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A mathematical study of Quantum Revivals and Quantum Fidelity

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Résumé

In this paper we present some results obtained recently, partly in collaboration with Didier Robert, about “Quantum Revivals” and “Quantum Fidelity”, mainly in the semiclassical framework. We also describe the exact properties of the Quantum Fidelity (also called Loschmidt Echo) for the case of explicit quadratic plus inverse quadratic time-periodic Hamiltonians and establish that the quantum fidelity equals one for exactly the times where the classical fidelity is maximal.

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1 Introduction

The quantum return probability is the modulus of the overlap between an initial wavepacket and its time evolution governed by Schrödinger equation. When this quantity happens to equal one for some time t , then the system is said to exhibit “Quantum Revivals”.

$$R(t) := |\langle \psi, U(t, 0)\psi \rangle|$$

When the Hamiltonian $\hat{H}(t)$ (possibly time-dependent) is assumed to be perturbed $\hat{H}_g(t) := \hat{H}(t) + gV$, then we can compare the evolutions generated by $\hat{H}(t)$ and $\hat{H}_g(t)$ respectively, acting on the **same** initial state ψ :

$$F(t) := \langle U(t, 0)\psi, U_g(t, 0)\psi \rangle$$

which is the measure of the “Quantum Fidelity” in the state ψ along the evolution. Of course $F(0) = 1$ and $F(t) \equiv 1$ if $g = 0$.

Thus the decrease in t of $F(t)$ measures the lack of fidelity due to the perturbation.

Both Quantum Revivals and Quantum Fidelity have attracted much recent interest in the physics literature (see references). Notably, it has been heuristically claimed that the decrease in time of the Quantum Fidelity allows to distinguish between systems having regular versus chaotic classical evolution. In our study, we consider the semiclassical regime for both quantities, using coherent states as initial wavepackets ψ .

We also perform exact calculations of the Quantum Fidelity in the case of the singular time-periodic Harmonic Oscillator, with initial wavepackets ψ being “generalized coherent states” in the sense of Perelomov, showing that they do not decrease to zero as time evolves, but present recurrences to 1 exactly at the values of times where the classical fidelity is maximal. More specifically we consider Hamiltonians :

$$\hat{H}(t) := \frac{P^2}{2} + f(t)\frac{Q^2}{2}$$

and

$$\hat{H}_g(t) := \hat{H}(t) + \frac{g^2}{Q^2}$$

where the real constant g is the size of the perturbation, and f is a T -periodic function of time, and we perform an exact calculation of :

$$F(t) := \langle U(t, 0)\psi, U_g(t)\psi \rangle$$

where $U(t, 0)$ (resp $U_g(t, 0)$) is the quantum evolution generated by $\hat{H}(t)$, (resp. $\hat{H}_g(t)$).

2 Semiclassical Quantum Revivals

We use the ‘‘Coherent States’’ of the Harmonic Oscillator : let φ_0 be the ground state of the Harmonic Oscillator $\hat{P}^2/2 + \hat{Q}^2/2$ in dimension n , where \hat{Q} (resp. \hat{P}) is the operator of multiplication by x (resp. the derivation operator $\hat{P} := -i\hbar\nabla$) in the Hilbert space of quantum states $\mathcal{H} = L^2(\mathbb{R}^n)$.

The Weyl-Heisenberg unitary translation operator by $\alpha := (q, p)$ is defined as :

$$\hat{T}(\alpha) := \exp\left(i\frac{p\cdot\hat{Q} - q\cdot\hat{P}}{\hbar}\right)$$

and the coherent state φ_α is defined as follows :

$$\varphi_\alpha := \hat{T}(\alpha)\varphi_0$$

Now consider a classical trajectory in phase space induced by the Hamiltonian $H := \frac{p^2}{2} + V(q)$:

$$\alpha \mapsto \alpha_t := (q_t, p_t)$$

Let $S(t)$ be the classical action along this trajectory, and M_t be the Hessian matrix of H taken at point α_t :

$$(M_t)_{i,j} := \frac{\partial^2 H}{\partial\alpha_i\partial\alpha_j}(q_t, p_t)$$

It is a real symmetric $2n \times 2n$ matrix. The linearized flow (or Stability Matrix) is obtained by solving the differential equation :

$$\dot{F}_t = JM_t F_t$$

where J is the symplectic matrix :

$$J = \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix}$$

with initial data

$$F_0 = \mathbb{1}$$

Clearly F_t is a symplectic matrix, ie satisfies for any t

$$\tilde{F}_t J F_t = J$$

Namely

$$\frac{d}{dt}(\tilde{F}_t J F_t) = -\tilde{F}_t M_t J J F_t + \tilde{F}_t J J M_t F_t = \tilde{F}_t M_t F_t - \tilde{F}_t M_t F_t = 0$$

and therefore $\tilde{F}_t J F_t$ is a constant $2n \times 2n$ matrix equal to J since $F_0 = \mathbb{1}$.

To this symplectic matrix can be associated a unitary operator $\hat{R}(F)$ in the Hilbert space \mathcal{H} , acting as expected :

$$\hat{R}(F)^{-1} \begin{pmatrix} \hat{Q} \\ \hat{P} \end{pmatrix} \hat{R}(F) = F \begin{pmatrix} \hat{Q} \\ \hat{P} \end{pmatrix}$$

Now we are in a position to give the semiclassical approximation for the quantum evolution of coherent states (see [8])

Proposition 2.1 *Under suitable assumptions on V , there exists a constant $\epsilon(\hbar, t)$, small as \hbar goes to zero, such that if $U(t, 0)$ is the quantum evolution operator associated to the Weyl quantization of H one has :*

$$\|U(t, 0)\varphi_\alpha - e^{i\delta_t/\hbar}\hat{T}(\alpha_t)\hat{R}(F_t)\varphi_0\| \leq \epsilon(\hbar, t)$$

where $\delta_t := S(t) - (p_t \cdot q_t - p \cdot q)/2$

Thus, up to a controlable error, the recurrence probability $|\langle \varphi_\alpha, U(t, 0)\varphi_\alpha \rangle|$ can be replaced with

$$\tilde{R}(t) := |\langle \hat{T}(\alpha)\varphi_0, \hat{T}(\alpha_t)\hat{R}(F_t)\varphi_0 \rangle| = |\langle \hat{T}(\alpha - \alpha_t)\varphi_0, \hat{R}(F_t)\varphi_0 \rangle|$$

We shall now make use of a beautiful result by Mehlig and Wilkinson [22], that gives the Weyl covariant symbol of the metaplectic operators. For a complete mathematical proof see our paper [9].

Proposition 2.2 *Let F be a symplectic $2n \times 2n$ matrix not having 1 as eigenvalue. Then the associated metaplectic operator $\hat{R}(F)$ can be written as*

$$\hat{R}(F) = \frac{h^{-n}\gamma_F}{|\det(\mathbb{1} - F)|^{1/2}} \int_{\mathbb{R}^{2n}} dz \hat{T}(z) e^{iz \cdot Az/2\hbar}$$

where γ_F is a complex number of modulus 1, and

$$A := \frac{J}{2}(F + \mathbb{1})(F - \mathbb{1})^{-1}$$

We now set

$$z_t := \alpha - \alpha_t$$

Then using Proposition 2.2, we have :

$$\tilde{R}(t) = h^{-n} |\det(\mathbb{1} - F_t)|^{-1/2} \left| \int dz \langle \hat{T}(z_t)\varphi_0, \hat{T}(z)\varphi_0 \rangle e^{iz \cdot Az/2\hbar} \right|$$

$$\begin{aligned}
&= h^{-n} |\det(\mathbb{1} - F_t)|^{-1/2} \left| \int dz \exp \left(\frac{iz \cdot Az}{2\hbar} + \frac{iz \cdot Jz_t}{2\hbar} - \frac{1}{4\hbar} (z - z_t)^2 \right) \right| \\
&= h^{-n} |\det(\mathbb{1} - F_t)|^{-1/2} e^{-z_t^2/4\hbar} \left| \int dz \exp \left(-\frac{1}{2\hbar} z \cdot \left(\frac{\mathbb{1}}{2} - iA \right) z + \frac{1}{2\hbar} z \cdot (J + i\mathbb{1}) z_t \right) \right|
\end{aligned}$$

Now by using the calculus of Fourier Transforms of Gaussians, we get :

$$\tilde{R}(t) = |\det(\mathbb{1} - F_t) \left(\frac{\mathbb{1}}{2} - iA \right)|^{-1/2} \left| \exp \left(-\frac{z_t^2}{4\hbar} - \frac{1}{4\hbar} z_t \cdot K z_t \right) \right|$$

where the matrix K is given by

$$K := (J - i\mathbb{1})(\mathbb{1} - 2iA)^{-1}(J + i\mathbb{1})$$

Now we have the following remarkable result :

$$\frac{\mathbb{1}}{2} - iA = N(F - \mathbb{1})^{-1}$$

where

$$N := \frac{1}{2}(F(\mathbb{1} - iJ) - (\mathbb{1} + iJ))$$

so that

$$|\det(\mathbb{1} - F) \left(\frac{\mathbb{1}}{2} - iA \right)|^{-1/2} = |\det N|^{-1/2}$$

But N has the important following property

Lemma 2.3

$$|\det N| \geq 1$$

and

$$|\det N| = 1 \iff F \text{ is unitary}$$

This result has been established in full generality in [9], but we shall here indicate the calculus in dimension $n = 1$. The symplectic matrix F has now the simple form

$$F = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

with $ad - bc = 1$ to ensure the symplecticity. Using the form given above for N we easily get

$$\det N = -\frac{1}{2}(a + d + i(b - c))$$

so that

$$|\det N|^2 = \frac{1}{4}((a + d)^2 + (b - c)^2) = \frac{1}{4}(4 + (a - d)^2 + (b + c)^2) \geq 1$$

with equality to 1 $\iff a = d, b = -c$, in which case F is just unitary (rotation).

Thus we get the following result

Theorem 2.4 Denoting by z_t the following distance $z_t := \alpha - \alpha_t$, the complete return probability has the following semiclassical estimate :

$$\left| R(t) - |\det N|^{-1/2} \exp \left(-\frac{z_t^2}{4\hbar} - \frac{1}{4\hbar} z_t \cdot K z_t \right) \right| \leq \epsilon(\hbar, t)$$

Thus if α lies on a classical periodic orbit γ with period T_γ , the exponential is just 1 ; furthermore the prefactor is 1 $\iff F_{T_\gamma}$ is unitary, in which case we have almost semiclassical recurrence.

$$R(T_\gamma) \geq 1 - \epsilon(\hbar, t)$$

Note that $\epsilon(\hbar, T_\gamma) = O(\hbar^\epsilon)$ provided $T_\gamma \leq \lambda |\log \hbar|$ for some λ given by the classical dynamics.

3 The Quantum Fidelity

Let \hat{H} be a quantum Hamiltonian, and $\hat{H}_g := \hat{H} + gV$ be a perturbation of it (g small). The quantum fidelity at time t in the state ψ is given as

$$F(t) := |\langle e^{-it\hat{H}/\hbar} \psi, e^{-it\hat{H}_g/\hbar} \psi \rangle|$$

Remark 3.1 If $\psi = \psi_j$ is an eigenstate of \hat{H} (resp. \hat{H}_g), then the fidelity is nothing else than the “return probability” $|\langle \psi, e^{-it\hat{H}/\hbar} \psi \rangle|$ (resp. $|\langle \psi, e^{-it\hat{H}_g/\hbar} \psi \rangle|$)

Remark 3.2 Clearly $F(0) = 1$, and $F(t) \equiv 1$ if $g = 0$. One expects that if $g \neq 0$ then $F(t)$ will decrease as time increases. Furthermore it is believed that this decrease could be significantly different for an associated classical dynamics being regular versus chaotic. No exact result has been established up to now.

One can semiclassically estimate this quantum fidelity along the same lines as the return probability above.

Theorem 3.3 Let α_t (resp. $\alpha'(t)$) be the classical phase-space point reached by the trajectory governed by Hamiltonian H (resp. H_g), starting from the **same** point α at time 0. Then the following estimate holds true :

$$|F(t) - C_t \exp \left\{ -\frac{1}{4\hbar} (F'_t(\alpha'_t - \alpha_t))^2 K(\alpha, g, t) \right\}| \leq \hbar^{1/2} L(\alpha, g, t)$$

where F'_t is the stability matrix for the dynamics generated by H_g , and C_t , $K(\alpha, g, t)$, $L(\alpha, g, t)$ are positive controllable constants.

Remark 3.4 The proof of this statement is contained in our paper [9]. The important fact to notice is that the classical **infidelity** $\alpha_t - \alpha'_t$ is an important quantity to estimate in t and in g .

We now come to an interesting particular case where the quantum fidelity can be computed exactly. The Hamiltonians considered are time-periodic and have the following form :

$$H_0(t) := \frac{p^2}{2} + f(t)\frac{x^2}{2}$$

$$H_g(t) := H_0(t) + \frac{g^2}{x^2}$$

where $t \mapsto f$ is a T-periodic function, and g a real constant.

We denote by $\hat{H}_0(t)$ and $\hat{H}_g(t)$ the corresponding selfadjoint operators in $\mathcal{H} = L^2(\mathbb{R})$. The initial states we shall consider are generalized coherent states in the sense of Perelomov ([23]) adapted to the underlying algebra $SU(1,1)$. Let

$$K_0 := \frac{\hat{Q}^2 + \hat{P}^2}{4} + \frac{g^2}{2\hat{Q}^2} = 2\hat{H}_g$$

$$K_{\pm} = \frac{\hat{P}^2 - \hat{Q}^2}{4} \mp i \frac{\hat{Q} \cdot \hat{P} + \hat{P} \cdot \hat{Q}}{4} - \frac{g^2}{2\hat{Q}^2}$$

They satisfy :

$$[K_0, K_{\pm}] = \pm K_{\pm}$$

$$[K_-, K_+] = 2K_0$$

$$K_- = K_+^*$$

Let ψ be the ground state of \hat{H}_g :

$$\hat{H}_g\psi = (\alpha + \frac{1}{2})\psi$$

with $\alpha = \frac{1}{2} + \sqrt{\frac{1}{4} + 2g^2}$. It also annihilates K_- :

$$K_-\psi = 0$$

and has the following form :

$$\psi(x) = cx^{\alpha}e^{-x^2/2}$$

where c is a normalization constant such that $\|\psi\| = 1$.

For $\beta \in \mathbb{C}$, we define the unitary operator

$$\hat{S}(\beta) := \exp(\beta K_+ - \bar{\beta} K_-)$$

and the generalized coherent states as

$$\psi_{\beta} := \hat{S}(\beta)\psi$$

Let $U_0(t, 0)$ and $U_g(t, 0)$ be the quantum unitary evolution operators generated by $\hat{H}_0(t)$ and $\hat{H}_g(t)$. We shall study the quantum fidelity (without absolute value) :

$$F(t) := \langle U_0(t, 0)\psi_\beta, U_g(t, 0)\psi_\beta \rangle$$

We shall first study the particular case $g = 1$ (whence $\alpha = 2$). Then ψ is obviously a simple linear combination of the eigenstates φ_0 and φ_2 of the harmonic oscillator.

We have the following explicit result (see [7]) :

Proposition 3.5 *Let $z(t)$ be a complex solution of the linear differential equation (Hill's equation) :*

$$\ddot{z} + fz = 0$$

We define its polar decomposition by

$$z(t) = e^{u+i\theta}$$

where $t \mapsto u$ and $t \mapsto \theta$ are real, and consider the following initial data :

$$u(0) = u_0$$

$$\dot{u}(0) = \dot{u}_0$$

$$\theta(0) = \theta_0$$

$$\dot{\theta}(0) = e^{-2(u_0-\epsilon)}$$

Let $\hat{H}_g = \frac{\hat{P}^2 + \hat{Q}^2}{2} + \frac{g^2}{\hat{Q}^2}$. Then we have :

$$U_g(t, 0) = e^{i\dot{u}\hat{Q}^2/2} e^{-i(u-\epsilon)(\hat{Q}\cdot\hat{P}+\hat{P}\cdot\hat{Q})/2} e^{-i(\theta-\theta_0)\hat{H}_g} e^{i(u_0-\epsilon)(\hat{Q}\cdot\hat{P}+\hat{P}\cdot\hat{Q})/2} e^{-i\dot{u}_0\hat{Q}^2/2}$$

The same formula holds true for $U_0(t, 0)$ with $\hat{H}_g(t)$ replaced by $\hat{H}_0(t)$.

An important fact to notice is that the constants u_0 , \dot{u}_0 , θ_0 , ϵ can be adjusted, given any $\beta \in \mathbb{C}$ such that

$$\hat{S}(\beta)\psi = e^{i\dot{u}_0\hat{Q}^2/2} e^{-i(u_0-\epsilon)(\hat{Q}\cdot\hat{P}+\hat{P}\cdot\hat{Q})/2} e^{-i\theta_0\hat{H}_g}\psi$$

Then $U_g(t, 0)\psi$ and $U_0(t, 0)\psi$ have simple explicit form (see [7]), leading to a very simple form of the fidelity :

Theorem 3.6 *Let $g = 1$. Then the fidelity is just given by*

$$F(t) = \frac{2}{3} + \frac{1}{3}e^{-2i(\theta(t)-\theta(0))}$$

Let us remark that $\theta(t)$ is just given by the formula

$$\theta(t) - \theta(0) = e^{2\epsilon} \int_0^t e^{-2u(s)} ds$$

Study of the Hill's equation

$$\ddot{z} + fz = 0$$

• **Stable case :**

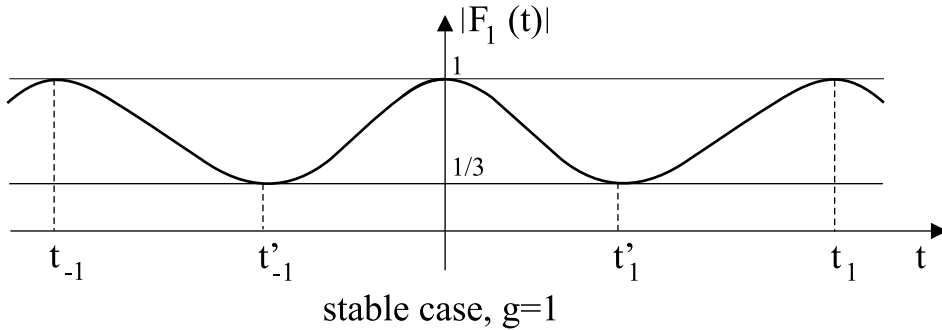
u is T -periodic, and thus $\theta(t)$ grows from $-\infty$ to $+\infty$ when t varies from $-\infty$ to $+\infty$. Therefore there exists an infinite sequence $(t_k)_{k \in \mathbb{Z}}$ such that

$$\theta(t_k) - \theta(0) = 2k\pi$$

in which case $F(t_k) = 1$, ie the quantum fidelity is perfect. Moreover there exists an infinite sequence $(t'_k)_{k \in \mathbb{Z}}$ such that

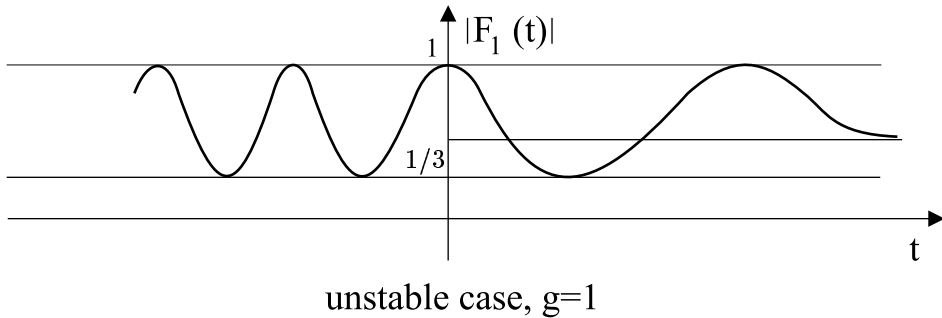
$$\theta(t'_k) - \theta(0) = (2k + 1)\pi$$

in which case $F(t'_k) = 1/3$. We thus have the following picture :



• **unstable case :**

$u(t) \simeq \pm \lambda t$ with, say, $\lambda > 0$ (Lyapunov exponent) in which case $\theta(t) \rightarrow cst$ as $t \rightarrow +\infty$ and $\theta(t) \rightarrow -\infty$ exponentially as $t \rightarrow -\infty$ (or vice-versa). We thus have the following picture :



General case

We assume now that g is an arbitrary real constant. Then ψ no longer has a finite linear decomposition on the Hermite functions, but instead

$$\psi = \sum_0^{\infty} \lambda_n \varphi_n$$

with $\sum |\lambda_n|^2 = 1$. Then the following result holds true :

Theorem 3.7

$$\langle U_0(t, 0)\psi_\beta, U_g(t, 0)\psi_\beta \rangle = e^{-i(\theta(t)-\theta(0))\alpha} \sum_0^{\infty} |\lambda_n|^2 e^{-in(\theta(t)-\theta(0))}$$

Therefore $|F(t)| = 1$ if $\theta(t) - \theta(0) = 0 \pmod{2\pi}$

If g is such that $\alpha := \frac{1}{2} + \sqrt{\frac{1}{4} + 2g^2} = \frac{p}{q} \in \mathbb{Q}$, then
 $F(t) = 1$ if $\theta(t) - \theta(0) = 0, \pmod{2q\pi}$

Exact Classical Fidelity implies Exact Quantum Fidelity

Let $x(t)$ and $y(t)$ be real classical solutions for the Hamiltonians $H_0(t)$ and $H_g(t)$ respectively such that $x(0) = y(0)$ and $\dot{x}(0) = \dot{y}(0)$ which means that the trajectories **merge from the same point in phase space at $t = 0$**

t is said a time of **classical fidelity** if $x(t) = y(t)$ and $\dot{x}(t) - \dot{y}(t) = 0$, and the **classical infidelity** at time t is measured by the distance $|x(t) - y(t)|$.

Theorem 3.8 *Let $x(t)$ be a real solution of Hill's equation $\ddot{x} + fx = 0$. We write it as*

$$x(t) = e^{u(t)} \cos \tilde{\theta}(t)$$

with u and $\tilde{\theta}$ real functions and $\tilde{\theta}(t) = g\sqrt{2} \int_0^t ds e^{-2u(s)}$. Then $y(t) := e^{u(t)}$ is a solution of equation

$$\ddot{y} + fy - \frac{2g^2}{y^3} = 0$$

such that $x(0) = y(0)$ and $\dot{x}(0) = \dot{y}(0)$. This means that $y(t)$ is a real trajectory for Hamiltonian $H_g(t)$, merging from the same point in phase space as $x(t)$.

We clearly have

$$|x(t) - y(t)| = y(t)(1 - \cos \tilde{\theta}(t))$$

which vanishes for $\tilde{\theta}(t) = 0 \pmod{2\pi}$

Remark 3.9 By choosing ϵ such that $e^{2\epsilon} = g\sqrt{2}$, we clearly have

$$\tilde{\theta}(t) = \theta(t) - \theta(0)$$

Corollary 3.10 If $\theta(t) - \theta(0) = 0 \pmod{2\pi}$, then the classical fidelity is zero, and the Quantum Fidelity equals 1 (at least in absolute value in the case of general g).

The proof is very elementary. Let $z(t)$ be a complex solution of Hill's equation of the form

$$z(t) := e^{u(t)+i\tilde{\theta}(t)}$$

with u and $\tilde{\theta}$ **real** functions. Since f is real the wronskian of z and z^* is constant, and we assume that it equals $4i\sqrt{g}$:

$$\dot{z}z^* - \dot{z}^*z = 2i\tilde{\theta}'|z|^2 = 4i\sqrt{g}$$

Therefore

$$\frac{d}{dt}\tilde{\theta} = g\sqrt{2}e^{-2u}$$

and therefore Hill's equation for z implies :

$$\ddot{z} + fz = 0 = [\ddot{u} + \dot{u}^2 - (\frac{d}{dt}\tilde{\theta})^2 + i(\frac{d^2}{dt^2}\tilde{\theta} + 2\dot{u}\frac{d}{dt}\tilde{\theta})]z$$

whence the equation for u :

$$\ddot{u} + \dot{u}^2 - 2g^2e^{-4u} = -f$$

and thus for $y := e^u$:

$$\ddot{y} + fy - \frac{2g^2}{y^3} = 0$$

which is nothing but Newton's equation for Hamiltonian $H_g(t)$. Furthermore assuming that $\tilde{\theta}(0) = 0$, $x(t) := e^{u(t)} \cos \tilde{\theta}(t)$ and $y(t)$ have the same initial data. This completes the proof, noting that

$$\tilde{\theta}(t) = \theta(t) - \theta(0) = g\sqrt{2} \int_0^t ds e^{-2u(s)}$$

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