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Abstract

It is argued that mass anomalies at the $N \sim Z$ line are associated with SU(4) isospin–spin symmetry. Drawing on these arguments, a Weizsäcker-type nuclear mass formula is investigated which has the eigenvalue of the quadratic Casimir operator of SU(4) as a Wigner term. This SU(4)-based mass formula yields a better agreement than the one with the usual Wigner term $|N - Z|/A$. In addition, the SU(4) eigenvalue expression adequately replaces the usual pairing term of the Weizsäcker formula giving a lower overall rms deviation than the latter.

I. SU(4) SYMMETRY

In 1932, shortly after the discovery of the neutron by Chadwick, Heisenberg proposed the idea of isospin symmetry in nuclei [1]. At the basis of this symmetry is the observed near-equality of the masses of the neutron and proton, and the additional assumption that nuclear forces are independent of the character of the nucleon. The idea can be cast in a mathematically elegant form by assigning to a nucleon an isospin of $T = 1/2$ with the isospin projections $M_T = \pm 1/2$ for the two charge states of the nucleon. These seemingly innocuous premisses have far-reaching consequences, in particular as regards the classification of the

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eigenstates of the nuclear many-body system. Introducing the isospin generators

$$\vec{T} = \frac{1}{2}\sum_i \vec{\tau}(i), \tag{1}$$

where $\vec{\tau}$ are Pauli matrices acting in isospin space and the summation extends over all particles in the nucleus, the condition of isospin symmetry is conveniently summarised as

$$[H_{\text{nuclear}}, \vec{T}] = 0. \tag{2}$$

It follows that the eigenstates of H_{nuclear} are characterised by a total isospin T and its projection M_T and, furthermore, that states with the same T but different M_T are degenerate. This degeneracy is lifted by the Coulomb interaction among the protons since

$$[H_{\text{Coulomb}}, T_{\pm}] \neq 0. \tag{3}$$

The breaking of isospin symmetry caused by the Coulomb interaction is predominantly of *dynamical character*, that is, the degeneracy is lifted but T remains an approximate quantum number. The justification of the approximate validity of the isospin quantum number, the Coulomb interaction notwithstanding, has been extensively discussed, especially by Lane and Soper in the sixties (see, e.g., [2] and references therein).

Five years after Heisenberg's suggestion Wigner and, independently of him, Hund proposed a beautiful extension by assuming nuclear forces to be *isospin* as well as *spin* invariant [3,4]. This requires

$$[H_{\text{nuclear}}, \vec{T}] = [H_{\text{nuclear}}, \vec{S}] = [H_{\text{nuclear}}, \vec{\vec{Y}}] = 0, \tag{4}$$

where $\vec{S}$ is a spin vector and $\vec{\vec{Y}}$ is a spin–isospin (Gamow–Teller) double vector,

$$\vec{S} = \frac{1}{2}\sum_i \vec{\sigma}(i), \qquad \vec{\vec{Y}} = \frac{1}{2}\sum_i \vec{\sigma}(i)\vec{\tau}(i). \tag{5}$$

Eigenstates of a nuclear hamiltonian satisfying the invariances (4) are classified by the labels

$$|(\lambda\mu\nu)LSJM_JTM_T\rangle. \tag{6}$$

One recognises the isospin labels T and M_T introduced previously and the labels from an LS coupling scheme (the total orbital angular momentum L, the total spin S and their coupling to total angular momentum J and projection M_J), familiar from the electron classification in atomic physics. In (6) also appear the quantum numbers $(\lambda\mu\nu)$; these are associated with the isospin–spin SU(4) algebra generated by $\vec{S}$, $\vec{T}$ and $\vec{\vec{Y}}$ and are of pivotal importance in Wigner's consideration.

To understand the relevance of the labels $(\lambda\mu\nu)$ one should note that the invariances (4) imply that any A-particle eigenstate can be written as

$$\Psi(1,\ldots,A) = \sum_{M_L M_S} (LM_L\, SM_S|JM_J)\Psi_{LM_L}(\vec{r}_1,\ldots,\vec{r}_A)\Psi_{TM_TSM_S}(1,\ldots,A), \tag{7}$$

where $\Psi_{LM_L}(\vec{r}_1,\ldots,\vec{r}_A)$ depends on the *spatial* coordinates while $\Psi_{TM_TSM_S}(1,\ldots,A)$ is dependent on *isospin and spin* coordinates. The complete wavefunction $\Psi(1,\ldots,A)$ is antisymmetric but $\Psi_{LM_L}(\vec{r}_1,\ldots,\vec{r}_A)$ and $\Psi_{TM_TSM_S}(1,\ldots,A)$ separately are not. For two particles, for example, the spatial part of the wavefunction can be symmetric and the isospin–spin part antisymmetric or *vice versa.* It turns out in this case that the $(\lambda\mu\nu)$ labels can be either (010) or (200) of which the first is symmetric (antisymmetric) while the second is antisymmetric (symmetric) in space (isospin–spin). The generalisation of this example to many particles leads to the result that the SU(4) quantum numbers $(\lambda\mu\nu)$ *specify the way in which the overall antisymmetry is distributed over the spatial and spin–isospin parts of the nuclear wavefunction.*

States that are spatially symmetric are energetically favoured due to the short-range nature of the residual interaction. To see this point one may return to the example of two particles and consider an extreme form of a short-range interaction, namely a delta interaction. It can be shown that (see, e.g., chapter 11 of [5])

$$\langle(200)LSJM_JTM_T|\delta(\vec{r}_1-\vec{r}_2|(200)LSJM_JTM_T\rangle = 0, \tag{8}$$

that is, the interaction matrix element vanishes identically for $(\lambda\mu\nu) = (200)$ while it is attractive in the spatially symmetric case $(\lambda\mu\nu) = (010)$. Again, this result can be generalised to A particles. The space exchange operator $\sum_{i<j} P_{ij}$, where

$$P_{ij}\Psi(\ldots,\vec{r}_i,\ldots,\vec{r}_j,\ldots) = \Psi(\ldots,\vec{r}_j,\ldots,\vec{r}_i,\ldots), \tag{9}$$

measures the symmetry of the spatial part of the wavefunction since it gives $+1$ in a symmetric and -1 in an antisymmetric two-particle state. Its eigenvalue acting on a general A-particle state is (see chapter 29 of [5])

$$a + b\,g(\lambda\mu\nu), \tag{10}$$

where $g(\lambda\mu\nu)$ is the eigenvalue of $C_2[\mathrm{SU}(4)]$, the quadratic operator of SU(4),

$$g(\lambda\mu\nu) \equiv 3\lambda(\lambda+4) + 3\nu(\nu+4) + 4\mu(\mu+4) + 4\mu(\lambda+\nu) + 2\lambda\nu, \tag{11}$$

and a and b are coefficients which depend on the particle number A.

The procedure to test SU(4) symmetry from binding energies is as follows [3,6]. For a given nucleus one assumes that its ground state has $T = |T_z| = \frac{1}{2}|N-Z|$ (with the possible exception of $N = Z$ odd–odd nuclei) and from there one works out the $(\lambda\mu\nu)$ values that maximise spatial symmetry [or minimise $g(\lambda\mu\nu)$] and are compatible with T. In summary, Wigner's supermultiplet theory thus predicts the nuclear binding energy $B(N,Z)$ in lowest order to be

$$B(N,Z) = a + b\,g(\lambda\mu\nu), \tag{12}$$

where a and b are coefficients (b is negative) depending smoothly on mass. The labels $(\lambda\mu\nu)$ determine the favoured SU(4) representation and depend solely on the isospin of the nucleus in its ground state (see, for example, chapter 29 of [5]). They are given by

$$(\lambda\mu\nu) = \begin{cases} (0T0) & \text{if } N \text{ even and } Z \text{ even} \\ \left.\begin{matrix} (1,T-\frac{1}{2},0) \\ (0,T-\frac{1}{2},1) \end{matrix}\right\} & \left\{\begin{matrix} \text{if } N \text{ even and } Z \text{ odd} \\ \text{if } N \text{ odd and } Z \text{ even} \end{matrix}\right. \\ (010) & \text{if } N \text{ odd and } Z \text{ odd}, \quad N = Z \\ (1,T-1,1) & \text{if } N \text{ odd and } Z \text{ odd}, \quad N \neq Z \end{cases} \quad . \tag{13}$$

where it is assumed that $T = |T_z| = \frac{1}{2}|N-Z|$. For odd-mass nuclei the two combinations $(1,T-\frac{1}{2},0)$ and $(0,T-\frac{1}{2},1)$ occur; since they give the same eigenvalue $g(\lambda\mu\nu)$ we need not distinguish here between the two cases.

II. SU(4) SYMMETRY AND MASSES OF $N \sim Z$ NUCLEI

When Wigner and Hund made their suggestion, insufficient data were available on nuclear masses to test the SU(4) symmetry hypothesis. Many years later, in 1963, Franzini and Radicati [7] suggested the use of a ratio $R(T_z)$ of mass differences involving four isobaric nuclei with different isospin projections T_z and showed that the values agree rather well with the SU(4) predictions for nuclei up to $A \sim 110$. However, it was demonstrated subsequently [8] that this ratio $R(T_z)$ is not very sensitive to SU(4) symmetry breaking and hence it does not provide a critical test of this symmetry. Recently, a different combination of masses was proposed as a measure of SU(4) symmetry [9]. For even–even nuclei it is the double binding energy difference

$$\delta V_{\rm np}(N,Z) \equiv \frac{1}{4}\Big([B(N,Z) - B(N-2,Z)] - [B(N,Z-2) - B(N-2,Z-2)]\Big), \qquad (14)$$

which was used previously by Brenner *et al.* [10] to extract the empirical neutron–proton (np) interaction strength of the last neutron with the last proton. A remarkable feature of the mass combination (14) is its enhancement for $N = Z$ nuclei. The main purpose of [9] was to show that the $N = Z$ enhancement of $|\delta V_{\rm np}|$ is an unavoidable consequence of Wigner's SU(4) symmetry and that the degree of the enhancement provides a sensitive test of the quality of the symmetry itself.

Since it is central to the nuclear mass formula presented in section III, it is worthwhile to reiterate here the gist of the argument of [9] by way of an example. A representative sample of the data is shown in figure 1a which gives $\delta V_{\rm np}(N,Z)$ (where known) for the *sd* shell. While for $N \neq Z$ the np interaction strength—which equals $-\delta V_{\rm np}(N,Z)$—is roughly constant and of the order of -1 MeV, the dramatic enhancement of $|\delta V_{\rm np}|$ occurring for $N = Z$ is clearly evident. Its explanation in terms of Wigner's supermultiplet theory is as follows. For even–even nuclei the favoured SU(4) representation is $(0T0)$, where T is the isospin of the ground state and, in the approximation (12), the binding energy is then $a + b\,g(0T0)$. Assuming constant coefficients a and b for the four nuclei in (14), a simple expression is found for $\delta V_{\rm np}$, namely

$$\delta V_{\rm np}(N,Z)/b = \begin{cases} \frac{1}{4}[g(000) - g(010) - g(010) + g(000)] = -10, & N = Z \\ \frac{1}{4}[g(0T0) - g(0,T-1,0) - g(0,T+1,0) + g(0T0)] = -2, & N \neq Z \end{cases} . \quad (15)$$

Wigner's supermultiplet theory in its *simplest* form (i.e., without symmetry breaking—dynamical or otherwise—in isospin and/or spin) therefore predicts $|\delta V_{\rm np}|$ to be five times bigger for $N = Z$ than for $N \neq Z$. This result is displayed in figure 1b.

The relevance of this result for the subsequent construction of an additional term in the nuclear mass formula is twofold. First, it shows that masses of $N \sim Z$ nuclei have peculiar properties which—as will be discussed below—often elude standard mass formulas if no additional Wigner term is considered. Second, judging from double binding energy differences, the Wigner interaction C_2[SU(4)] seems to account for these peculiar $N \sim Z$ properties, albeit in an approximate manner since it is evidently clear from figure 1 that the data substantially deviate from the simple SU(4) picture. The combination of these two observations leads in a natural way to the consideration of a C_2[SU(4)] Wigner term in the nuclear mass formula.

III. MASS FORMULAS WITH A WIGNER TERM

Two standard approaches exist that attempt to give a systematic account of all nuclear masses [or nearly all since the very light nuclei ($A \leq 16$) are often excluded]: the Extended Thomas–Fermi (ETF) approximation of [12] and the finite-range droplet model (FRDM) of Möller and Nix (a recent version is described in [13]). The ETF model gives a description of average trends of static nuclear properties and provides a link between Hartree–Fock theory and phenomenological models such as the droplet model. A Wigner term cannot be included in the ETF approach. This is recognised by the authors of [14] who claim this to be the reason that even–even $N = Z$ nuclei are systematically underbound in the ETF theory by about 2 MeV, an effect that persists to a lesser extent in odd–odd $N = Z$ or odd-mass $N = Z \pm 1$ nuclei. The FRDM mass formula is in essence an emanation of the Weizsäcker nuclear formula, refined to take account of shell effects etc. In this case a Wigner term can be considered and it consists of a function $f_W(N, Z)$ that peaks at $N = Z$. Two forms are routinely taken, namely

$$a'_W \frac{|N-Z|}{A} \tag{16}$$

and

$$a''_{W_1} \exp\left(-a''_{W_2} \frac{|N-Z|}{A}\right), \tag{17}$$

containing one and two parameters, respectively. It is clear that adding either (16) or (17) to a nuclear mass formula will improve its description of $N \sim Z$ nuclei. Both terms lack, however, a clear physical justification and, because of that, an extrapolation to unknown nuclei based on fits to known binding energies, is perilous.

We propose here to use instead a Wigner term of the type

$$a_W \frac{g(\lambda\mu\nu)}{A^{\gamma_W}}, \tag{18}$$

where $g(\lambda\mu\nu)$ is given by (11) and $(\lambda\mu\nu)$ is the favoured SU(4) representation specified in (13). The difference with the previous Wigner terms is that the origin of $g(\lambda\mu\nu)$ is understood as a result of the short-range character of the residual interaction. Since the goodness of SU(4) symmetry decreases as the nuclear mass increases, an A-dependent coefficient in front of $g(\lambda\mu\nu)$ is taken in (18). Although it is expected therefore that $\gamma_W > 0$, the exact mass dependence is not known at this point and γ_W is taken as a parameter.

In this paper the results obtained with the SU(4) Wigner term (18) are compared to those found with the standard Wigner terms, specifically the linear one, (16). The following expression is used for the binding energy $B(N, Z)$ of a nucleus with N neutrons and Z protons:

$$B(N,Z) = a_V A + a_S A^{2/3} + a_C \frac{Z^2}{A^{1/3}} + a_I \frac{(N-Z)^2}{A} + a_P \frac{\delta(A)}{A^{3/4}} + \begin{cases} a_W \dfrac{g(\lambda\mu\nu)}{A^{\gamma_W}} \\ \\ a'_W \dfrac{|N-Z|}{A} \end{cases} . \tag{19}$$

Equation (19) contains the usual terms of the Weizsäcker formula, that is, the volume, surface, Coulomb, symmetry and pairing terms [with $\delta(A) = +1, 0, -1$ for even–even, odd-mass and odd–odd nuclei, respectively], complemented with a choice of two different Wigner terms. It is clear that (19) can only give a rudimentary description of nuclear masses and is no match for the models proposed in [12] or [13]. Most conspicuously, shell effects are absent from (19). The primary purpose of the work presented here is not so much to fit nuclear masses with high accuracy but rather to compare the relative merits of the different Wigner terms. This is discussed in the next section.

IV. RESULTS

To evaluate the merits of the different Wigner terms, we have carried out four different fits to measured nuclear masses:

- W0: Weizsäcker formula without Wigner term ($a_{\mathrm{W}} = a'_{\mathrm{W}} = 0$);
- W1: Weizsäcker formula with a linear Wigner term ($a'_{\mathrm{W}} \neq 0$);
- W2: Weizsäcker formula with a mass-independent SU(4) Wigner term ($a_{\mathrm{W}} \neq 0$ but $\gamma_{\mathrm{W}} = 0$);
- W3: Weizsäcker formula with a mass-dependent SU(4) Wigner term ($a_{\mathrm{W}} \neq 0$ and $\gamma_{\mathrm{W}} \neq 0$).

As already stated in the previous section, no shell corrections are considered here, inevitably reducing the accuracy of the fits. In order to account to some degree for the absence of any shell-correction terms, several regions of nuclei have been fitted separately. These are: (i) *sd*-shell nuclei ($8 \leq N, Z \leq 20$); (ii) *pf*-shell nuclei ($20 \leq N, Z \leq 40$); (iii) *sd*- and *pf*-shell nuclei ($8 \leq N, Z \leq 40$); (iv) all nuclei with known mass.

Results (coefficients in the mass formula and rms deviations) of the different fits to the different regions of nuclei are shown in table I. If a coefficient is kept fixed at zero, this is indicated by —. A few observations are in order. In the pf shell the rms deviation is the same (1.59 MeV) for the linear and the SU(4) Wigner terms but otherwise the latter is significantly superior. The fits W2 and W3 yield approximately the same rms deviation although W3 involves the additional parameter $\gamma_{\rm W}$. Thus, surprisingly, on the basis of measured nuclear binding energies, no A dependence of the SU(4) term in the mass formula seems required. This might be a consequence of the interdependence of the various terms in (19). Specifically, it is seen from table I that the addition of the SU(4) Wigner term to the usual Weizsäcker formula has little effect on the values of $a_{\rm V}$, $a_{\rm S}$ and $a_{\rm C}$. The volume, surface and Coulomb terms describe effects that have nothing to do with SU(4) (which is a spatial symmetry effect) and consequently the influence of SU(4) on these terms is small. On the other hand, the symmetry and pairing coefficients do change substantially when the SU(4) term is added and particularly so if the latter is made mass dependent.

The preceding observation suggests, therefore, that the symmetry and/or pairing terms conceivably might be replaced by the SU(4) Wigner term. Concerning these replacements, three different fits have been carried out:

- W4: Weizsäcker formula without a symmetry term ($a_{\rm I} = 0$) and with a mass-dependent SU(4) Wigner term ($a_{\rm W} \neq 0$ and $\gamma_{\rm W} \neq 0$);
- W5: Weizsäcker formula without a pairing term ($a_{\rm P} = 0$) and with a mass-dependent SU(4) Wigner term ($a_{\rm W} \neq 0$ and $\gamma_{\rm W} \neq 0$);
- W6: Weizsäcker formula without a symmetry nor pairing term ($a_{\rm I} = a_{\rm P} = 0$) and with a mass-dependent SU(4) Wigner term ($a_{\rm W} \neq 0$ and $\gamma_{\rm W} \neq 0$).

The results of the different fits to the different regions of nuclei are shown in table II, where also the results of the fit with the Weizsäcker formula without Wigner term (W0) are repeated for easy comparison. It is seen (fit W5) that the SU(4) Wigner term indeed provides an adequate replacement of the usual pairing term in the Weizsäcker formula. If a mass dependence of $\gamma_{\rm W} \approx 0.5$ is taken (i.e., an approximate $A^{-1/2}$ dependence) comparable or significantly lower rms deviations are obtained (e.g., for all nuclei, 2.88 MeV instead of 3.46 MeV for the usual Weizsäcker formula).

The results of replacing the symmetry term by the SU(4) Wigner term are shown in fit W4 of Table II. In this case the rms deviation is comparable with the one obtained with the simple Weizsäcker formula. Finally, the rms deviation in W6, where the symmetry and pairing terms are replaced by the SU(4) Wigner term, is usually higher than in W0. This finding contradicts the study of Cauvin *et al.* [15] who claim that both the symmetry and the pairing term can be replaced by the SU(4) Wigner term while maintaining a good description of nuclear binding energies. This difference in conclusion might be a consequence of (i) the greater number of nuclei included in the present fit (since the analysis of [15], the number of known masses has almost doubled) and (ii) the fixed coefficient in the Coulomb term taken in [15], $a_C = 0.72$ MeV, which is different from the fitted values found here.

In figures 2 and 3 selected results of the fits are shown in two representative cases: the nickel ($Z = 28$) and tin ($Z = 50$) isotopes. To amplify differences between the fits, deviations from the standard Weizsäcker formula W0 are given in three cases: (i) for measured masses, (ii) for the Weizsäcker formula W1 with a linear Wigner term, and (iii) for the Weizsäcker formula W5 without pairing but with an SU(4) Wigner term. The extent that a particular fit reproduces the solid curve thus indicates the goodness of that fit. Two points are noteworthy. First, extrapolation towards the lines of particle stability can be significantly different depending on which Wigner term is taken. Since at this point in neither case the N, Z dependence of the coefficient is known, both extrapolations should be taken with extreme care. Second, although all mass formulas exhibit large deviations from the data in particular at shell closures, the mass formula W5 with the SU(4) Wigner term is seen to reproduce best the shape of the 'experimental' line.

V. SUMMARY AND OUTLOOK

We have stressed in this contribution the ideas at the basis of SU(4) symmetry and have shown how these lead in a natural way to an SU(4)-type Wigner term in the nuclear mass formula. A Weizsäcker mass formula with an SU(4) Wigner term gives a lower rms deviation than the one obtained with the traditional Wigner term of the type $|N-Z|/A$. More importantly, however, is that the SU(4) term has a microscopic foundation which is lacking for other Wigner terms. This conceivably might be of use to find its mass-dependence: it is well-known that SU(4) is increasingly broken as A increases and one expects, therefore, the gradual disappearance of the SU(4) term. Moreover, since SU(4) is an spatial-symmetry effect arising from the short-range character of the residual interaction, a gradual decrease is expected as the nuclear interaction diludes with the increase in size of the nucleus. Although these arguments are supported by the results of the fits, $\gamma_{\mathrm{W}} > 0$, a microscopic estimate of this coefficient is still lacking.

The currently proposed mass formula is too simple to give an accurate description of nuclear masses. In particular, shell corrections and deformation effects are required in addition to what is proposed here, such as has been done in [15]. There is, however, a difference between the approach advocated by Cauvin *et al.*, who associate SU(4) with a quartetting effect, and ours, where the association with the underlying fermion symmetry has been a direct inspiration for the construction of the mass formula. We believe, therefore, that before considering in detail shell corrections and the like, it would be more appropriate to explore the existence of 'hybrid' SU(4) symmetries (such as a pseudo-SU(4) symmetry constructed from isospin and pseudo-spin [9] or boson SU(4) symmetries as they arise in the interacting boson model [16]) and their bearing on nuclear masses. What we have shown so far demonstrates, once more, how arguments based on simple principles of symmetry lead to concrete results in the field of nuclear structure.

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TABLES

TABLE I. Coefficients in the mass formula and associated rms deviation

Fit	a_V	a_S	a_C	a_I	a_P	a'_W	a_W	γ_W	rms
				$8 \leq N, Z \leq 20$ (123 nuclei)					
W0	15.05	−16.46	−0.60	−19.10	22.32	—	—	—	1.58
W1	14.81	−15.62	−0.59	−16.48	22.80	−23.75	—	—	1.43
W2	15.19	−16.43	−0.61	−10.65	9.76	—	−0.17	—	1.14
W3	15.17	−16.37	−0.61	−10.52	9.55	—	−0.19	0.03	1.14
				$20 \leq N, Z \leq 40$ (200 nuclei)					
W0	14.56	−14.42	−0.62	−20.77	29.33	—	—	—	1.62
W1	14.87	−15.04	−0.65	−20.03	29.78	−26.70	—	—	1.59
W2	15.02	−15.57	−0.66	−17.24	23.71	—	−0.051	—	1.59
W3	15.32	−16.06	−0.70	−12.34	14.61	—	−1.15	0.56	1.56
				$8 \leq N, Z \leq 40$ (394 nuclei)					
W0	14.85	−15.66	−0.61	−20.04	24.43	—	—	—	1.88
W1	14.74	−15.24	−0.61	−18.65	24.97	−16.45	—	—	1.82
W2	15.29	−16.66	−0.66	−17.59	21.15	—	−0.046	—	1.76
W3	15.16	−15.98	−0.66	−12.09	12.91	—	−1.04	0.56	1.65
				All nuclei with known mass (1909 nuclei)					
W0	15.47	−16.74	−0.70	−22.68	19.49	—	—	—	3.46
W1	15.76	−17.91	−0.71	−23.88	20.74	28.45	—	—	3.15
W2	15.24	−16.44	−0.67	−18.92	17.73	—	−0.015	—	2.87
W3	15.26	−16.43	−0.67	−15.52	12.95	—	−0.35	0.46	2.84

[a]All parameters are in MeV, except γ_W which is dimensionless.

[b]W0: Weizsäcker formula without Wigner term; W1: Weizsäcker formula with a linear Wigner term; W2: Weizsäcker formula with a mass-independent SU(4) Wigner term; W3: Weizsäcker formula with a mass-dependent SU(4) Wigner term.

TABLE II. Coefficients in the mass formula without symmetry and/or pairing term, and associated rms deviation

Fit	a_V	a_S	a_C	a_I	a_P	a_W	γ_W	rms
				$8 \le N, Z \le 20$ (123 nuclei)				
W0	15.05	−16.46	−0.60	−19.10	22.32	—	—	1.58
W4	14.64	−14.46	−0.60	—	− 6.42	−1.43	0.41	1.55
W5	15.00	−15.78	−0.61	− 7.65	—	−0.45	0.21	1.21
W6	14.67	−14.56	−0.60	—	—	−1.38	0.40	1.60
				$20 \le N, Z \le 40$ (200 nuclei)				
W0	14.56	−14.42	−0.62	−20.77	29.33	—	—	1.62
W4	16.35	−18.27	−0.81	—	− 6.35	−2.58	0.54	1.68
W5	15.73	−16.94	−0.74	− 7.74	—	−1.78	0.56	1.59
W6	16.33	−18.24	−0.80	—	—	−2.46	0.53	1.70
				$8 \le N, Z \le 40$ (394 nuclei)				
W0	14.85	−15.66	−0.61	−20.04	24.43	—	—	1.88
W4	15.17	−15.36	−0.69	—	− 4.46	−3.37	0.64	2.06
W5	15.14	−15.78	−0.67	− 9.48	—	−1.58	0.60	1.72
W6	15.17	−15.38	−0.69	—	—	−3.36	0.64	2.07
				All nuclei with known mass (1909 nuclei)				
W0	15.47	−16.74	−0.70	−22.68	19.49	—	—	3.46
W4	14.70	−14.53	−0.63	—	−13.79	−3.11	0.66	3.42
W5	15.22	−16.27	−0.67	−13.99	—	−0.58	0.52	2.88
W6	14.70	−14.60	−0.63	—	—	−2.96	0.65	3.46

[a]All parameters are in MeV, except γ_W which is dimensionless.

[b]W0: Weizsäcker formula without Wigner term; W4: Weizsäcker formula without symmetry term and with a mass-dependent SU(4) Wigner term; W5: Weizsäcker formula without pairing term and with a mass-dependent SU(4) Wigner term; W6: Weizsäcker formula without symmetry and pairing terms, and with a mass-dependent SU(4) Wigner term.

FIGURES

FIG. 1. Barchart representation of double binding energy differences (a) as observed in even–even *sd*-shell nuclei and (b) as predicted by Wigner SU(4). The data are taken from [11]; an empty square indicates that the datum is lacking. The x and y coordinates of the centre of a cuboid define N and Z and its height z defines $\delta V_{\mathrm{np}}(N, Z)$.

FIG. 2. Deviations in the masses of nickel isotopes from the simple Weizsäcker formula W0 fitted to all nuclei. Three deviations are shown: from (i) measured masses (solid), (ii) the Weizsäcker formula W1 with a linear Wigner term (dashed), and (iii) the Weizsäcker formula W5 without pairing but with an SU(4) Wigner term (dotted).

FIG. 3. Deviations in the masses of tin isotopes from the simple Weizsäcker formula W0 fitted to all nuclei. Three deviations are shown: from (i) measured masses (solid), (ii) the Weizsäcker formula W1 with a linear Wigner term (dashed), and (iii) the Weizsäcker formula W5 without pairing but with an SU(4) Wigner term (dotted).

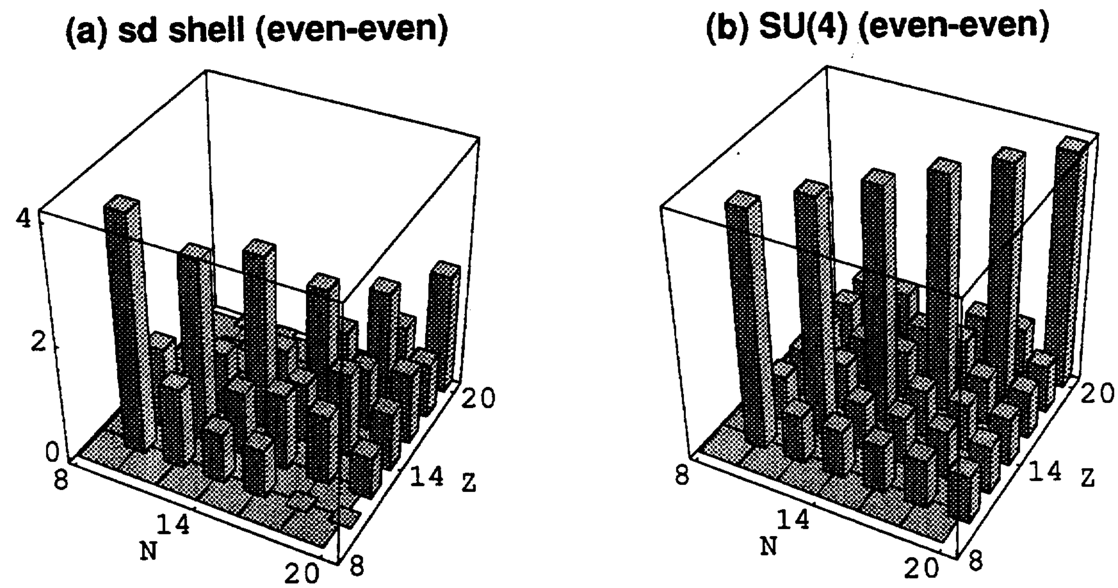
(a) sd shell (even-even)
(b) SU(4) (even-even)
4
2
0
8
14
N
20
8
14
Z
20
8
14
N
20
8
14
Z
20

Fig 1

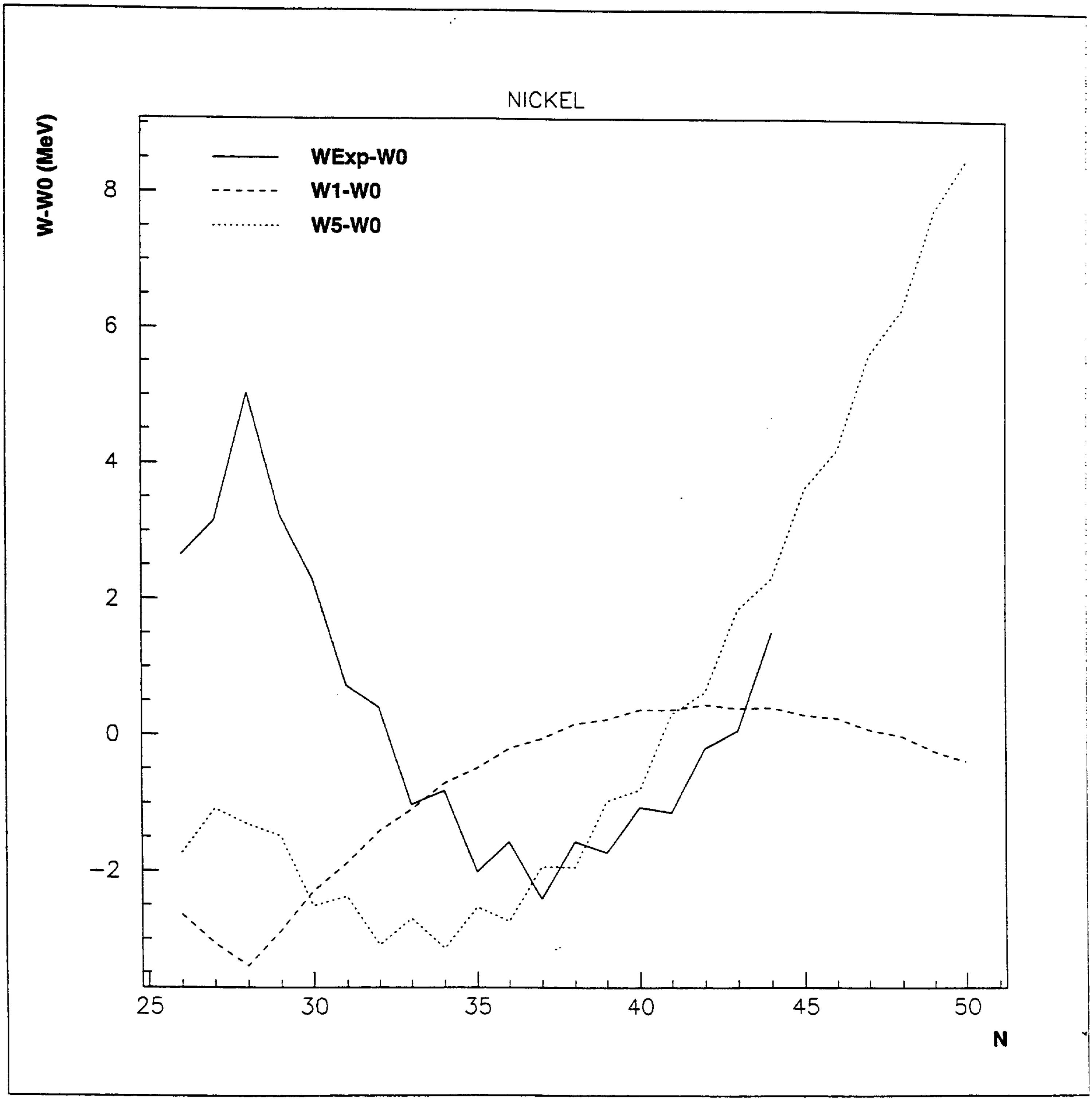

Fig. 2

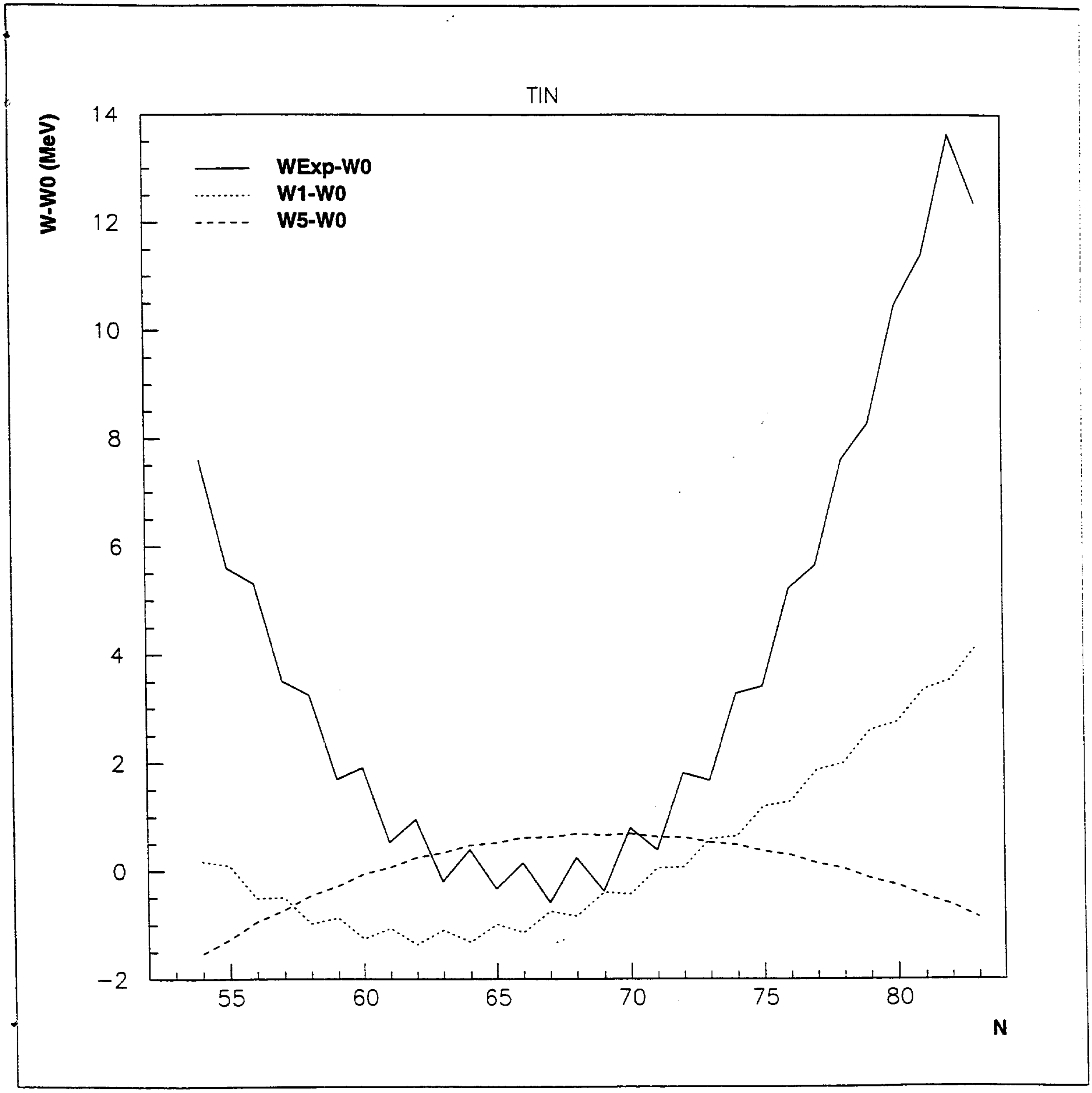

Fig. 3