Measurements of the CKM angle α/ϕ_2 at the B Factories

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We describe measurements related to the CKM angle α/ϕ_2, using exclusive decays of B mesons to charmless hadronic final states. Time-dependent CP asymmetries, branching fractions and direct CP asymmetries in the $B \to \pi\pi$, $B \to \rho\pi$ and $B \to \rho\rho$ modes are considered. Constraints on the CKM angle α are obtained, and perspectives on future constraints are discussed.

In the Standard Model (SM), CP violation occurs as a consequence of a complex phase in the 3×3 Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix [1]. The unitarity of the CKM matrix imposes the condition $V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$, where V_{ij} are the CKM matrix elements. This condition can be conveniently illustrated as a triangular relation in the $(\bar{\rho}, \bar{\eta})$ complex plane. A non-vanishing phase in the CKM matrix results in a non-zero area for the Unitarity Triangle (UT). Various measurements in the B meson system are sensitive to the CKM angles α, β, and γ of the UT [1]. In particular, the $\alpha = \pi - \beta - \gamma$ condition is better constrained using $b \to u$ processes.

In the $B^0 - \bar{B}^0$ system, information on the CKM angles can be obtained by measuring the time dependence of B^0 or $\bar{B}^0$ decays to CP eigenstates f_{CP}. The time distribution is given by

$$\frac{d}{dt} N(B^0(\bar{B}^0) \to f_{CP}) \propto$$
$$e^{-t/\tau} \left[1 - (\pm S_f \sin \Delta m t \mp C_f \cos \Delta m t)\right], \qquad (1)$$

where τ is the B^0 meson lifetime, and Δm is the $B^0 - \bar{B}^0$ oscillation frequency. The CP-violating coefficients S_f and C_f are functions of the parameter λ_f:

$$\lambda_f = \frac{q}{p} \frac{A(\bar{B}^0 \to f)}{A(B^0 \to f)} \, ,$$
$$S_f = \frac{2Im(\lambda_f)}{1 + |\lambda_f|^2} \, , \; C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}. \qquad (2)$$

[1]The CKM angles are also called ϕ_2, ϕ_1 ϕ_3, respectively. In this report we will indistinctly use either convention.

In this expression $A(B^0 \to f)$ (resp. $A(\bar{B}^0 \to f)$) is the decay amplitude of B^0 (resp. $\bar{B}^0$) to the final state f_{CP} respectively, and the q/p ratio ratio is given by the admixture of flavour eigenstates B^0 and $\bar{B}^0$ in the neutral B mass eigenstates. The equivalent notation $A_f = -C_f$ is also used. The SM predicts $|q/p| \simeq 1$. If only one weak phase enters the decay amplitude, $\lambda_f = \eta_f e^{2\theta}$, where $\eta_f = \pm 1$ is the CP of the final state f.

1. Extraction of α/ϕ_2 from $b \to u\bar{d}d$ decays

Techniques for accessing the CKM angle α/ϕ_2 are based on the use of isospin relations and measurements of branching ratios and CP asymetries in $B \to \pi\pi$, $B \to \rho\pi$ and $B \to \rho\rho$ decays. The $B \to \rho\pi$ system also exploits interferences owing to the resonant structure of the $\pi^+\pi^-\pi^0$ final state. This section describes the case of the $B \to \pi\pi$ system, whose general features are common to all cases, and following sections will refer to specificities in the $\rho\pi$ and $\rho\rho$ cases.

The $B^0 \to \pi^+\pi^-$ decay amplitude A^{+-}, can be expressed in a completely general form as

$$A^{+-} = T e^{i\gamma} + P e^{-i\beta} \qquad (3)$$

where the T and P terms are often referred to as the "tree" and "penguin" amplitudes, owing to the fact that they correspond, in a given convention, to the topologies given in Figure 1. as examples. It is understood that T and P receive contributions from all possible topologies. It is important to stress that, while the parametrisation (3) is trivial to be derived as a consequence

Table 1
Counting of degrees of freedom in the $B \to \pi\pi$ isospin system. One extra observable, the time-dependent CP asymmetry parameter S_{00} in $B^0 \to \pi^0\pi^0$ decays, in in principle available, but being out of experimental reach in the present, it is not considered in this account.

Unknowns	Observables	Constraints	Account
α	B^{+-}, B^{+0}, B^{00}	2 isospin triangles	13 unknowns
T^{+-}, P^{+-}	S_{+-}, C_{+-}	one common side	-7 observables
T^{+0}, P^{+0}	$A_{CP}(\pi^+\pi^0)$		-5 constraints
T^{00}, P^{00}	C_{00}		-1 global phase

of the CKM unitarity, it nevertheless holds even in presence of non-SM physics.

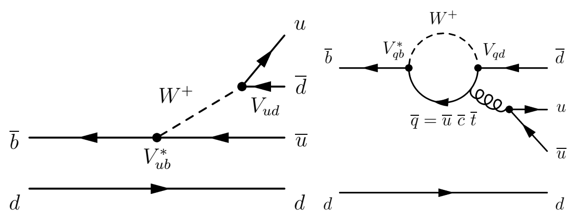

Figure 1. Tree (left) and penguin (right) diagrams for the $B^0 \to \pi^+\pi^-$ decay amplitudes.

In absence of penguin contributions, the time-dependent decay rate asymmetry is governed by a single weak phase, and (2) read

$$\lambda_{\pi\pi} = e^{2i\alpha},\ C_{\pi\pi} = 0\ ,\ S_{\pi\pi} = \sin\left[-2(\beta+\gamma)\right],$$

that is, no direct CP violation can occur if only one weak phase is present, and a measurement of the CP violation in the interference between mixing and decay gives direct access to the CKM angle α. In the realistic case, the phase of $\lambda_{\pi\pi}$ deviates from the 2α value as consequence of the interference between the T and P contributions to the amplitude, and the measurement of the time-dependent parameters S and C in the $B^0 \to \pi^+\pi^-$ decays is not sufficient to constrain the CKM angle $\alpha = -2(\beta+\gamma)$. One defines then

$$\sin 2\alpha_{\text{eff}} = \frac{S_{\pi\pi}}{\sqrt{1-C^2_{\pi\pi}}} \neq \sin 2\alpha, \qquad (4)$$

where α_{eff} is called the "effective angle". However, isospin invariance of the strong interactions can provide additional information on the decay amplitudes. Gronau and London[2] have demonstrated that this can be obtained from the isospin relations of the three $B \to \pi\pi$ decay amplitudes

$$A(\pi^+\pi^-) - \sqrt{2}A(\pi^+\pi^0) + \sqrt{2}A(\pi^0\pi^0) = 0, \qquad (5)$$

plus a similar relation for the CP-conjugated amplitudes.

Measurements of the CP-averaged branching ratios of the $B^0 \to \pi^+\pi^-$, $B^+ \to \pi^+\pi^0$ and $B^0\pi^0\pi^0$ decays, plus the CP-violating asymmetries $S_{\pi^+\pi^-}$, $C_{\pi^+\pi^-}$ and $C_{\pi^0\pi^0}$ allow the extraction, up to an irreducible eith-fold ambiguity, of the CKM angle α. Table 1. summarises the number of unknowns and observables in the isospin analysis of the $B \to \pi\pi$ system. Isospin analysis can provide useful information, even in absence of some observables; as an example, Grossmann and Quinn[3] showed that an upper limit on the $B^0 \to \pi^0\pi^0$ branching fraction translates into a bound on the angle shift $|\alpha - \alpha_{\text{eff}}|$. More stringent bounds have also been derived[4].

The isospin constraints can be expressed in an equivalent way in the following parametrisation:

$$\begin{aligned} A^{+-} &= Te^{i\gamma} + Pe^{-i\beta} \\ \sqrt{2}A^{+0} &= (T+C)e^{i\gamma} + P_{EW}e^{-i\beta} \\ \sqrt{2}A^{00} &= Ce^{i\gamma} + (P_{EW} - P)e^{-i\beta} \end{aligned} \qquad (6)$$

where C is referred to as the "colour-suppressed" tree amplitude, and P_{EW} are the electroweak penguins. Within the SM, electroweak penguins in the two-pion modes are expected to be small; furthermore, it has been shown[5] that P_{EW} carries, the same phase as the tree amplitude; thus no direct CP violation can occur in the $B^+ \to$

Table 2
Results on branching ratios and CP asymmetries in the $B \to \pi\pi$ system.

Parameter	BaBar	Belle
$C_{+-} = -A_{+-}$	$-0.09 \pm 0.15 \pm 0.04$	$-0.56 \pm 0.12 \pm 0.06$
S_{+-}	$-0.30 \pm 0.17 \pm 0.03$	$-0.67 \pm 0.16 \pm 0.06$
$B(B^0 \to \pi^+\pi^-)$	$5.5 \mp 0.4 \pm 0.3$	$4.4 \pm 0.6 \pm 0.3$
$C_{00} = -A_{00}$	$-0.12 \pm 0.56 \pm 0.06$	$+0.44 \pm 0.53 \pm 0.17$
$B(B^0 \to \pi^0\pi^0)$	$1.17 \mp 0.32 \pm 0.10$	$2.3 \pm 0.5 \pm 0.3$
A_{+0}	$-0.01 \pm 0.10 \pm 0.02$	$+0.02 \pm 0.08 \pm 0.01$
$B(B^+ \to \pi^+\pi^0)$	$5.8 \mp 0.6 \pm 0.4$	$5.0 \pm 1.2 \pm 0.5$

$\pi^+\pi^0$ mode (this refers to the "one common side" constraint in Table 1.).

Within this report, the hypothesis of vanishing electroweak penguins will be assumed; it has been shown[6] that a more proper treatment would bring a small shift on the CKM angle α. In the same spirit, potential isospin-breaking effects, like $\pi^0/\eta/\eta'$ mixing, will also be neglected.

2. Results on α/ϕ_2 from $B \to \pi\pi$ decays

The results from BaBar [7] and Belle[8] on branching ratios and CP asymmetries are summarised in Table 2. At the time of this workshop, the reported measurements of branching ratios did not include a globally coherent treatment of QED corrections due to final-state radiation. This issue was eventually addressed by the two experiments in results made public shortly after this workshop, and are thus not discussed here. On a similar basis, it is worth mentioning that the BaBar and Belle results on the CP parameters S_{+-}, C_{+-} in $B^0 \to \pi^+\pi^-$ were $\sim 2.3\sigma$ away, the BaBar measurements being compatible with no CP violation, and the Belle results constitute an observation of CP violation at the 5.4σ level, and an evidence of direct CP violation at a level of 4.0σ. This situation also evolved later on: results made public after the workshop show an improved compatibility between results from the two experiments.

Figure 2. shows the constraints on the CKM angle α obtained from the isospin analysis for the BaBar and Belle experiments. Both experiments have used the technique described in [16] to obtain confidence curves. The eight mirror solutions on α are clearly visible in the BaBar curve, while for Belle, the eight solutions are quasi-degenerated, four being merged by pairs and the remaining four clustered into a single peak. Close solutions are separated by less than half of a degree, so they are difficult to resolve in the figure but nevertheless present. Data from Belle exclude values of ϕ_2 in the $[19^o, 72^o]$ interval (95% C.L.), and BaBar excludes α in the $[29^o, 61^o]$ range (90% C.L.). The question of exclusion for the value $\alpha \equiv 0$ was raised, in view of the Belle evidence for CP violation in $B^0 \to \pi^+\pi^-$. Indeed, if evaluated precisely at that singular value, strict isospin analysis of the Belle results exclude the single point $\alpha \equiv 0$ at 5.4σ, but non-zero values of α cannot be excluded[15]. Equivalent facts hold for the other singular point $\alpha \equiv 0$.

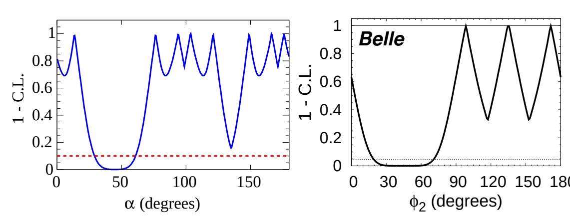

Figure 2. Constraints on the CKM angle α (resp. ϕ_2) from isospin analysis of the BaBar (resp. Belle) results on $B \to \pi\pi$ decays.

The BaBar measurements provide a bound

Table 3
Results on branching ratios, CP asymmetries and polarisation fractions in the $B \to \rho\rho$ system.

Parameter	*BABAR*	Belle
$B(B^0 \to \rho^+\rho^-)$	$30 \pm 4 \pm 5$	$22.8 \pm 3.8^{+2.3}_{-2.6}$
f_L^{+-}	$0.978 \pm 0.014^{+0.021}_{-0.029}$	$0.941^{+0.034}_{-0.040} \pm 0.030$
S_L^{+-}	$-0.33 \pm 0.24^{+0.08}_{-0.14}$	$0.08 \pm 0.41 \pm 0.09$
$C_L^{+-} = -A_L^{+-}$	$-0.03 \pm 0.18 \pm 0.09$	$0.00 \pm 0.30 \pm 0.09$
$B(B^+ \to \rho^+\rho^0)$	$17.2 \pm 2.5 \pm 2.8$	$31.7 \pm 7.1^{+3.8}_{-6.7}$
f_L^{+0}	$0.96 \pm 0.04 \pm 0.05$	$0.95 \pm 0.11 \pm 0.02$
$B(B^0 \to \rho^0\rho^0)$	< 1.1 (90% C.L.)	

$|\alpha - \alpha_{\rm eff}| < 35^{\rm o}$ (90% C.L.), and the Belle measurements give $|\alpha - \alpha_{\rm eff}| < 38^{\rm o}$ (95% C.L.). The $B \to \pi\pi$ isospin analysis is limited by the relatively large value of the $B^0 \to \pi^0\pi^0$ branching ratio. It is neither large enough to allow for a precision measurement of the $\alpha - \alpha_{\rm eff}$ phase shift, nor small enough to provide a sensible bound on the magnitude of this shift.

These conclusions can also be illustrated by a closer look at the amplitude parameters P, C, T. The *BABAR* results being compatible with no CP violation, *BABAR* data favours relatively small values for the module of the penguin-to-tree ratio $|P/T|$, and sets no stringent constraint on its phase; in contrast, Belle data requires larger, non-vanishing values for $|P/T|$, with a phase excluding the $0^{\rm o}$ and $180^{\rm o}$ values. Concerning colour-suppression, the relatively large value of the $B^0 \to \pi^0\pi^0$ branching fraction can be stated in terms of the decay amplitudes as $2 \times |A^{00}| \simeq |A^{+-}|$. This condition disfavours vanishingly small values for the colour-suppressed amplitude C. Two distinct solutions are observed, both with values of $|C/T|$ on the range $[0.5, 1.0]$. Confidence curves for amplitude ratios and phases can be found in [14].

3. Results from the $B \to \rho\rho$ system

When compared to the $B \to \pi\pi$ system, the $B \to \rho\rho$ system shows some specific features, that are to be addressed both in the experimental and isospin analysis techniques.

In contrast to the $B^0 \to \pi^+\pi^-$ case, the CP content of the vector-vector decay $B^0 \to \rho^+\rho^-$ is complicated by the presence of three helicity states: the $h = \pm 1$ states, corresponding to transverse polarisation, are an admixture of CP-even and CP-odd states, while the $h = 0$ state corresponds to longitudinal polarisation, and is a CP-even final state.

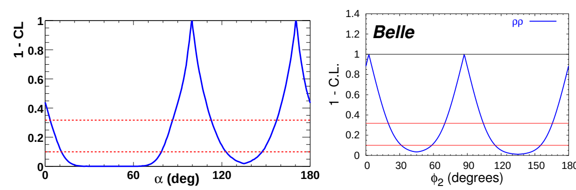

Figure 3. Constraints on the CKM angle α (resp. ϕ_2) from isospin analysis of the *BABAR* (resp. Belle) results on $B \to \rho\rho$ decays.

The experimental study of $B \to \rho\rho$ decays is also complicated by the lower detection efficiencies on the four-body final state, plus the presence of one or two π^0's for the $B^+ \to \rho^+\rho^0$ and $B^0 \to \rho^+\rho^-$. Also, several classes of B backgrounds, sometimes with poorly known rates and CP content, need to be characterised and properly subtracted from the signal samples. On the positive side, the $B^0 \to \rho^0\rho^0$ decay has four charged tracks in the final state, and thus B vertexing can be performed, in contrast to the $B^0 \to \pi^0\pi^0$ mode; as a consequence, the measurement of the time-dependent CP violation parameters is possible in principle. The addition of this extra observable on the isospin analysis should help removing some of the mirror solutions in α.

Table 3. summarises the results from *BABAR* [9] and Belle[10] on the $B \to \rho\rho$ system. While a complete angular analysis is in principle required to disentangle the different CP components, the fact that $B \to \rho\rho$ decays are dominated by longitudinal polarisation simplifies noticeably the analysis, as this component is a pure CP-even state. Figure 3. shows the constraints on the CKM angle α from the measurements of CP asymmetries, branching ratios and polarisation fractions from *BABAR* and Belle. The smallness of the $B^0 \to \rho^0\rho^0$ branching ratio is also helpful to the isospin analysis, as this translates into stronger constraints on the penguin and colour-suppressed amplitudes. In this way, although the number of available observables is unsufficient for a complete isospin analysis, the Grossmann-Quinn bound on $|\alpha - \alpha_{eff}| < 17^o$ translates in a stringent constraint of the CKM angle α: the SM-preferred solution lies in the $[79^o, 113^o]$ interval (90% C.L.). It is worth mentioning that before the latest *BABAR* measurement of the $B^+ \to \rho^+\pi^0$ branching ratio, the central values did not satisfy triangle inequalities for the $\rho\rho$ isospin triangles. This feature translated into an artificially stringent constraint on the penguin pollution.

The penguin and colour-suppressed amplitude ratios for $B \to \rho\rho$ are bounded at $|P/T|, |C/T| < 0.3$, and zero values for these amplitudes are not excluded by experimental data, reflecting that CP violation parameters in $B^0 \to \rho^+\rho^-$ decays are compatible, at the current level of statistics, with the hypothesis of no CP violation.

4. Extraction of α from $B^0 \to (\rho\pi)^0$ decays

The decay $B^0 \to \pi^+\pi^-\pi^0$ proceeds predominantly through the isospin-related, resonant $\rho^\pm\pi^\mp$, $\rho^0\pi^0$ intermediate states. Interference between these modes occurs mostly on the edges of the $\pi^+\pi^-\pi^0$ Dalitz plane, and provides further information on the dynamics of the decay. Snyder and Quinn have shown[11] that the time-dependent interference patterns in the Dalitz plane allows to perform an ambiguity-free extraction of the CKM angle α, by constraining the strong phases. A more straightforward technique is the so-called "quasi-2-body" approach, that consists on removing interference areas from the ρ bands in the Dalitz plot, perform a time-dependent analysis, and use the pentagonal isospin relations for the $B \to \rho\pi$ system to extract information on α. In the quasi-2-body approach, the parametrisation of time-dependent rates for $B^0 \to \rho^\pm\pi^\mp$ requires three additional CP parameters: the charge asymmetry $A_{\rm CP}$ between $\rho^+\pi^-$ and $\rho^-\pi^+$, and two dilution parameters ΔS and ΔC. The $A_{\rm CP}$ and C parameters are usually expressed into the more convenient ones $A^{+-}_{\rho\pi}$ and $A^{-+}_{\rho\pi}$ The combined results[17] from the quasi-2-body analyses performed by *BABAR* and Belle, gives $A^{+-} = (-0.15 \pm 0.09)$ and $A^{-+} = (-0.47^{+0.13}_{-0.14})$, and significance of direct CP violation is 3.4σ.

BABAR has also performed a time-dependent amplitude (Dalitz plot) analysis. The main output of the amplitude analysis is a confidence curve for the CKM angle α, that is shown in Figure 4. The 68% C.L. constraint is $\alpha = (113^{+27}_{-17} \pm 6)^o$. Also shown is the constraint on the relative phase δ_{+-} between the $B^0 \to \rho^+\pi^-$ and the (complex conjugate of the) $B^0 \to \rho^-\pi^+$ decay amplitudes.

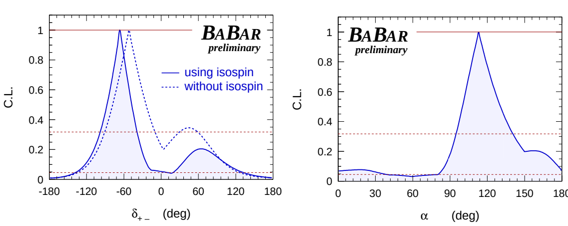

Figure 4. Confidence Level curves from the *BABAR* Dalitz analysis of $B^0 \to (\rho\pi)^0$. Left: Constraints on the relative phase δ. Right : Constraint on the CKM angle α.

5. Conclusions

The *BABAR* and Belle experiments have measured decay rates and CP asymmetries in the $B \to \pi\pi$, $B \to \rho\pi$ and $B \to \rho\rho$ systems. *BABAR* has also performed a time-dependent Dalitz-plot analysis of $B^0 \to (\rho\pi)^0$ decays. The combined result on the CKM angle α/ϕ_2, shown in Figure 5 is in good agreement with the global fit to the CKM matrix, that gives an indirect measurement of $(100.2^{+15.0}_{-8.8})^{o}$. The constraint on α is dominated by the isospin analysis of $B \to \rho\rho$, followed by the $\rho\pi$ Dalitz result, that allows to suppress the mirror solution at high values of α. The isospin analysis on the $B \to \pi\pi$ system suffers from the relatively large value of the $B^0 \to \pi^0\pi^0$ branching ratio. An improved constraint from $B \to \pi\pi$ will require significantly higher statistics. The LHCb experiment has studied the potential for precision measurements of the CKM angle α. The most promising approaches are the amplitude analysis of $B^0 \to (\rho\pi)^0$ decays, and the measurement of time-dependent CP asymmetries in $B^0 \to \rho^0\rho^0$ decays.

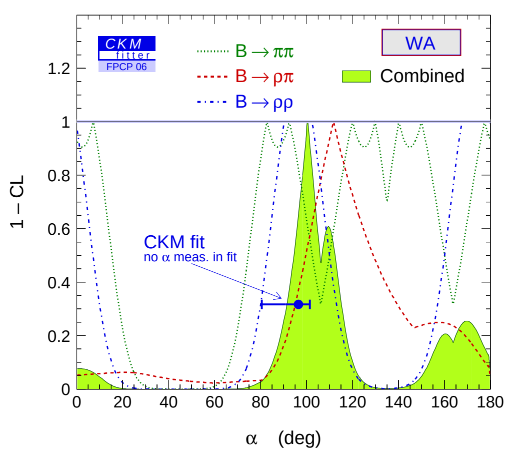

Figure 5. Confidence Level curves on the CKM angle α/ϕ_2 from the combined $B \to \pi\pi$, $B \to \rho\pi$ and $B \to \rho\rho$ systems. The point with error bars corresponds to the 68% C.L. interval obtained from the global CKM fit in exclusion of direct measurements of α.

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