\lambda_T 0)T \times (\lambda_S 0)S | C_2[\mathrm{SU}_{TS}(4)] | [N](\lambda'_T 0)T \times (\lambda'_S 0)S \rangle, \tag{6}$$

are given by

$$\begin{aligned} V^{NTS}_{\lambda_T\lambda_S\lambda_T\lambda_S} &= 2\lambda_T\lambda_S + 3N + T(T+1) + S(S+1), \\ V^{NTS}_{\lambda_T\lambda_S\lambda_T-2\,\lambda_S+2} &= [(\lambda_T - T)(\lambda_T + T + 1)(\lambda_S - S + 2)(\lambda_S + S + 3)]^{1/2}, \\ V^{NTS}_{\lambda_T\lambda_S\lambda_T+2\,\lambda_S-2} &= [(\lambda_T - T + 2)(\lambda_T + T + 3)(\lambda_S - S)(\lambda_S + S + 1)]^{1/2}. \end{aligned} \tag{7}$$

We note that these expressions can be used to find transformation brackets from the $\mathrm{SU}_{TS}(4)$ to the $\mathrm{SU}_T(3) \otimes \mathrm{SU}_S(3)$ basis in closed form [17].

The results of the diagonalisation are shown in Fig. 1(a-c). Quantities are plotted as a function of the ratio b/a of coefficients in (5) with $b/a = 0$ corresponding to the SU(4) limit. Figure 1(a) shows the $T = 0$ and $T = 1$ single boson energies. The expectation value of $C_2[\mathrm{SU}_S(3)]$ is zero for $S = 0$ and hence the $T = 1$ boson energy is unaffected by the chosen symmetry breaking term, while the energy of the $T = 0$ boson, with $S = 1$, has a linear dependence on b/a. Thus a shift between the energies of the $T = 0$ and $T = 1$ correlated pair states has been introduced. In Fig. 1(b) the effect of this shift on the lowest $T = 0$ and $T = 1$ states in an odd-odd nucleus with $N = 5$ is illustrated. The ground-state configuration changes at $b/a = 0$. The corresponding pair structure of the two states is shown in Fig. 1(c) in terms of the quantity $f(T = 0)$ which represents the number of $T = 0$ bosons in the state as a fraction of the total boson number N, $f(T = 0) \equiv \langle \hat{N}(T = 0) \rangle / N$.

The change in ground state which occurs at $b/a = 0$ is accompanied by a discontinuous change in $f(T=0)$, which jumps from $(3N-3)/8N$ to $(5N+3)/8N$.

A more complete picture of the pair structure in $N = Z$ nuclei is given in Fig. 2 where the quantity $f(T=0)$ is shown as a function of both the boson number N and the ratio b/a. The seesaw structure of the surface is an even-even/odd-odd effect. It is always present in the SU(4) limit ($b/a = 0$) irrespective of whether the $T = 0$ or $T = 1$ states are considered in the odd-odd nuclei. If one of the pairing modes becomes dominant the even-even/odd-odd staggering disappears for the ground state which corresponds, in the odd-odd nucleus, to $T = 0$ for $b/a \ll 0$ and to $T = 1$ for $b/a \gg 0$.

In summary, the pair structure of the ground state of $N = Z$ nuclei depends on the relative energies of the $T = 0$ and $T = 1$ collective pairs; if one mode is greatly favoured, an almost pure pair structure, in terms of the isospin of the pairs, results. More realistically, if the two basic modes compete and have comparable energy, the resulting pair structure is a mixture of $T = 0$ and $T = 1$. In the particular case of degeneracy, corresponding to SU(4) symmetry, the ground state of $N = Z$ nuclei contains 50% of each pair type.

This result differs from that of the earlier HFB treatments [3] where the different pairing solutions for $T = 1, M_T = \pm 1$, $T = 1, M_T = 0$ and $T = 0$ did not mix. However, the current conclusion is supported by a study of the mixing of nn, np, and pp $T = 1$ pairs in the framework of a simple seniority-like model of the pairing interaction [5], which showed that both the simple model and more detailed shell-model Monte-Carlo calculations for some specific $N = Z$ nuclei in the $f_{7/2}$ subshell also contain the mixing of pair types and the characteristic even-even to odd-odd staggering in the expectation values for np pairs. Most importantly, the results presented here have been obtained in a fully isospin invariant collective model, incorporating both $T = 0$ and $T = 1$ modes, and provide a quantitative link between the energy separation of the elemental pair states and the eventual pair structure of the nuclear ground state. Moreover, the method used to forge this link has also established, for the first time, a one-to-one correspondence between the IBM-3 and IBM-4 bases. Thus a natural way has been found to break the Wigner isospin-spin symmetry contained in IBM-4.

Over the past two decades, the IBM-1 and IBM-2 frameworks have been applied extensively to describe the collective structure of a wide range of nuclei albeit, in common with other models, without the explicit inclusion of isospin. One of the most powerful and characteristic features of the approach has been the existence of dynamical symmetries and their use as benchmarks in defining the structure and the hamiltonian in a realistic calculation. The current study now points the way to developing a similar capability to treat the collective structure along the $N \sim Z$ line in a quantitative framework based on dynamical symmetries which incorporate isospin.

We wish to thank J.P. Elliott and F. Iachello for inspiring discussions. This work is supported by the Franco-British Alliance grant PN 96-058 and by the EU grant CHGE-CT-94-00-56.

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TABLES

TABLE I. IBM-3 isospin and IBM-4 isospin-spin classifications for one and two bosons.

	IBM-3			IBM-4				
					$SU_{TS}(4)$		$SU_T(3)\otimes SU_S(3)$	
N	$[N_1N_2N_3]$	$(\lambda_T\mu_T)$	T	$[N_1\ldots N_6]$	$(\lambda\mu\nu)$	(TS)	$(\lambda_T\mu_T)\times(\lambda_S\mu_S)$	(TS)
1	[1]	(10)	1	[1]	(010)	(01)(10)	(00)×(10)	(01)
							(10)×(00)←	(10)
2	[2]	(20)	0,2	[2]	(000)	(00)	(00)×(20)	(00)(02)
					(020)	(00)(02)(11)(20)	(10)×(10)	(11)
							(20)×(00)←	(00)(20)
	[11]	(01)	1	[11]	(101)	(01)(10)(11)	(00)×(01)	(01)
							(01)×(00)←	(10)
							(10)×(10)	(11)

FIGURE CAPTIONS

1. (a) Energy (in units a) of the lowest $T = 0$ and $T = 1$ eigenstates as a function of the ratio b/a of coefficients in (5) for $N = 1$ boson. (b) Same for $N = 5$ bosons. (c) The pair structure $f(T = 0) \equiv \langle \hat{N}(T = 0) \rangle / N$ as a function of b/a for the lowest $T = 0$ and $T = 1$ eigenstates of (5) for $N = 5$ bosons.

2. The pair structure $f(T = 0) \equiv \langle \hat{N}(T = 0) \rangle / N$ as a function of b/a and boson number N for the lowest $T = 0$ and $T = 1$ eigenstates of (5). In the upper part the entire surface is shown for $1 \leq N \leq 10$ and $-1 \leq b/a \leq 1$; in the lower part intersections at $b/a = 1$, 0, and -1 are displayed. For odd-odd nuclei (N odd) the pair structure of the lowest $T = 0$ state is given on the left while that of the $T = 1$ state is shown on the right; for even-even nuclei (N even) both sides give the pair structure of the lowest $T = 0$ state.

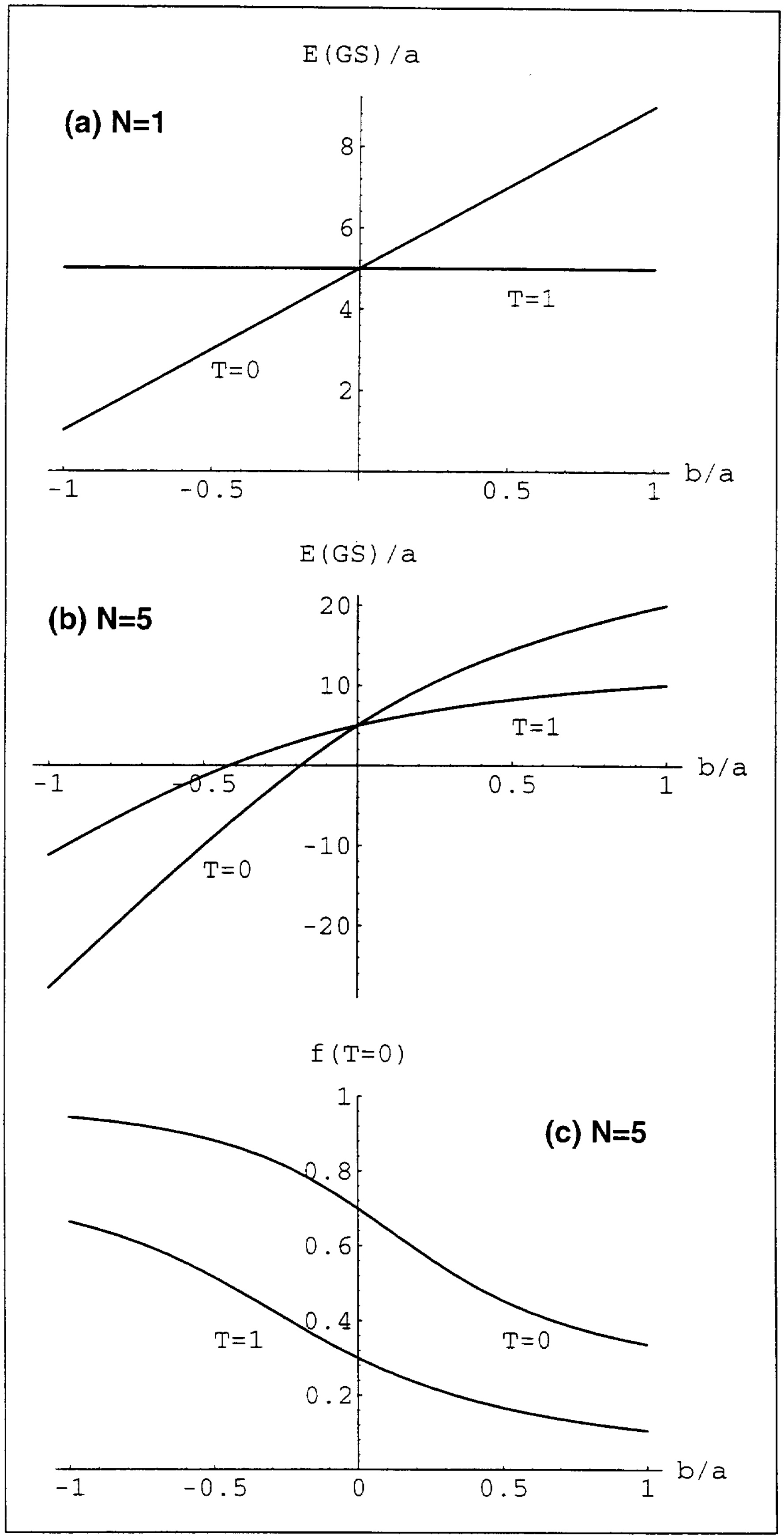
E(GS)/a
(a) N=1
8
6
4
2
T=1
T=0
-1
-0.5
0.5
1
b/a
E(GS)/a
(b) N=5
20
10
T=1
-1
-0.5
0.5
1
b/a
-10
T=0
-20
f(T=0)
1
(c) N=5
0.8
0.6
0.4
T=1
T=0
0.2
-1
-0.5
0
0.5
1
b/a

Figure 1

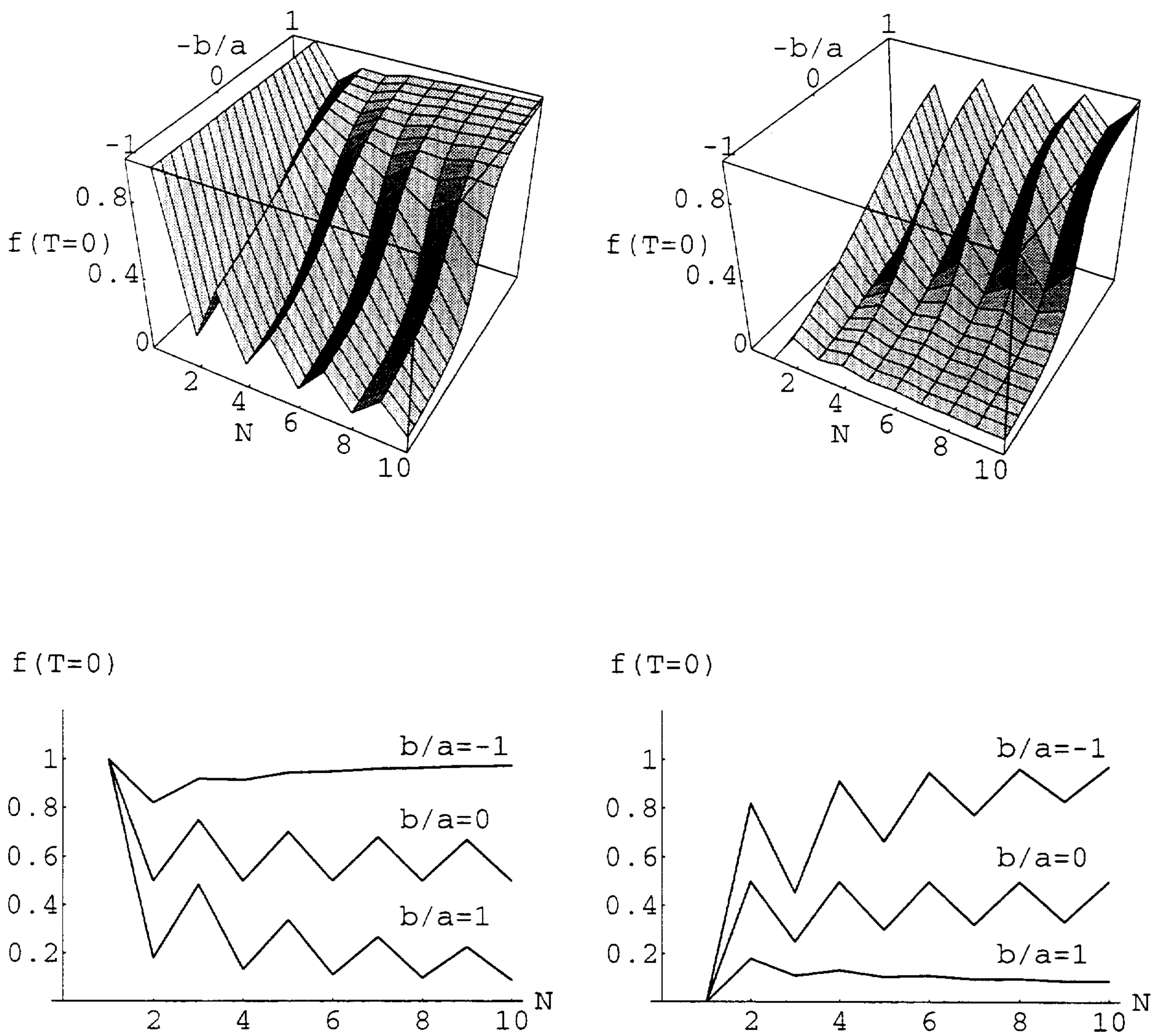

Figure 2