Mysteries of the lightest nuclear systems

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Abstract

In this contribution current advances and challenge in low energy few-nucleon scattering problem are discussed. Some original results obtained when solving Faddeev-Yakubovski equations in configuration space are presented both for three and four-nucleon systems.

1 Introduction

Theoretical nuclear physics aims to describe the systems of interacting nucleons. It faces two evident frontiers: first devoted to the origins of the strong force between the nucleons; the second one is microscopic quantum mechanical treatment of many-body systems (nuclei) with the strong force as dynamical input. In order to link these two frontiers few-nucleon physics plays very important role:

- Since nuclear interaction can not be determined from nucleon structure underlying theory (QCD)– one is obliged to rely on the phenomenological models. To test these models efficiently it is required to develop powerful tools, which enables description of nuclear processes without acquiring additional approximations.
- It is evident however that such exhaustive approach can account only for negligibly small part of nuclear processes – the ones in the very lightest and simplest structures. Numerous approximations are however necessary and unavoidable for description of the more complex systems. Therefore it is of fundamental importance to have an accurate tool, which is capable to evaluate the effect of different approximations one relies on.

At present, a variety of high-precision NN-potentials become available. Beyond the longest range one-pion-exchange part, which all these potentials contain, the medium and short range region is parametrized either purely phenomenologically or semiphenomenologically. These models describe bound and scattering states (up to 350 MeV laboratory energy) of two-nucleon system with magnificent accuracy, having a χ^2 per datum of about one.

However, an important new features appears in the systems with more than two particles. Such systems can exhibit off-energy shell behavior, which is prohibited in two-body ones. These of-energy shell effects turns to play important role in nuclear systems and reveal themselves already in three-nucleon bound state. It have been remarked that A$\geq$3 systems cannot be described using NN-forces alone, it is necessary to include three-nucleon forces (3NF) to account for seizable 10% underbinding of the triton. The origin and the explicit form of the 3NF, which probably is not unique and certainly depends on NN partner in use, becomes a central issue of few-nucleon physics [1].

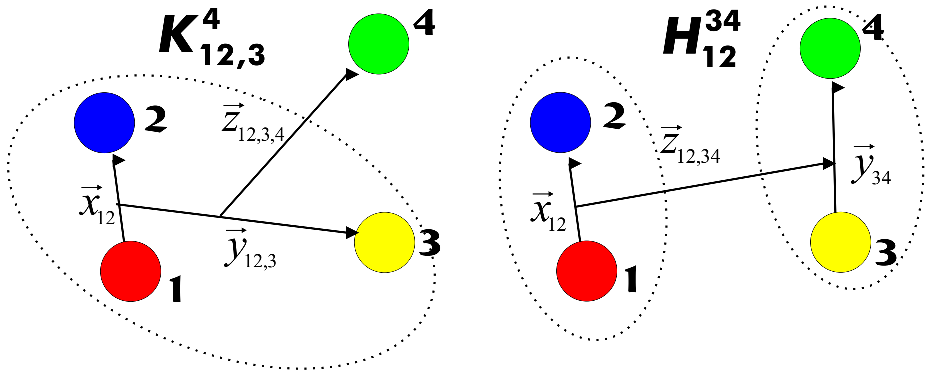

Figure 1: FY components K and H. Asymptotically, as $z \to \infty$, components K describe 3+1 particle channels, whereas components H contain asymptotic states of 2+2 channels.

Recently big progress has been made in describing the structure of stable nuclei: two very powerful methods have been developed Green's function Monte Carlo [2] and No-Core Shell Model [3], which permits to treat nuclear binding of up to 12 nucleons. There are several other methods, which can provide rigorous description of $4 \leq A \leq 6$ bound states [4]. These achievements permit to study in more rigor structure of collective nuclear forces (3NF, 4NF,..), however nuclear binding energies being strongly correlated does not permit to form a full picture. The richest information source about the nature of the nuclear interaction is scattering experiment, making particle collision theory of fundamental importance. Unfortunately description of already the simplest nuclear reactions meets serious theoretical drawbacks on the formal level, resulting that advancement in this domain is much slower [1]. There are only very few methods, which can go beyond A=3 system and these at the time being limited to A=4 case [5]. This subject will be briefly discussed in this contribution, having aim to present recent challenges and achievements in few-nucleon scattering calculations. In particular, the formalism of Faddeev-Yakubovski equations will be highlighted [6], which enables consistent description of scattering and bound states for non-relativistic systems. Some results, obtained employing this formalism for describing scattering process (including break-up and rearrangement reactions) in three and four-nucleon systems will be presented.

2 Theoretical grounds

The Schrödinger equation is a founding ground of non-relativistic quantum mechanics. However this equation is not able to provide an unique solution for multiparticle problems with asymptotically non-vanishing wave functions (N≥3 particle scattering problem). Faddeev [7] have succeeded to show that these equations can be reformulated by introducing some additional physical constraints, which leads to mathematically rigorous and unique solution of the three-body scattering problem. Faddeev's pioneering work was followed by Yakubovsky [8], who generalized these equations for any number of particles.

We solve four-particle problem using the Faddeev-Yakubovski (FY) equations in configuration space. In this formalism systems wave function is expressed as a sum of 18 FY components. One has twelve components K, which describe asymptotes of elastic 3+1 particle channels; six H components intend to describe 2+2 ones (see

Figure ??). If system is composed of four identical particles[1] different components K (or H) become formally identical and they can be related using particle permutation operators:

$$P^+ = (P^-)^- = P_{23}P_{12}; \quad Q = \varepsilon P_{34}; \quad \tilde{P} = P_{13}P_{24} = P_{24}P_{13}. \tag{1}$$

Then systems wave function is expressed

$$\Psi = \left[1 + (1 + P^+ + P^-)Q\right](1 + P^+ + P^-)K^4_{12,3} + (1 + P^+ + P^-)(1 + \tilde{P})H^{34}_{12}, \tag{2}$$

using two nonreducible components $K^4_{12,3}$ and H^{34}_{12}. These components are coupled by two differential FY equations:

$$\begin{array}{rcl} (E - H_0 - V_{12})\, K^4_{12,3} & = & V_{12}(P^+ + P^-)\left[(1+Q)K^4_{12,3} + H^{34}_{12}\right] + V_{12,3}\Psi \\ (E - H_0 - V_{12})\, H^{34}_{12} & = & V_{12}\tilde{P}\left[(1+Q)K^4_{12,3} + H^{34}_{12}\right] \end{array} \tag{3}$$

here V_{12} and $V_{12,3}$ are respectively two and three nucleon potential energy operators.

One should mention, that equations (3) become non appropriate once long range interaction, in particular Coulomb, is present. In fact, FY components remain coupled even in far asymptotes, thus making numerical implementation of asymptotic conditions hardly possible. The way to circumvent this problem is in detail described in [6].

In this short presentation of the formalism we have skipped three particle FY equations. In fact, four-particle equations comprise all three-particle physics in them. Three particle equations can be obtained from 4-particle ones simply by separating degrees of freedom of the particle four.

Equations (3) in conjunction with the appropriate boundary conditions [6] are solved by making partial wave decomposition of amplitudes $K^4_{12,3}$ and H^{34}_{12}:

$$K_i(\vec{x}_i, \vec{y}_i, \vec{z}_i) = \sum_{LST} \frac{\mathcal{K}_i^{LST}(x_i, y_i, z_i)}{x_i y_i z_i} \left[L(\hat{x}_i, \hat{y}_i, \hat{z}_i) \otimes S_i \otimes T_i\right] \tag{4}$$

$$H_i(\vec{x}_i, \vec{y}_i, \vec{z}_i) = \sum_{LST} \frac{\mathcal{H}_i^{LST}(x_i, y_i, z_i)}{x_i y_i z_i} \left[L(\hat{x}_i, \hat{y}_i, \hat{z}_i) \otimes S_i \otimes T_i\right] \tag{5}$$

Equations (3) are then projected on this basis of angular momentum, spin and isospin. The partial components $\mathcal{K}_i^{LST}$ and $\mathcal{H}_i^{LST}$are further expanded in the basis of three-dimensional splines. One thus converts integro-differential equations into a system of linear equations. More detailed discussion on the technical issues can be found in [6].

3 3N scattering

Three nucleon scattering is theoretically the most explored few-body system. Nowadays numerical solution of neutron-deuteron scattering, both for elastic and break-up observables, becomes almost a routine problem. On the other hand description of proton-deuteron break-up process causes severe theoretical drawbacks due to inability to predefine asymptotic form of the systems wave function. Only very recently [9] this problem has been overcommed numerically, thus permitting to analyse 3N system in the full extent.

It was believed that, due to its simplicity, 3N system is a perfect testing ground for the 3NF. However deuteron being very extended structure results, what three-nucleon get seldom close to each other in 3N reactions and that some scattering

[1] We consider proton and neutron being two different sates of the same particle – nucleon. These states are discriminated by the isospin quantum number.

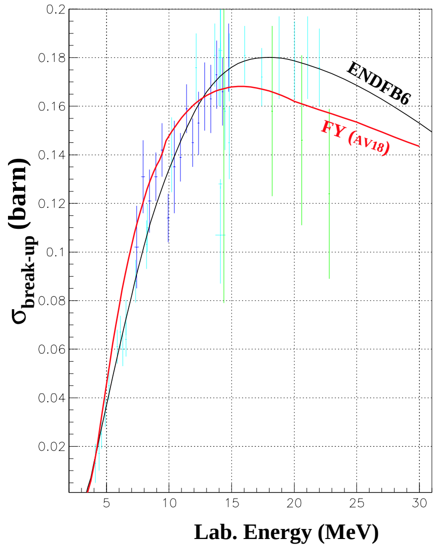

Figure 2: Comparison of n-d total break-up cross sections obtained solving Faddeev equations and AV18 NN interaction model(FY$_{(AV18)}$ curve) with experimental data points and experimental data based R-matrix evaluation (ENDFB6).

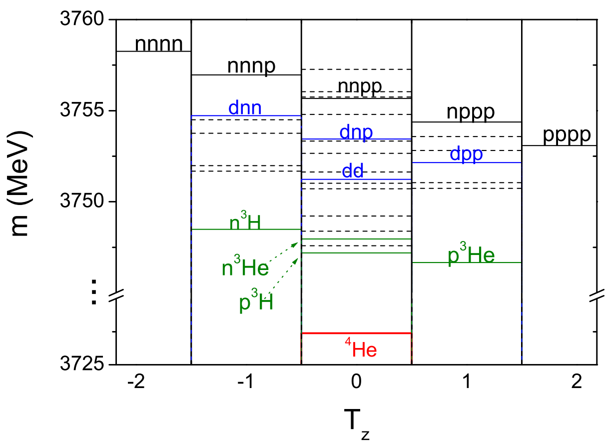

Figure 3: In left figure experimental spectra of *4N* bound and resonant states is presented. Positions of resonant states (dashed lines) are taken from R-matrix analysis [10].

observables – as elastic differential or total break-up cross sections – over a broad range of energy show little sensitivity to the inclusion of the 3NF. Unfortunately one is obliged to seek for 3NF effects in more complex systems. On the other hand in view of insensibility of N-deuteron cross sections to nuclear interaction model in use, accurate evaluations can be provided for this system based only on theoretical description – paramount due to lack of accurate n-d break-up cross-section data (see Fig. 2).

4 4N scattering

The study of the 4N system is particularly interesting as a "theoretical laboratory" to test new nuclear force models. Unlike the $A = 3$ case, $A = 4$ shows a delicate and rich structure of excited states in the continuum, see Fig. 3, whose position and width depends critically on the underlying nucleon-nucleon (NN) interaction. These resonant states, noticeably ones having negative parity, should be very sensible to

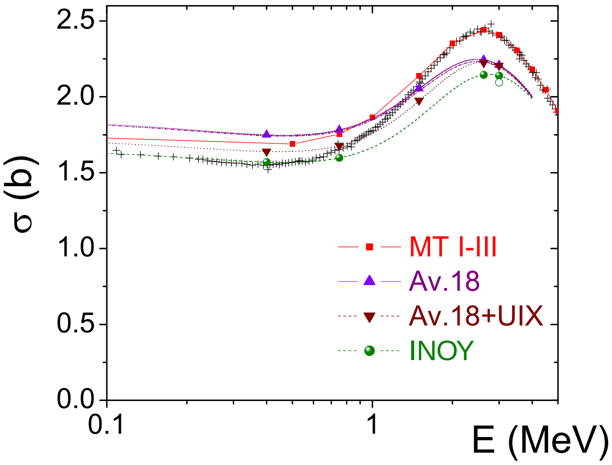

Figure 4: Comparison of calculated n-^{3}H cross sections with experimental data. Realistic interaction models, also including 3NF and non-local NN terms, underestimate these cross sections in resonance region.

NN P-waves, which is believed to be responsible for some unexplained discrepancies in 3N polarization observables. Furthermore the impact of three-nucleon (3N) force are believed to be larger than in the $A = 3$ system. Apart all, it is the simplest system where the 3N interaction in the channels of total isospin $\mathcal{T} = 3/2$ can be studied – permitting one fully explore charge dependence of NN and NNN interactions.

n-^{3}H and p-^{3}He elastic scattering

n-^{3}H elastic channel represents the simplest *4N* reaction. It is almost pure isospin $\mathcal{T} = 1$ state, free of Coulomb interaction in the final state as well as in the target nucleus. This system however has very large neutron excess, as large as in the neutron richest stable nucleus – ^{8}He. Furthermore four negative parity resonances, two spin degenerated doublets, are present in n-^{3}H continuum, which strongly contributes to enhance elastic cross section in around E$_{cm}$ ≈3 MeV. Ability of realistic nuclear interaction models to describe n-^{3}H resonant cross sections was recently put in doubt [5, 11].

At very low energy pure *NN* local interaction models overestimate n-^{3}H zero energy cross sections [12, 13]. This is not surprising however, these models lacking 3NF underestimate triton (target nucleus) binding energy, also resulting it to be too large. Once triton binding energy and size are adjusted, as example using UIX 3NF in conjunction with AV18 NN interaction, zero energy cross sections are reduced to gain overall agreement with experimental data, see Fig. 4. Nevertheless some discrepancy still remains for coherent scattering length [11], indicating that scattering cross section is not well redistributed between $\mathcal{J}^\pi = 0^+$ and 1^+ states.

Recently phenomenological non-local INOY NN interaction model was constructed [14], which is capable to reproduce the triton binding energy as well as improve description of low-energy 3N polarization observables, without requiring 3NF. This model seems also give better agreement for n-^{3}H coherent scattering length than AV18+UIX complex. Nevertheless all realistic interaction models, also including INOY, underestimate by more than 10% elastic cross sections in the resonance region.

By analyzing differential n-^{3}H cross section in this resonances region one can observe that underestimation of cross section is most significant for forward and backward scattered neutron, see Fig. 5. These regions are dominated by scattering in negative parity states, i.e. ones containing resonances. The similar discrepancy can be observed when describing p-^{3}He differential cross-sections, note, this system is isospin partner of n-^{3}H compound and thus has similar structure. However due

J^π	R-matrix [10]	S-matrix	
		MT I-III	AV18
2^-	3.19-2.71i		1.1-2.2i
1^-	3.50-3.37i	1.03-2.14i	0.8-2.1i
0^-	5.27-4.46i		≈0.4-2.8i
1^-	6.02-6.50i	0.35 -2.31i	≈-0.1-2.3i

Table 1: Comparison of n-^{3}H resonance positions: R-matrix evaluated values and theoretical calculation of S-matrix poles for two different NN interaction models. Energies are given in respect to n-^{3}H threshold.

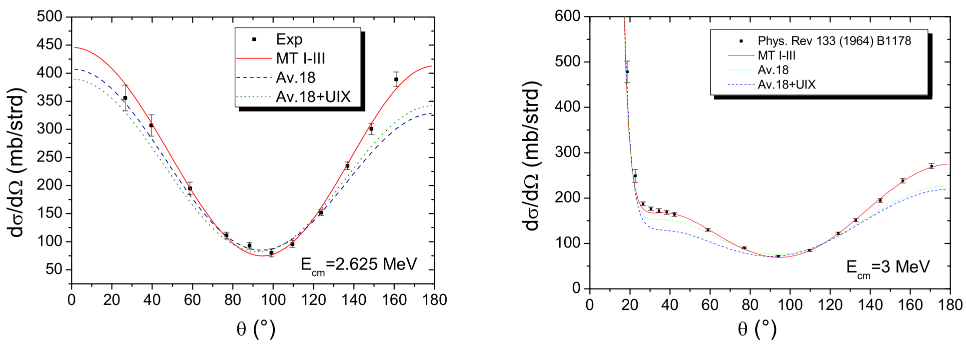

Figure 5: Comparison of calculated n-^{3}H (figure in the left) and p-^{3}He (figure in the right) differential cross sections with experimental data.

to presence of Coulomb interaction p-^{3}He effective interaction is less attractive than one for n-^{3}H case – therefore resulting resonances are less pronounced.

There is one NN interaction model, which can surprisingly well describe both differential and total cross sections in these two compounds. It is Malfiet-Tjon potential (MT I-III). However this purely phenomenological model is very simplified, as example it ignores presence of tensor force and therefore is not capable to describe accurately NN data or account for polarization observables. This model thus serves to us only as reference to study nuclear scattering at very low energies and give gross predictions.

The underlying reason of the realistic interaction model failure are still to be learned better. On part it can be due to the fact that neutron-neutron interaction is not well known, and that charge dependent terms can be sizeable in NN force. However this reasoning can not explain failure in p-^{3}He cross section description, nor explain the fact that MT I-III potential lacking charge symmetry breaking terms do rather well in both cases. The origin of these discrepancies then can lie either in P-wave of NN interaction – to which negative parity cross sections are extremely sensible, either be affected by the presence of 3NF with complicated structure.

Finally, few words should be mentioned about actual positions of these simplest hadronic resonances. R-matrix analysis [10], based on n-^{3}H experimental data, predicts these resonances to be situated below the peak region in total cross section. They have widths, $\Gamma = 2 * Im(E_{res})$, as large as 13 MeV (see Table 1). Using Faddeev-Yakubovski equations in conjunction with Complex Scaling method we can calculate resonance parameters – real energy and width – directly starting from realistic Nuclear Hamiltonians, provided these resonances are not very broad[2] $Im(E_{res}) < Re(E_{res})$. However using realistic interaction models we could not find any resonance in the vicinity of R-matrix ones.

[2]energies here are taken relatively to the closest threshold: n-^{3}H one.

Alternatively, n-^{3}H scattering phase shifts can be used to continue S-matrix analytically into the complex energy plane and search for its poles. Such procedure results resonances situated close to n-^{3}H threshold, with imaginary energy parts much larger than real ones. Positions of these resonances clearly depends on underlying interaction model, however they retain the same qualitative feature – having real energies scattered around n-^{3}H threshold, whereas their imaginary energies trapped inside $[-3, -2]$ MeV range.

p-^{3}H scattering at very low energies

^{4}He continuum is the most complex *4N* system, its spectrum contains numerous resonances (see Fig. 3) and thresholds. Only the first steps are made in describing this compound. In this contribution we demonstrate only very low energy results below n-^{3}He threshold, fully described by proton scattering in S-waves. Nevertheless already in this region excitation function – $\frac{d\sigma}{d\Omega}(E)\big|_{\theta=120^\circ}$ – has complicated structure due to existence of the $\mathcal{J}^\pi = 0^+$ resonance, often called α-particle breathing mode. This resonance is located at $E_R \approx 0.4\ MeV$ above p-^{3}H threshold and with its width $\Gamma \approx 0.5\ MeV$ covers almost the entire region below n-^{3}He.

Separation of n-^{3}He and p-^{3}H channels requires proper treatment of Coulomb interaction, the task is furthermore burden since both thresholds are described by the same isospin quantum numbers. When ignoring Coulomb interaction or treating it effectively, as was a case in the large number of nuclear scattering calculations, n-^{3}He and p-^{3}H thresholds coincide. In this case 0^+ resonant state moves below the joint threshold and becomes a bound state. Former fact is reflected in low energy scattering observables (see Fig. 6 dashed line): excitation function decreases smoothly with incident particles energy and does not demonstrate any resonant behavior. Only by properly taking Coulomb interaction into account, thus separating n-^{3}He and p-^{3}H thresholds, the ^{4}He excited state is placed in between.

In order to reproduce the shape of experimental excitation function, *NN* interaction model is furthermore obliged with high accuracy situate ^{4}He excited state inbetween two thresholds. In fact, width of the resonance is strongly correlated with its relative position to p-^{3}H threshold. If this resonance is slightly 'overbound' the peak in excitation curve becomes too narrow and is situated at lower energies. This is a case for MT I-III model prediction, see Fig. 6. If one 'underbinds' this resonance too flat excitation function is obtained, which furthermore underestimates cross sections. This is a case when AV18 NN interaction is considered without 3NF. Only once implementing UIX *3NF* in conjunction with Av18 *NN* model one obtains singlet scattering length as well as the excitation function in agreement with experimental data.

Evidently this system requires to be studied at higher energies. Already just above n-^{3}He threshold narrow $\mathcal{J}^\pi = 0^-$ resonance is situated, representing challenge for realistic nuclear interaction models. On the other hand this system, representing admixture of three different isospin states (namely $\mathcal{T}$=0, $\mathcal{T}$=1 and $\mathcal{T}$=2) also contains $\mathcal{T}$=1 resonances, reflecting ones in n-^{3}H and p-^{3}He systems and thus enabling us to explore charge dependence of nuclear interaction. We have undertaken this study and the first results above n-^{3}He threshold are already obtained.

In more distant perspective study of ^{2}H+^{2}H$\rightarrow \alpha + \pi^0$ reaction presents a great interest. In this reaction isospin conservation is broken, therefore its cross section is directly related to charge symmetry breaking term in nuclear Hamiltonian. First measurements of this reaction have been already reported [15]. Theoretical description of this process requires knowledge of accurate ^{2}H+^{2}H wave function at energies above 110 MeV, where four and three particle break-up channels are open. One is urged to find a plausible approximation to analyse these highly non-trivial processes.

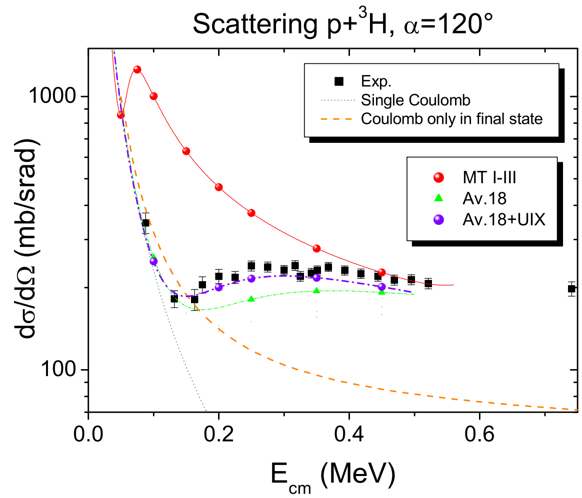

Figure 6: Various model calculations for p-^{3}H excitation function $\frac{d\sigma}{d\Omega}(E)\big|_{\theta=120^\circ}$ compared to experimental data.

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