Boltzmann-Langevin approaches

P. Chomaz

GANIL

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Philippe CHOMAZ
GANIL, BP 5027, 14076 CAEN Cedex 5, France
E-mail: chomaz@ganil.fr

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In this paper we summarise generalised mean-field approaches and we justify the stochastic extensions of these theories. Then, we present several test of Boltzmann-Langevin approach and we give results of first applications of this theory.

1) Introduction

In this paper we present an overview of semi-classical stochastic mean-field approaches of the Boltzmann-Langevin type. The philosophy of these approaches is based on the fact that a physical theory should be a tool to predict results of measurements performed on a wide ensemble of systems always prepared in the same way. The main problem we will face is that most of the time the information we are able to control (prepare and measure) is not enough or even is much to small compared with a full description of the system which often require an infinite number of observations. Therefore the disregarded information will strongly influence the dynamics of the pertinent degrees of freedom not only in a deterministic way by making the whole dynamics non linear but also in a stochastic way the unknown observation introducing some random terms in the evolution.

A widely used stochastic mean-field approach is the Boltzmann-Langevin theory[1-7] which can be applied in classical and semi-classical dynamics. This approach is very close from the Boltzmann theory and leads to a fluctuating collision term. The average value of this fluctuating term is nothing but the standard Boltzmann collision term while the difference in the actual collisions experienced by the various trajectories introduces a large degree of stochasticity.

In the mean-field approximation only the average evolution is considered. In the Boltzmann approaches, the collisions are taken into account but the fluctuations are averaged out regularly after each collision to be able to defined a unique trajectory. Finally, in the Boltzmann-Langevin approaches every branching induced by the collision is followed over the whole time interval. Therefore one deals with a large ensemble of trajectories.

2) Exact simulations of Boltzmann-Langevin theory.

The authors of ref.[4,6] developed an approach simply based upon the Boltzmann picture of collisions. In addition to the usual mean field evolution they consider that the inclusion of direct two-body collisions subjects the individual nucleons to irregular forces which in turn produce random changes in the one-body phase-space density f. Consequently, in such a scenario, the evolution exhibits a continual branching and the temporal distribution of f is no longer a single trajectory but an ever widening bundle of trajectories. The task is to treat this diffusive behaviour within the framework of transport theory.

The stochastic contribution to the evolution of the phase-space occupancy is relatively easy to treat when the random force is given in terms of the Uehling-Uhlenbeck collision term, which is generated by a stochastic sequence of individual transitions in the system. The mean number of transitions in which nucleons are scattered from two phase-space elements around the locations s_1, s_2, $s \equiv (\mathbf{r}, \mathbf{p})$ into two other phase-space elements around $s_{1'}$, $s_{2'}$, is given by

$$dv_{1,2;1',2'} = f_1 f_2 \bar{f}_{1'} \bar{f}_{2'} \delta(\mathbf{r}) W(\mathbf{p}) ds_1 ds_2 ds_{1'} ds_{2'} \qquad (1)$$

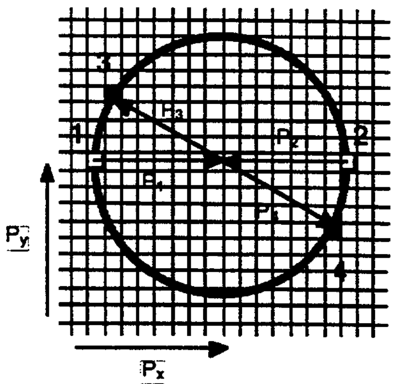

Figure 1: Schematic illustration of the kinematics of the collision of the cell 1 with the cell 2 leading to the cells 3 and 4 on the sphere of constant momentum and energy.

Here $f_i \equiv f(\mathbf{r}_i, \mathbf{p}_i)$ is the phase-space occupancy at one of the initial locations and $\bar{f}_i \equiv 1 - f(\mathbf{r}_i, \mathbf{p}_i)$ are the Pauli blocking factors expressing the availability at the final locations. Moreover, the phase-space elements $ds \equiv d\mathbf{r}d\mathbf{p} / h^D$ where D is the dimension of the physical space. The elementary transition rate W(p) incorporates energy-momentum conservation and can be related to the cross section of the colliding nucleons

$$\int W(\mathbf{p}) \frac{d\mathbf{p}_{1'} d\mathbf{p}_{2'}}{h^{2D}} = v_{1,2} \int d\Omega_{1',2'} \frac{d\sigma}{d\Omega} \qquad (2)$$

where $\Omega_{1',2'}$ represents the direction of relative motion after the collision and $v_{1',2'}$ is the relative speed of the two colliding nucleons. Since the collisions are regarded as random processes analogous to a random walk, the number of transitions actually occurring during a time step Δt fluctuates according to the corresponding Poisson distribution:

$$\nu^{(n)}_{1,2;1',2,} = \nu_{1,2;1',2,} + \Delta\nu^{(n)}_{1,2;1',2,} \quad (3)$$

where the average is nothing but the Boltzmann collision term

$$\nu_{1,2;1',2,} = \int_{\Delta 1,\Delta 2;\Delta 1',\Delta 2,} d\nu_{1,2;1',2,} \Delta t \quad (4)$$

while the fluctuation are following the Poisson relation[1]

$$\prec \Delta\nu^{(n)}_{1,2;1',2,} \Delta\nu^{(n)}_{1'',2'';1'',2''} \succ = \nu_{1,2;1',2,} \delta_{(1,2;1',2,),(1'',2'';1'',2'')} \quad (5)$$

3) Test of the lattice simulations.

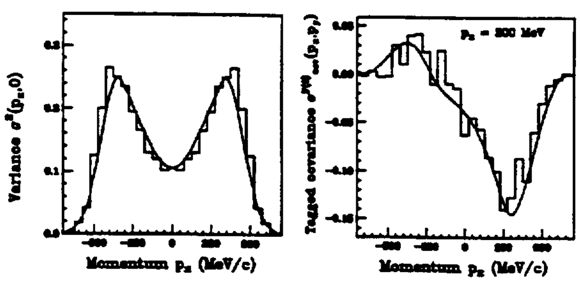

Figure 2: extracted from references[4-6] showing that the Boltzmann-Langevin approach correctly describes the thermalisation of a fermion gas. The left part shows the fluctuations of the occupation of the phase space while the left part presents the correlations of a second particle with a particle detected in the forward direction. The histogram is the asymptotic result of our Boltzmann-Langevin simulations starting from a highly non equilibrated system while the solid line presents the thermodynamical predictions.

In the case of a fermion gas we have demonstrated that the Boltzmann Langevin approach provides a good statistical limit as far as mean values, correlations and fluctuations are concerned[4,6].

Moreover, it has been shown that the Boltzmann-Langevin simulations correctly describe the dynamics of fluctuations. For example, references[4,5] study the thermalisation of two colliding Fermi fluids. We show that the fluctuations of $f(p,t)$ in phase space follows the relation

$$\sigma^2(p,t) = f(p,t)(1 - f(p,t)) \quad (6)$$

which is what is expected in the case of a fermion fluid.

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[1] It should be noticed the relation between the average and the variance of the actual collision number can be considered as a microscopic realisation of the fluctuation-dissipation theorem.

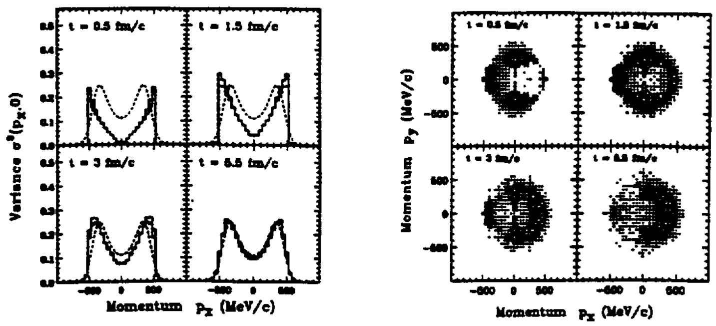

Figure 3: Boltzmann-Langevin simulation of the collision of two Fermi fluids. The left part shows the fluctuations in phase space as function of time. The thick histograms are the result of the simulations while the dashed line represents the thermodynamical limit. The thin histograms correspond to the fluctuation $\sigma^2(p,t) = f(p,t)(1 - f(p,t))$. The right part presents the correlation in phase space associated with the detection of a forward particle.

4) Application of the Boltzmann-Langevin approaches to phase transition

The Boltzmann-Langevin theory appears therefore well founded and promising as far as the possibility to describe fluctuations in a dynamical system is concerned. In the chapters devoted to the problem of instabilities and phase transitions we will apply this theory to the catastrophic dynamics associated with spinodal decomposition. Indeed, we can consider the problem of liquid-gas phase transitions, spontaneous symmetry breaking, multifragmentation and nuclear collisions.

During the multifragmentation of atomic nuclei it seems that identical (or almost identical) initial conditions are leading to very different partitions of the system in interaction . In such a case it is necessary to develop approaches that are able to describe the observed extra-ordinary diversity of the final channels. On the other hand the multifragmentation being characterised by the formation of relatively large fragments, one may think that the mean field plays an important role to organise the system in nuclei. Indeed, the mean field (i.e., the long range part of the bar nucleon-nucleon interaction) is at the origin of the cohesion of the nuclei. Moreover, it has been shown that mean-field approaches or one-body approaches including "à la Boltzmann" collision terms were providing excellent descriptions of many aspects of heavy ion reactions around the Fermi energy (see for example[2] and references therein).

The problem with bare mean-field approaches is that they are unable to break spontaneously symmetries. Therefore, they cannot describe phenomena where bifurcations, instabilities or chaos occurs. However, since few years, many tests and studies have been performed showing that the stochastic extensions of mean-field approaches were good candidates for the description of catastrophic processes. Indeed, the presence of a source of stochasticity allows to explore a large variety of evolutions. Therefore, such approaches may provide valuable descriptions of the dynamics of phase transitions (at least in the case of first-order phase transitions for which the mean-field is known to give a reasonable description of equilibrium properties).

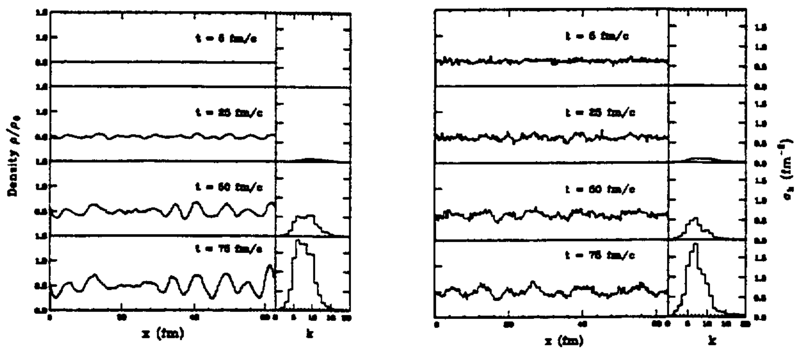

Figure 4: Simulation of the evolution of a piece of nuclear matter in two-dimension[8,9]. The left part presents a simulation performed according to the Boltzmann-Langevin theory solved on a lattice grid. It should be notice that in absence of a fluctuation source the initial symmetry by translation is always preserved and the system remains in an unstable equilibrium situation. When the stochastic source is taken into account the symmetry is broken and the system can develop clusters after a catastrophic evolution towards multifragmentation. The right part presents the simulation using a simplified stochastic approach as described in ref.[10,11]. This comparison shows that in the case of strong instabilities the detailed structure of the source of fluctuation is not important the dynamics being dominated by the amplification of the unstable modes.

To describe this spontaneous symmetry breaking one can use the recently developed stochastic approaches. Figure 4 shows some results obtained using two versions of such approaches: an application of the Boltzmann-Langevin theory[8,9] and a simplified treatment (see refs.[10,11]) in which the only a simple noise is added to the dynamics.

From these different studies, one can conclude that the fragmentation of a system initialised in the spinodal region is initiated by the amplification of unstable zero sound waves. These modes are characterised by typical size and time scales. These modes tend to favour the partition of the system in close mass fragments with the absence of light clusters.

5) Test of the approach on the case of a classical system:

These ideas can be easily tested in the case of a classical Van der Waals gas. In ref.[12]

the stochastic simulations are compared with an exact numerical solution of the classical N-body problem. This comparison has been performed by fitting an effective density dependent mean-field potential to the volume energy of the considered classical gas. This allows to define a phase diagram in the mean field approach which was found to be rather closed from the exact one. As far as the dynamical evolution is concerned an important ingredient is the existence of a finite range for the interaction. Indeed far from the critical point this gives a scale to the fragmentation process. This range in the mean field potential has been fitted on the surface energy of the classical gas.

Figure 5: Classical dispersion relation computed for a classical Van der Waals gas in two dimensions initialised deep in its spinodal region: at the critical density and at the half of the critical temperature. The squares correspond to the exact simulation while the dots are the mean-field simulations. The shade area corresponds to the predictions of the mean-field linear response theory. The inserted figure shows the time evolution of the most unstable mode both for the exact simulation and for the mean-field approach.

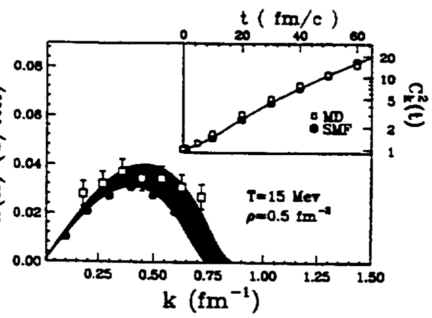

Figure 5 presents an example of a classical dispersion relation extracted from the exact many-body dynamics. As in the nuclear case, we observe that the dispersion relation is dominated by a typical scale directly related to the range of the considered forces. As can be seen from the comparison presented on figure 5, the mean-field simulation and the linear response approach give a very accurate description of the observed mode which should be then interpreted as unstable zero-sound waves.

The dynamics of the phase transition can be studied solving the exact many-body dynamics as shown in figure 6 in the case of one event. It can be also simulated using a classical version of the Boltzmann-Langevin theory and such an example is also presented in figure 6. The qualitative comparison of these two events shows striking similarities:

To study quantitatively these dynamics, one should consider an ensemble of events and compute statistical quantities. Figure 5 and 6 present the fluctuations associated with the different modes, i.e., the ensemble average of the square of the Fourier transform of the density fluctuations. Again one can observe a very good agreement between the exact simulations and the stochastic mean-field approximation. This shows that the stochastic mean-field approaches can also be used in the case of very strong instabilities.

It is important to notice that the classical liquid-gas phase transition is initiated by the presence of unstable collective zero-sound waves which lead to a self organisation of the system in fragments. This type of evolution is already well known for many physical systems such as for example the binary alloys[13,14]. The presented results provide a microscopic understanding of the dynamics of a phase transition and they demonstrate the validity of the stochastic mean-field approaches.

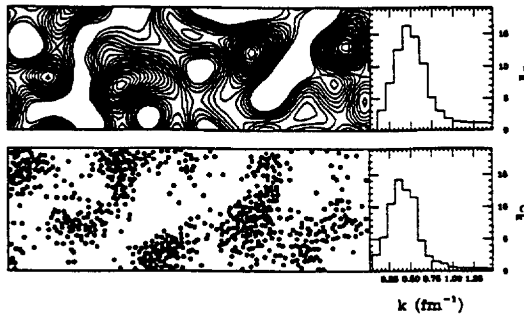

Figure 6: Left part: distribution of the matter for two dynamical simulations of the spinodal decomposition of a piece of infinite nuclear matter; top: an exact classical many-body evolution; bottom: a stochastic mean field simulation. The left part shows the Fourier analysis of the fluctuations normalised to the time 0 and computed over 100 events such as those presented on the left.

6) Conclusion: A first comparison with experiments.

We can now study the fragmentation of a hot and diluted nucleus lying deep inside the spinodal zone of instabilities. We have considered masses, charges, densities, temperatures, spins, expansion,... as predicted by one-body dynamic approaches[15,16]. To describe the spontaneous symmetry breaking associated with the fragmentation of hot spherical sources we have used the recently developed stochastic approaches. Using the stochastic mean field approaches we can now directly compute the various characteristics of the multifragmentation events such as the various partitions or the fragment velocities.

Some experimental data are already pleading in favour of the spinodal decomposition scenario for the time scales, for the favoured partition in equal mass fragments and even for the quenching. However, before entering this discussion, we would like to stress that more experimental and theoretical studies are needed prior to conclude about the observation of spinodal decomposition in nuclei. However, almost all the one-body approaches are predicting that the composite system should enter the spinodal zone. We will see that stochastic mean-field simulations of subsequent spinodal decomposition are able to describe correctly various aspects of multifragmentation events from central collisions.

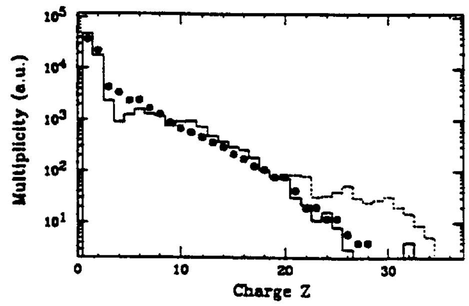

Figure 7: The fragment charge distribution Xe+Sn reaction at 50 MeV per nucleon . The dots are the experimental results from ref.[17] and the solid line the theoretical predictions of the stochastic mean-field simulations filtered using the experimental selection. The dashed line being the unfiltered theoretical result .

We have also performed a comparison with the recent results of the INDRA collaboration[17] concerning events with the formation of a composite source in the Xe+Sn reaction at 50 MeV per nucleon. Indeed, also in this case our one body approaches are predicting the formation of a composite system diving deep in the spinodal region. Figure 7 presents the fragment charge distribution associated with these events while Figure 8 displays the individual charge distributions of the 3 largest fragments. One can see a rather good agreement between experiment and theory. In particular the tail at large Z is well reproduced by the theory. We would like to recall that this tail is coming from both the mode beating and the final state interaction between fragments. On the other hand, the charge distributions of the 3 largest fragments are well reproduced both in centre position and in global shape (and width). In conclusion, while more studies are certainly needed to compare more characteristics of the multifragmention events with the spinodal decomposition scenario, the presented results are very encouraging. Stochastic mean-field approaches can be now applied for realistic simulation in 3D. These dynamic approaches are now able to compete with multifragmentation models and can be directly compared with experiments.

We have then presented the Boltzmann-Langevin approach and we have discussed its properties. Through an ensemble of simulation we have illustrate the power of such a new kind of approaches . Finally we have discussed the implications of these concepts for first order phase transitions and spontaneous symmetry breaking.

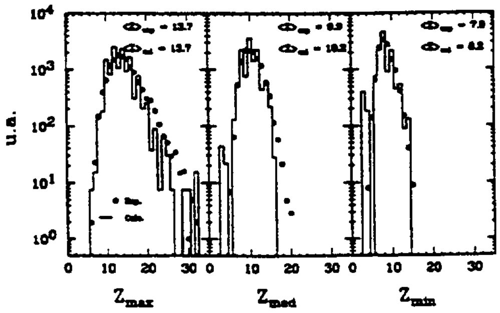

Figure8 Charge distribution of the three largest fragments of each event associated with a spinodal decomposition from the left to the right in increasing size order.

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