CKM Fits: What the Data Say
(focused on B-Physics)

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Charm is interesting in several special areas, but I will concentrate on b’s
The Unitary Wolfenstein Parameterization

- The standard parameterization uses Euler angles and one CPV phase → unitary!

\[ V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}e^{i\delta} & s_{23} \\ s_{12}s_{23} - c_{12}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \]

- Now, define \( s_{12} \equiv \lambda \), \( s_{23} \equiv A\lambda^2 \), \( s_{13}e^{-i\delta} \equiv A\lambda^3(\rho - i\eta) \)

- And insert into \( V \rightarrow V \) is still unitary! With this one finds (to all orders in \( \lambda \)):

\[
\rho + i\eta = \frac{\sqrt{1 - A^2\lambda^4(\bar{\rho} + i\bar{\eta})}}{\sqrt{1 - \lambda^2[1 - A^2\lambda^4(\bar{\rho} + i\bar{\eta})]}}
\]

where: \( \bar{\rho} + i\bar{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \)

\[
\lambda^2 \equiv \frac{|V_{us}|^2}{|V_{ud}|^2 + |V_{us}|^2}, \quad A^2\lambda^4 \equiv \frac{|V_{cb}|^2}{|V_{ud}|^2 + |V_{us}|^2}
\]

- Physically meaningful quantities are phase-convention invariant

- Four unknowns [unitary-exact and phase-convention invariant]: \( A, \lambda, \bar{\rho}, \bar{\eta} \)
The CKM Matrix: Four Unknowns

Measurement of Wolfenstein parameters:

- $\lambda$ from $|V_{ud}|$ (nuclear transitions) and $|V_{us}|$ (semileptonic $K$ decays)
  - combined precision: 0.5%

- $A$ from $|V_{cb}|$ (inclusive and exclusive semileptonic $B$ decays)
  - combined precision: 2%

- $\bar{\rho}$, $\eta$ from (mainly) CKM angle measurements:
  - combined precision: 20% ($\rho$), 7% ($\eta$)
All measurements must agree.

Pre B-Factory:

Can the KM mechanism describe flavor dynamics of many constraints from vastly different scales?

This is what matters and not the measurement of the CKM phase’s value \textit{per se}. 

\textbf{Predictive Nature of KM Mechanism}
The (rescaled) Unitarity Triangle: The $B_d$ System

Convenient method to illustrate (dis-)agreement of observables with CKM predictions

\[
\frac{V_{ud}V_{ub}^*}{V_{ub}V_{ub}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{ub}V_{ub}^*} = 0
\]

phase invariant: $\rho + i\eta$

"There is no such thing as $\alpha/\phi_2$"

$[\alpha = \pi - (\beta+\gamma)]$

$B \to D^{(*)}K^{(*)}$

$B \to D_{K_S^0\pi^+\pi^-}$

$B^0 \to DK_S^0, ...$

$B^0 \to D^*\pi(\rho)$

$B^0 \to \pi\pi, \rho\pi, \rho\rho$

$B^0 \leftrightarrow \bar{B}^0 : \Delta m_d$

$B \to \rho(\omega)_g / B \to K^{(*)}_g$

$b \to u\ell\nu$

$b \to c\bar{c}s$

$b \to s\bar{s}s$

$B^0 \to J/\psi K_S^0, ...$

$B^0 \to \phi K_S^0, ...$
The Unitarity Triangle: The $B_s$ System (hadron machines)

(sb) triangle ("$B_s$ triangle"): 

$$ V_{us} V_{ub}^* + V_{cs} V_{cb}^* + V_{ts} V_{tb}^* = 0 $$

$$ O(\lambda^4) + O(\lambda^2) + O(\lambda^2) = 0 $$

$\Rightarrow$ squashed triangle

$$ \chi = \beta_s = \arg \left[ -\frac{V_{cs} V_{cb}^*}{V_{ts} V_{tb}^*} \right] $$

Attention: sign

(ut) triangle:

$$ V_{td} V_{ud}^* + V_{ts} V_{us}^* + V_{tb} V_{ub}^* = 0 $$

$$ O(\lambda^3) + O(\lambda^3) + O(\lambda^3) = 0 $$

$\Rightarrow$ non-squashed triangle

$B_s^0 \rightarrow D_s K$

$B_s^0 \rightarrow K^+ K^-$

$B_s^0 \rightarrow J/\psi K_s^0$

$B_s^0 \rightarrow D^+_s D^-_s, \ldots$

$B_s^0 \leftrightarrow \bar{B}_s^0 : \Delta m_s / \Delta m_d$

$B_s^0 \rightarrow \mu^+ \mu^-$ (BR for $B_s^0 \sim 1 \times 10^{-10}$)

$B_s^0 \rightarrow \phi \phi, \ldots$

$B_s^0 \rightarrow J/\psi, \ldots$

$b \rightarrow s \bar{s}s$

$b \rightarrow c \bar{c}s$
Probing short distance (quarks) but confined in hadrons (what we observe)

- QCD effects must be under control (various tools: HQET, SCET, QCDF, LQCD, ...)
- “Theoretical uncertainties” have to be controlled quantitatively in order to test the Standard Model. There is however no systematic method to do that.
Digression: Statistics
Statistics tries answering a wide variety of questions ➔ two main different frameworks:

**Frequentist:** probability *about the data* (randomness of measurements), given the model

\[ P(\text{data}|\text{model}) \]  

*Hypothesis testing:* given a model, assess the **consistency** of the data with a particular parameter value ➔ 1-CL curve (by varying the parameter value)

**Bayesian:** probability *about the model* (degree of belief), given the data

\[ P(\text{model}|\text{data}) \propto \text{Likelihood}(\text{data},\text{model}) \times P(\text{prior}) \]

\[ P(\text{data}|\text{model}) \neq P(\text{model}|\text{data}): \]

- model: Male or Female
- data: pregnant or not pregnant

\[ P(\text{pregnant} | \text{female}) \sim 3\% \]

but

\[ P(\text{female} | \text{pregnant}) >>> 3\% \]

Although the graphical displays appear similar: the meaning of the “Confidence level” is not the same. It is especially important to understand the difference in a time where one seeks deviation of the SM.
The Bayesian approach in physical science fails in the sense that nothing guarantees that my uncertainty assessment is any good for you - I'm just expressing an opinion (degree of belief). To convince you that it's a good uncertainty assessment, I need to show that the statistical model I created makes good predictions in situations where we know what the truth is, and the process of calibrating predictions against reality is inherently frequentist."

**Digression: Statistics (cont.)**


**How to read a Posterior PDF?**

- updated belief (after seeing the data) of the plausible values of the parameter
- it's a bet on a proposition to which there is no scientific answer

My talk is about “**What the Data say**”, thus I will stick to the frequentist approach.
I) Direct Measurement: magnitude
\[ |V_{ud}| \text{ and } |V_{us}| \text{ [not discussed here]} \]
\[ |V_{ub}| \text{ and } |V_{cb}| \]
\[ B^+ \rightarrow \tau^+ \nu \]

CPV in \( K^0 \) mixing [not discussed here]
\( B_d \) and \( B_s \) mixing

II) Angle Measurements:
\[ \sin 2\beta \]
\[ \alpha : (B \rightarrow \pi \pi, \rho \rho, \rho \pi) \]
\[ \gamma : \text{ADS, GLW, Dalitz (GGSZ)} \]
\(|V_{cb}| \text{ and } |V_{ub}|\)
\[ |V_{cb}| (\rightarrow A) \text{ and } |V_{ub}| \]

For \( |V_{cb}| \) and \( |V_{ub}| \) exist exclusive and inclusive semileptonic approaches (complementary)

<table>
<thead>
<tr>
<th>exclusive</th>
<th>inclusive</th>
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<tbody>
<tr>
<td>( b \rightarrow u )</td>
<td>( b \rightarrow u )</td>
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<tr>
<td>( B \rightarrow \pi \ell \nu )</td>
<td>( B \rightarrow X_u \ell \nu )</td>
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<tr>
<td>( b \rightarrow c )</td>
<td>( B \rightarrow D^* \ell \nu )</td>
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<tr>
<td>( B \rightarrow X_c \ell \nu )</td>
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**Complication for charmless decays:**

\[
\frac{\Gamma(b \rightarrow ul\nu)}{\Gamma(b \rightarrow cl\nu)} \approx \frac{|V_{ub}|^2}{|V_{cb}|^2} \approx \frac{1}{50}
\]

→ need to apply kinematic cuts to suppress \( b \rightarrow cl\nu \) background
→ measurements of partial branching fractions in restricted phase space regions
→ theoretical uncertainties more difficult to evaluate

OPE parameters measured from data (spectra and moments of \( b \rightarrow s\gamma \) and \( b \rightarrow c\ell\nu \) distributions)

\( |V_{ub}| \) is crucial for the SM prediction of \( \sin(2\beta) \)
\( |V_{cb}| \) is important in the kaon system (\( \varepsilon_K, \text{BR}(K \rightarrow \pi \nu\nu), \ldots \) )
$|V_{cb}|$ and $|V_{ub}|$

$|V_{cb}|$: Precision measurement: 1.7%!

$|V_{cb}|_{\text{incl.}} [10^{-3}] = 41.70 \pm 0.70$  
[PDG06]

$|V_{cb}|_{\text{excl.}} [10^{-3}] = 39.7 \pm 2.0$  
[w/ FF=0.91±0.04  
[ICHEP06]

$|V_{ub}|$:

- SF params. from $b \rightarrow c l \nu$, OPE from BLPN
- BR precision ~8%, $|V_{ub}|$ excl. ~ 16%: theory dominated
- HFAG with our error budget

$|V_{ub}| [10^{-3}] = 4.10 \pm 0.09_{\text{exp}} \pm 0.39_{\text{theo}}$
\( B^+ \rightarrow \tau^+ \nu_\tau \)

- Helicity-suppressed annihilation decay sensitive to \( f_B \times |V_{ub}| \)
- Powerful together with \( \Delta m_d \): removes \( f_B \) (Lattice QCD) dependence
- Sensitive to charged Higgs replacing the \( W \) propagator

\[
\text{BR}(B^+ \rightarrow \tau^+ \nu) = \frac{G_F^2 m_B m_{\tau}}{8\pi} \left(1 - \frac{m_{\tau}^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2
\]

**ICHEP06**

\( \text{BR}[10^{-4}]=0.88^{+0.68}_{-0.67} \text{ (stat)} \pm 0.11 \text{ (syst)} \)

(320m)

\( \text{BR}[10^{-4}]=1.79^{+0.56}_{-0.49} \text{ (stat)}^{+0.39}_{-0.46} \text{ (syst)} \)

(447m)

- Prediction from global CKM fit :

\[
\text{BF}(B^+ \rightarrow \tau^+ \nu_\tau) = (0.87^{+0.13}_{-0.20}) \times 10^{-4}
\]
\[ \Delta m_d \text{ and } \Delta m_s \]
\( \Delta m_d \) and \( \Delta m_s \): constraints in the (\( \rho \)-\( \eta \)) plane

\[
\Delta m_s = \frac{G_F^2}{6\pi^2} m_{B_s} m_W^2 \eta_B S_0(x_t) f_{B_s}^2 B_s \left| V_{ts} V_{tb}^* \right|^2
\]

The point is:

\[
f_{B_s}^2 B_s = \frac{f_{B_d}^2 B_d}{f_{B_d}^2 B_d} \xi^2 f_{B_d}^2 B_d
\]

\( \xi \): SU(3)-breaking corrections

Very weak dependence on \( \bar{\rho} \) and \( \bar{\eta} \)

Measurement of \( \Delta m_s \) reduces the uncertainties on \( f_{B_d}^2 B_d \) since \( \xi \) is better known from Lattice QCD

\[
\sigma_{\text{rel}} \left( f_{B_d}^2 B_d / f_{B_d}^2 B_d \right) = 36\% \quad \rightarrow \quad \sigma_{\text{rel}} \left( \frac{\xi^2}{f_{B_s}^2 B_s / f_{B_d}^2 B_d} \right) = 10\%
\]

\( \Rightarrow \) Leads to improvement of the constraint from \( \Delta m_d \) measurement on \( |V_{td} V_{tb}^*|^2 \)

\[
\Delta m_d = \frac{G_F^2}{6\pi^2} m_{B_d} m_W^2 \eta_B S_0(x_t) f_{B_d}^2 B_d \left| V_{td} V_{tb}^* \right|^2 \propto A^2 \lambda^6 \left[ (1 - \bar{\rho})^2 + \bar{\eta}^2 \right]
\]
$\Delta m_s$:

$17 < \Delta m_s < 21 \text{ ps}^{-1} @90 \text{ C.L.}$

$\Delta m_s : 17.77 \pm 0.10 \text{(stat.)} \pm 0.07 \text{ (syst.) ps}^{-1}$

The signal has a significance of $5.4\sigma$
Constraint on $|V_{td}/V_{ts}|$

\[
\frac{\Delta m_d}{\Delta m_s} = \frac{m_{Bs}}{m_{Bd}} \xi^{-2} \frac{|V_{td}|^2}{|V_{ts}|^2}
\]

⇒ First strong indication that $B_s$-$B_s$ mixing is probably SM-like.
angle $\beta$
"The" raison d’etre of the B factories:

\[ \sin(2\beta) \equiv \sin(2\phi_1) \]

Conflict with \( \sin 2\beta_{\text{eff}} \) from s-penguin modes? (New Physics (NP)?)

NP can contribute differently among the various s-penguin modes (Naive average: 0.52±0.05).

NB: a disagreement would falsify the SM. The interference NP/SM amplitudes introduces hadronic uncertainties

Cannot determine the NP parameters cleanly
angle $\alpha$
angle $\alpha$

Tree : dominant

Penguin : competitive ?

Time-dependent CP observable :

$$A_{h^+h^-}(t) = S_{h^+h^-} \sin(\Delta m_d t) - C_{h^+h^-} \cos(\Delta m_d t)$$

$$= \sqrt{1 - C_{h^+h^-}^2} \sin(2\alpha_{\text{eff}}) \cdot \sin(\Delta m_d t) - C_{h^+h^-} \cos(\Delta m_d t)$$

Time-dependent $CP$ analysis of $B^0 \rightarrow \pi^+\pi^−$ alone determines $\alpha_{\text{eff}}$: but, we need $\alpha$!

Isospin analysis

($\alpha$ can be resolved up to an 8-fold ambiguity within $[0,\pi]$)
Isospin Analysis: $B \to \pi\pi$

<table>
<thead>
<tr>
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<th>BABAR (347m)</th>
<th>Belle (532m)</th>
<th>Average</th>
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</thead>
<tbody>
<tr>
<td>$S_{\pi\pi}$</td>
<td>$-0.53 \pm 0.14 \pm 0.02$</td>
<td>$-0.61 \pm 0.10 \pm 0.04$</td>
<td>$-0.58 \pm 0.09$</td>
</tr>
<tr>
<td>$C_{\pi\pi}$</td>
<td>$-0.16 \pm 0.11 \pm 0.03$</td>
<td>$-0.55 \pm 0.08 \pm 0.05$</td>
<td>$-0.39 \pm 0.07$</td>
</tr>
</tbody>
</table>

"agreement": 2.6$\sigma$
Isospin Analysis: $B \rightarrow \rho \rho$

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<thead>
<tr>
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<th>BABAR (347m)</th>
<th>Belle (275m)</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{\rho \rho}$</td>
<td>$-0.19 \pm 0.21^{+0.05}_{-0.07}$</td>
<td>$0.08 \pm 0.41 \pm 0.09$</td>
<td>$-0.13 \pm 0.19$</td>
</tr>
<tr>
<td>$C_{\rho \rho}$</td>
<td>$-0.07 \pm 0.15 \pm 0.06$</td>
<td>$0.0 \pm 0.3 \pm 0.09$</td>
<td>$-0.06 \pm 0.14$</td>
</tr>
</tbody>
</table>

**Isospin analysis:**

$\alpha = [94 \pm 21]^\circ$
Isospin Analysis: angle $\alpha_{\text{eff}}$ [B→ $\pi\pi/\rho\rho$]

- Isospin analysis B→$\pi\pi$:
  \[|\alpha - \alpha_{\text{eff}}| < 32.1^\circ \text{ (95\% CL)}\]

- Isospin analysis B→$\rho\rho$:
  \[|\alpha - \alpha_{\text{eff}}| < 22.4^\circ \text{ (95\% CL)}\]
The $B \rightarrow \rho \pi$ System

Dominant mode $\rho^+ \pi^-$ is not a $CP$ eigenstate

Amplitude interference in Dalitz plot

- simultaneous fit of $\alpha$ and strong phases
- Measure 26 (27) bilinear Form Factor coefficients
- correlated $\chi^2$ fit to determine physics quantities

Aleksan et al, NP B361, 141 (1991)

Snyder-Quinn, PRD 48, 2139 (1993)

Lipkin et al., PRD 44, 1454 (1991)
Isospin Analysis: angle $\alpha \ [B \rightarrow \pi\pi / \rho\pi / \rho\rho]$

$\alpha_{\text{B-Factories}} = [93^{+11}_{-9}]^\circ$ \quad $\alpha_{\text{Global Fit}} = [100^{+5}_{-7}]^\circ$

$B \rightarrow \rho\rho$: at very large statistics, systematics and model-dependence will become an issue

$B \rightarrow \rho\pi$ Dalitz analysis: model-dependence is an issue!
angle $\gamma$
angle $\gamma$ [next UT input that is not theory limited]

$b \to c\bar{u}s,u\bar{c}s$

$B^- \to b \bar{u} W^- s c \to K^{(*)-}
\begin{align*}
\{ & b \bar{u} W^- s c \to K^{(*)-} \\
D^{(*)0} & \to b \bar{u} W^- u c \to \bar{D}^{(*)0}
\end{align*}$

Tree: dominant
\[ \propto V_{cb}V_{us}^* \propto \lambda^3 \]

Tree: color-suppressed
\[ \propto V_{ub}V_{cs}^* \propto \lambda^3 \sqrt{\rho^2 + \eta^2} \]

relative CKM phase: $\gamma$

Several variants:

- GLW: $D^0$ decays into CP eigenstate
- ADS: $D^0$ decays to $K^-\pi^+$ (favored) and $K^+\pi^-$ (suppressed)
- GGSZ: $D^0$ decays to $K_S\pi^+\pi^-$ (interference in Dalitz plot)

All methods fit simultaneously: $\gamma$, $r_B$ and $\delta$ (different $r_B$ and $\delta$)

$\{ r_B, r_B^* \}$ how small?

$\sigma_\gamma$ depends significantly on the value of $r_B$
Constraint on $\gamma$

$\gamma_{B\text{-Factories}} = [60^{+38}_{-24}]^\circ$

$\gamma_{\text{Global Fit}} = [59^{+9}_{-4}]^\circ$

$r_B(DK) = 0.10^{+0.03}_{-0.04}$

$r_B(D*K) = 0.10^{+0.04}_{-0.06}$

$r_B(DK^*) = 0.11^{+0.09}_{-0.11}$
Putting it all together

the global CKM fit

Inputs:

- $|V_{ub}/V_{cb}|$
- $\Delta m_d$
- $\Delta m_s$
- $B \rightarrow \tau\nu$
- $|\epsilon_K|$
- $\sin 2\beta$
- $\alpha$
- $\gamma$

(excl. at CL > 0.95)
The global CKM fit: Testing the CKM Paradigm

CP Conserving

**CP-insensitive observables imply CP violation!**

Angles (no theory)

No angles (with theory)
The global CKM fit: Testing the CKM Paradigm (cont.)

Tree (NP-Free) "Reference UT"

[No NP in $\Delta l=3/2$ $b \rightarrow d$ EW penguin amplitude
Use $\alpha$ with $\beta$ (charmonium) to cancel NP amplitude]

Loop

CKM mechanism: dominant source of CP violation
The global fit is not the whole story: several $\Delta F=1$ rare decays are not yet measured
$\Rightarrow$ Sensitive to NP
The global CKM fit: selected predictions

**Wolfenstein parameters:**

\[
A = 0.806^{+0.014}_{-0.014} \quad \lambda = 0.2272^{+0.0010}_{-0.0010} \quad \bar{\rho} = 0.195^{+0.022}_{-0.055} \quad \bar{\eta} = 0.326^{+0.027}_{-0.015}
\]

**Jarlskog invariant:**

\[
J = (2.91^{+0.25}_{-0.14}) \times 10^{-5}
\]

**UT Angles:**

\[
\alpha = (99.0^{+4.0}_{-9.4})^\circ \quad \beta = (22.03^{+0.72}_{-0.62})^\circ \quad \gamma = (59.0^{+9.2}_{-3.7})^\circ \quad \Sigma_{\text{meas.}} = (175^{+40}_{-27})^\circ
\]

**UT sides:**

\[
R_u = 0.380^{+0.011}_{-0.009} \quad R_t = 0.868^{+0.060}_{-0.025}
\]

**B-B mixing:**

\[
\Delta m_s = (18.9^{+5.7}_{-2.8}) \text{ps}^{-1} \quad (\text{CKM Fit}) \quad \Delta m_s : 17.77 \pm 0.1\text{(stat.)} \pm 0.07 \text{ (syst.) ps}^{-1} \quad (\text{direct,CDF})
\]

**B\rightarrow\tau\nu**

\[
\text{BF}(B^+ \rightarrow \tau^+ \nu_\tau) = (0.87^{+0.13}_{-0.20}) \times 10^{-4} \quad (\text{CKM Fit}) \quad (1.45^{+0.46}_{-0.43}) \times 10^{-4} \quad (\text{direct,WA})^{36}
\]
New Physics?
No significant modification of the $B$-$\bar{B}$ mixing amplitude
Hypothesis: NP in loop processes only (negligible for tree processes)

Mass difference: \( \Delta m_s = (\Delta m_s)^{SM} r_s^2 \)

Width difference: \( \Delta \Gamma_s^{CP} = (\Delta \Gamma_s)^{SM} \cos^2(2\chi - 2\theta_s) \)

Semileptonic asymmetry:
\( A_s^{SL} = -\text{Re}(\Gamma_{12}/M_{12})^{SM} \sin(2\theta_s)/r_s^2 \)

\( S_{\psi\phi} = \sin(2\chi - 2\theta_s) \)

**UT of** \( B_d \) **system**: non-degenerated

\( (h_d, \sigma_d) \) strongly correlated to the determination of \( (\rho, \eta) \)

**UT of** \( B_s \) **system**: highly degenerated

\( (h_s, \sigma_s) \) almost independent of \( (\rho, \eta) \)

\( B_s \) mixing phase very small in SM: \( \chi = -1.02 \pm 0.06 \) (deg)

**Bs mixing**: very sensitive probe to NP

NP wrt to SM:
- reduces \( \Delta \Gamma_s \)
- enhances \( \Delta m_s \)
NP in $B_s$ System

First constraint for NP in the $B_s$ sector
Still plenty of room for NP
Large theoretical uncertainties: LQCD

$h_s \sim \leq 3$ ($h_d \sim \leq 0.3$, $h_K \sim \leq 0.6$)
\[ \beta_s = (-0.56^{+0.44}_{-0.41}) \text{ (stat+syst) [rad]} \]

Time-dependent angular distribution of untagged decays $B_s \to J/\psi \phi$ + charge asymmetry

Prediction from global CKM fit:
\[ \beta_s = (-0.0175^{+0.0015}_{-0.0008}) \text{ [rad]} \]

→ Precision prediction
→ Sensitive test to NP
NP in $b \rightarrow s$ transitions?
NP related solely to the third generations?
Conclusion

• CKM mechanism: success in describing flavor dynamics of many constraints from vastly different scales.

• Improvement of Lattice QCD is very desirable [Charm/tau factory will help]

• $B_s$: an independent chapter in Nature’s book on fundamental dynamics
  • there is no reason why NP should have the same flavor structure as in the SM
  • $B_s$ transitions can be harnessed as powerful probes for NP ($\chi$: “NP model killer”)

• With the increase of statistics, lots of assumptions will be needed to be reconsidered [e.g., extraction of $\alpha$ from $B \rightarrow 3\pi, 4\pi$, etc., $P_{EW}$, …]

• Before claiming NP discovery, be sure that everything is “under control”
  (assumptions, theoretical uncertainties, etc.)
  ➔ null tests of the SM

• There are still plenty of measurements yet to be done
BACKUP SLIDES
Radiative Penguin Decays: $\text{BR}(B \to \rho \gamma)/\text{BR}(B \to K^* \gamma)$

$B \to \rho \gamma \left( \propto |V_{td}|^2 \right)$ & $B \to K^* \gamma \left( \propto |V_{ts}|^2 \right)$ sensitive to New Physics

\[
\frac{\text{BF} (B^0 \to \rho^0 \gamma)}{\text{BF} (B^0 \to K^{*0} \gamma)} = 1.023 \left( \frac{1}{2} \left| \frac{V_{td}}{V_{ts}} \right| \right)^2 \bar{\xi}^{-2} (1 + \Delta)
\]

Additional amplitude contribution to charged modes $\Rightarrow$ less reliable

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<thead>
<tr>
<th>BABAR (347m)</th>
<th>Belle (386m)</th>
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<tbody>
<tr>
<td>$\rho^0 \gamma$</td>
<td>$0.77 , ^{+0.21}_{-0.19} , \pm 0.07$</td>
</tr>
<tr>
<td>$\rho^+ \gamma$</td>
<td>$1.06 , ^{+0.35}_{-0.31} , \pm 0.09$</td>
</tr>
</tbody>
</table>

$0.55 \, ^{+0.42}_{-0.36} \, \pm 0.09$

$1.25 \, ^{+0.37}_{-0.33} \, \pm 0.06$

$0.55 \, ^{+0.42}_{-0.36} \, \pm 0.08$
CLEO (endpoint)
4.09 ± 0.48 ± 0.36

BELLE (endpoint)
4.82 ± 0.45 ± 0.30

BABAR (endpoint)
4.39 ± 0.25 ± 0.39

BABAR (F_0, q^2)
4.57 ± 0.31 ± 0.41

BELLE m_{s2}
4.06 ± 0.27 ± 0.24

BELLE sum. ann. (m_{s2}, q^2)
4.37 ± 0.46 ± 0.29

BABAR (m_{s2}, q^2)
4.75 ± 0.38 ± 0.32

Average +/- exp +/- (mb, theory)
4.49 ± 0.19 ± 0.27

χ^2/df = 6/11 (CL = 40.7%)

OPE-HQET-SCET (BLNP)

Phys Rev D72, 073004, 2005
m_{b} input from b \to c l \nu and b \to s \gamma moments

HFAG (ICHEP06)

Ball-Zwicky &lt;16 GeV
3.38 ± 0.12 ± 0.56 - 0.37

HPQCD &gt;16 GeV
3.93 ± 0.26 ± 0.59 - 0.41

FNAL &gt;16 GeV
3.51 ± 0.23 ± 0.61 - 0.40

APE &gt;16 GeV
3.54 ± 0.23 ± 1.36 - 0.63

|V_{ub}| \ [\times 10^{-3}]

HFAG (ICHEP06)
## FLAVOR STRUCTURE

<table>
<thead>
<tr>
<th>ELECTROWEAK STRUCTURE</th>
<th>b → s</th>
<th>b → d</th>
<th>s → d</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta F=2 \text{ box} )</td>
<td>( \Delta M_{B_s} ) ( A_{CP}(B_s \rightarrow \psi(\phi)) )</td>
<td>( \Delta M_{B_d} ) ( A_{CP}(B_d \rightarrow \psi K) )</td>
<td>( \Delta M_K, \varepsilon_K )</td>
</tr>
<tr>
<td>( \Delta F=1 ) 4–quark box</td>
<td>( B_d \rightarrow \phi K, B_d \rightarrow K \pi, \ldots )</td>
<td>( B_d \rightarrow \pi \pi, B_d \rightarrow \rho \pi, \ldots )</td>
<td>( \varepsilon'/\varepsilon, K \rightarrow 3 \pi, \ldots )</td>
</tr>
<tr>
<td>gluon penguin</td>
<td>( B_d \rightarrow X_s \gamma, B_d \rightarrow \phi K, B_d \rightarrow K \pi, \ldots )</td>
<td>( B_d \rightarrow X_d \gamma, B_d \rightarrow \pi \pi, \ldots )</td>
<td>( \varepsilon'/\varepsilon, K_L \rightarrow \pi^0 \ell \bar{\ell}, \ldots )</td>
</tr>
<tr>
<td>( \gamma ) penguin</td>
<td>( B_d \rightarrow X_s \ell^+ \ell^-, B_d \rightarrow X_s \gamma )</td>
<td>( B_d \rightarrow X_d \ell^+ \ell^-, B_d \rightarrow X_d \gamma )</td>
<td>( \varepsilon'/\varepsilon, K_L \rightarrow \pi^0 \ell \bar{\ell}, \ldots )</td>
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<tr>
<td>( Z^0 ) penguin</td>
<td>( B_d \rightarrow X_s \ell^+ \ell^-, B_s \rightarrow \mu \mu )</td>
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<tr>
<td>( H^0 ) penguin</td>
<td>( B_s \rightarrow \mu \mu )</td>
<td>( B_d \rightarrow \mu \mu )</td>
<td>( K_{L,S} \rightarrow \mu \mu )</td>
</tr>
</tbody>
</table>
Bayes at work

Zero events seen

\[ P(n; \lambda) = e^{-\lambda} \frac{\lambda^n}{n!} \]

Posterior \( P(\lambda) \)

\( \int_0^3 P(\lambda) \, d\lambda = 0.95 \)

Same as Frequentist limit - Happy coincidence

\( P(0 \text{ events}|\lambda) \)

(Likelihood)

Prior: uniform

x
Bayes at work again

Is that uniform prior really credible?

Posterior $P(\lambda)$

$\int_0^3 P(\lambda) \, d\lambda \gg 0.95$

Prior: uniform in $\ln \lambda$

Upper limit totally different!
Bayes: the bad news

• The prior affects the posterior. It is your choice. That makes the measurement subjective. This is BAD. (We’re physicists, dammit!)

• A Uniform Prior does not get you out of this.

• Beware snake-oil merchants in the physics community who will sell you Bayesian statistics (new – cool – easy – intuitive) and don’t bother about robustness.
Hypersphere:

One knows nothing about the individual Cartesian coordinates $x,y,z$…

What do we known about the radius $r = \sqrt{x^2+y^2+\ldots}$?

One has achieved the remarkable feat of learning something about the radius of the hypersphere, whereas one knew nothing about the Cartesian coordinates and without making any experiment.