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THE SYMMETRY ENERGY IN NUCLEI AND IN NUCLEAR MATTER

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We discuss to what extent information on ground-state properties of finite nuclei (energies and radii) can be used to obtain constraints on the symmetry energy in nuclear matter and its dependence on the density. The starting point is a generalized Weizsäcker formula for ground-state energies. In particular effects from the Wigner energy and shell structure on the symmetry energy are investigated. Strong correlations in the parameter space prevent a clear isolation of the surface contribution. Use of neutron skin-information improves the situation. We find that inclusion of the Wigner effect leads to a larger value of the symmetry energy of nuclear matter at the saturation point.

1. Introduction

The nuclear symmetry energy is an important ingredient in the description of properties of proto-neutron stars. The equation of state, the proton fraction and the pressure are strongly affected by the density dependence of the symmetry energy in nuclear matter. Conventionally the symmetry energy is expanded around the saturation density as[1]

$$S(\rho) = a_4 + p_0(\rho - \rho_0) + \Delta K(\rho - \rho_0)^2. \tag{1}$$

Microscopic calculations are mostly based upon either models using realistic nucleon–nucleon interactions (based on Brueckner or variational techniques) or mean-field models using parameters fitted to data of finite nuclei. In practice predictions for the symmetry energy vary substantially: *e.g.*, $a_4 \equiv S(\rho_0) = 28$–$38$ MeV, whereas predictions for the slope p_0 can vary by a factor three. A few observations are in order. First, in general relativistic models—mean field as well as Dirac Brueckner–Hartree–Fock (BHF)—predict a substantially larger value for p_0 than non-relativistic ones. Secondly, in *ab initio* calculations there is also the uncertainty associated with a lack of a precise treatment of the effects from three-nucleon forces. Thirdly, even among the non-relativistic models (BHF and variational) there is no full agreement. A comparison of the results of the commonly used BHF approach with more general "in-medium T matrix" methods[2] (using the same nucleon–nucleon interaction) indicates that the BHF approach overestimates the symmetry energy.

In practice, only the empirical of value of $a_4 \sim 29$ MeV has been extracted with reasonable accuracy from finite nuclei by fitting ground-state energies using the Weizsäcker mass formula. Very little information (from data) on the slope p_0 is available. From a recent analysis[3] of the excitation of the giant isoscalar monopole resonance in the Sn isotopes a negative curvature ΔK (asymmetric compressibility) was obtained, $-380 < \Delta K < -580$ MeV.

Therefore the question arises whether one can obtain quantitative constraints from finite nuclei. Naturally one may distinguish two regions, those containing information on sub-saturation densities ($\rho < \rho_0$) and those on supranormal densities ($\rho > \rho_0$). While the latter requires a (model-dependent) interpretation of results from heavy-ion reactions[4] (diffusion of neutron–proton asymmetry), the former region can be addressed by analyzing static properties of nuclei. This will be the subject of the present paper.

To make contact with finite systems, one uses the semi-empirical mass formula[5,6] which contains information on the average values of bulk and surface binding energies (for isospin symmetric systems) and (bulk) symmetry and Coulomb energies. In the past it has been realized that the symmetry and Coulomb energies not only have a bulk contribution but one from the surface as well. This leads to a generalized "liquid-drop" description of finite nuclei (see Bohr and Mottelson[6], p. 621) and is akin to the treatment in nuclear collective models (*e.g.* describing properties of the giant dipole resonance) where it is equally essential to include a surface symmetry energy in addition to the bulk symmetry energy[7,8]. More recently, in connection with the semi-empirical mass formula, it has been stressed by Danielewicz[9] that in order to provide a consistent description of nuclei with neutron excess one must consider a *surface symmetry term* in addition to the bulk symmetry energy. The purpose of this paper is to show that in considering a surface symmetry term several other corrections to the liquid-drop model (LDM) should be dealt with as well.

After a brief review of the extended LDM in Sect. 2, its application to ground-state energies is discussed in Sect. 3 with particular attention being given to corrections due to shell structure and to the effect of neutron–proton correlations (Wigner energy). Section 4 then addresses the application of the LDM to radii. One of the main motivations of studies of this type is that it provides an additional constraint on the nuclear equation of state. Once the nuclear surface symmetry energy is determined, it can be related to the symmetry energy $S(\rho)$ of nuclear matter at a sub-saturation density point either via a semi-infinite nuclear matter or via a Thomas–Fermi approximation. This point is briefly recalled in Sect. 5. Finally, in Sect. 6 some concluding remarks are made.

2. The extended liquid-drop model

The conventional semi-empirical von Weizsäcker or liquid-drop mass (LDM) formula gives the binding energy of a nucleus as

$$a_{\rm v}A - a_{\rm s}A^{2/3} - a_4\frac{(N-Z)^2}{A} - a_{\rm c}\frac{Z(Z-1)}{A^{1/3}} + a_{\rm p}\frac{\Delta(N,Z)}{A^{1/2}}, \qquad (2)$$

with N and Z the number of neutrons and protons, and A the total number of nucleons, $A = N + Z$. The terms in (2) represent the bulk or volume, surface, symmetry, Coulomb and pairing energies, respectively, with $\Delta(N,Z)$ a simple parametrization of pairing which is 1 for even–even, 0 for odd-mass and -1 for odd–odd nuclei. The signs in Eq. (2) are chosen such that all a coefficients are positive. Furthermore, a_4 is the symmetry energy at saturation density, $a_4 = S(\rho_0)$, discussed in the introduction.

It has been pointed out[9] that in Eq. (2) volume and surface terms are not separated in a consistent way: one also needs to introduce separate volume and surface contributions to the symmetry energy. To accomplish this is not trivial. In a rigorous derivation one first introduces the concept of surface tension; the latter can then be decomposed into isospin symmetric and asymmetric contributions[9,10]. In practice the same result can be obtained in a more schematic way by decomposing the total particle asymmetry, $N-Z$, into volume (v) and surface (s) terms, $N-Z = N_{\rm v} - Z_{\rm v} + N_{\rm s} - Z_{\rm s}$, and requiring that the symmetry energy (quadratic in the asymmetry) scales with particle numbers as

$$S_{\rm v}\frac{(N_{\rm v}-Z_{\rm v})^2}{A} + S_{\rm s}\frac{(N_{\rm s}-Z_{\rm s})^2}{A^{2/3}}. \qquad (3)$$

Minimization of Eq. (3) under fixed $N - Z$ leads to a generalized formula for the binding energy of a nucleus in which the symmetry energy depends on two independent parameters, $S_{\rm v}$, the volume symmetry energy and the ratio $y_{\rm s} \equiv S_{\rm v}/S_{\rm s}$ with S_s the surface symmetry energy:

$$a_{\rm v}A - a_{\rm s}A^{2/3} - \frac{S_{\rm v}}{1+y_{\rm s}A^{-1/3}}\frac{(N-Z)^2}{A} - a_{\rm c}\frac{Z(Z-1)}{A^{1/3}} + a_{\rm p}\frac{\Delta(N,Z)}{A^{1/2}}. \qquad (4)$$

This expression for the nuclear binding energy forms the basis of the subsequent discussion. We will focus our attention in this paper on two further corrections: those due to shell-structure effects, for which a simple parametrization shall be developed, and those due to quantal corrections to the symmetry energy.

We note that the modified LDM (4) suggests[6] an expansion of the symmetry (binding) energy of the form

$$-s_{\rm v}\frac{(N-Z)^2}{A} + s_{\rm s}\frac{(N-Z)^2}{A^{4/3}} \qquad (5)$$

where $s_{\rm v} \approx S_{\rm v}$ and $s_{\rm s} \approx S_{\rm v}^2/S_{\rm s}$, which should be valid for sufficiently large values of A and/or sufficiently small values of $y_{\rm s}$. While this might have been an acceptable approximation for the data set of nuclei available in the past, in fitting the

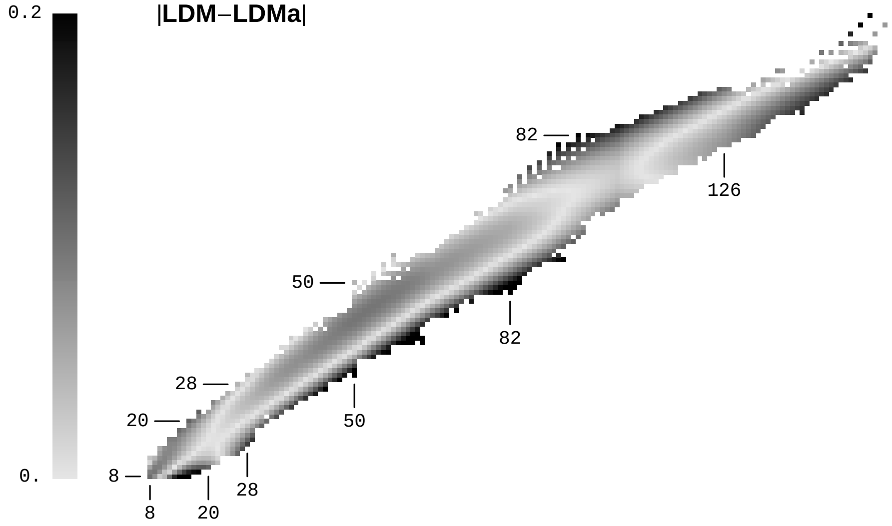

Fig. 1. Differences between calculated binding energies (in MeV) for nuclei with $N, Z \geq 8$. The binding energies are calculated with the extended LDM formula (4) and with its approximation (5). In each case shell and deformation corrections have been added. The parameters in both formulas are independently adjusted to the measured binding energies of all nuclei (with $N, Z \geq 8$) in the 2003 atomic mass evaluation[11].

currently known nuclear binding energies the use of the approximate formula (5) is not satisfactory. This is illustratred in Fig. 1 where the differences between the formulas (4) and (5) are plotted for all nuclei (with $N, Z \geq 8$) in the 2003 atomic mass evaluation[11]. The figure shows that data concerning exotic nuclei are crucial to discriminate between the extended LDM formula and its approximation.

Before turning to the results obtained with refinements of the extended LDM (4), we point out that, with use of a proportionality that exists between the neutron skin $\Delta R \equiv R_{\rm n} - R_{\rm p}$ and the nucleon surface asymmetry $N_{\rm s} - Z_{\rm s}$, the procedure leading to the result (4) also yields a direct relation between the skin ΔR and the symmetry energy parameters $(S_{\rm v}, S_{\rm s})$:

$$\frac{R_{\rm n} - R_{\rm p}}{R} = \frac{A(N_{\rm s} - Z_{\rm s})}{6NZ} = \frac{A}{6NZ} \, \frac{N - Z - a_{\rm c} Z A^{2/3} (12 S_{\rm v})^{-1}}{1 + y_{\rm s}^{-1} A^{1/3}}, \tag{6}$$

which is valid for the difference of sharp-sphere radii[9]. We note that, in the absence of the Coulomb contribution, the neutron skin depends on the ratio $y_{\rm s}$ only. However, except for very light nuclei, the Coulomb contribution cannot be neglected; for $N = Z$ nuclei it causes ΔR to be slightly negative (in agreement with data).

In practice Eq. (6) must be generalized for the case of rms radii. As pointed out by Danielewicz[9] this brings about an additional Coulomb correction (stemming from the polarization of the nuclear interior by the Coulomb force) to the right-hand side of Eq. (6); the final result is[9]

$$\frac{\langle r^2\rangle_{\rm n}^{1/2}-\langle r^2\rangle_{\rm p}^{1/2}}{\langle r^2\rangle^{1/2}}=\frac{A}{6NZ}\,\frac{N-Z-a_{\rm c}ZA^{2/3}(10/3+y_{\rm s}^{-1}A^{1/3})(28S_{\rm v})^{-1}}{1+y_{\rm s}^{-1}A^{1/3}}, \qquad (7)$$

where a (very small) surface diffuseness contribution is neglected.

3. Ground-state binding energies

In fitting nuclear binding energies with the extended LDM formula (4), special care has to be taken with the treatment of $T=0$ pairing effects (or Wigner energy)—with an important impact on the symmetry energy—and of shell effects.

3.1. *Wigner energy*

Nuclei with $N=Z$ are in general more strongly bound as compared to the LDM mass formula; this effect can be incorporated by including an additional term (known as the Wigner energy). The origin of the Wigner energy has been discussed by several groups, *e.g.* Satuła *et al.*[12,13,14], Zeldes[15], Neergåard[16] and Jänecke *et al.*[17,18]. The Wigner contribution to the binding energy is usually decomposed into two parts[19]

$$B_{\rm w}(N,Z)=-W(A)|N-Z|-d(A)\delta_{N,Z}\pi_{\rm np}, \qquad (8)$$

where $\pi_{\rm np}$ equals 1 for odd–odd nuclei and 0 otherwise. The origin of the Wigner energy [with $W(A)$ positive] can be understood microscopically as an effect from the overlap of neutron and proton wave functions which is maximal in $N=Z$ nuclei. The value of the parameter $W(A)$ can be determined in various ways, for example, from the double binding energy difference[20]

$$\delta V_{\rm np}(N,Z)=\frac{1}{4}[B(N,Z)-B(N-2,Z)-B(N,Z-2)+B(N-2,Z-2)], \qquad (9)$$

valid for even–even nuclei, and differences thereof[12],

$$W(A)=\frac{1}{2}[\delta V_{\rm np}(A/2,A/2-2)+\delta V_{\rm np}(A/2+2,A/2)]. \qquad (10)$$

To take into account the first term (linear in T) at the right-hand side of Eq. (8), in several recent applications of the LDM the term $(N-Z)^2$ is replaced (in an *ad hoc* manner) by $4T(T+r)$, where $T=|T_z|$ is the isospin of the nuclear ground state and r is a parameter.

The form of the Wigner energy can be understood in more general terms from supermultiplet theory[21]; the latter is based upon the assumption that nucleon–nucleon forces are spin and isospin independent, and that as a result of the net attraction of the residual interactions the ground state has maximum spatial symmetry [or, equivalently, maximum SU(4) anti-symmetry] consistent with the Pauli principle. As a result the correlation energy in the ground state is related to the expectation value of the quadratic SU(4) Casimir operator in the following way[22]

$$g(\lambda,\mu,\nu) = (N-Z)^2 + 8|N-Z| + 8\delta_{N,Z}\pi_{\rm np} + 6\Delta'(N,Z), \tag{11}$$

where the labels λ, μ, ν are functions of N, Z and $\Delta'(N,Z)$ is a pairing term which follows the somewhat unusual convention of being 0 in even–even, 1 in odd-mass and 2 in odd–odd nuclei. Note that the second and third terms exactly correspond to the Wigner energy (8) with the constraint $W = d$. Furthermore, it is seen that the first two terms in the expression (11) have the appearance of a symmetry energy $4T(T+r)$ with $r = 4$. [The correct A dependence is lacking in Eq. (11) since this information cannot be provided by supermultiplet theory.] If SU(4) symmetry is broken (as a result of the spin–orbit interaction) but assuming charge invariance is satisfied, an argument similar to that leading to Eq. (11) gives a symmetry energy of the form $T(T+1)$, that is, $r = 1$. One may thus expect the coefficient r to lie somewhere between $r = 1$ and $r = 4$.

As a final remark in this section on the Wigner energy, we note that this effect is also at the basis of the frequently observed "isospin inversion" in odd–odd nuclei which should be taken into account in the fit to the nuclear masses.

3.2. *Shell-stucture effects*

It is well known that shell corrections to the LDM mass formula play an important role. In the literature many methods have been proposed to deal with shell effects; for example, those developed by Möller and Nix[19]. Here we use a simple prescription which is closely related to the ideas used in the interacting boson approximation (IBA) model[23]. This model describes collective degrees of freedom in nuclei away from closed shells, and suggests that the relevant physics ingredient is the number of valence particles (neutrons and/or protons) with respect to the nearest closed shells (taken here to be N, Z = 20, 28, 50, 82, 126) where particles beyond mid-shell are counted as holes. In our present work we add to the LDM expression a two-parameter term

$$B_{\rm shell}(N_{\rm n}, N_{\rm p}) = -a_1 F_{\rm max} + a_2 F_{\rm max}^2, \tag{12}$$

where $F_{\rm max} = (N_{\rm n} + N_{\rm p})/2$ with N_i the number of valence neutrons or protons, of particle or hole character. This is equivalent to counting bosons in the neutron–proton IBA model where $F_{\rm max}$ is the maximum F spin[24]. We emphasize that Eq. (12) provides a parametrization of nuclear shell effects, and *not* a calculation of them, since the magic numbers for the shell closures are put in by hand.

3.3. *Correlations between parameters*

It has been noted by Danielewicz[9] and Steiner *et al.*[10] that in the fit to nuclear binding energies (assuming $r = 0$) the parameters $S_{\rm v}$ and $S_{\rm s}$ are correlated. In the correlation plot of $S_{\rm v}$ *versus* $y_{\rm s} = S_{\rm v}/S_{\rm s}$ one obtains for the rms deviation a narrow

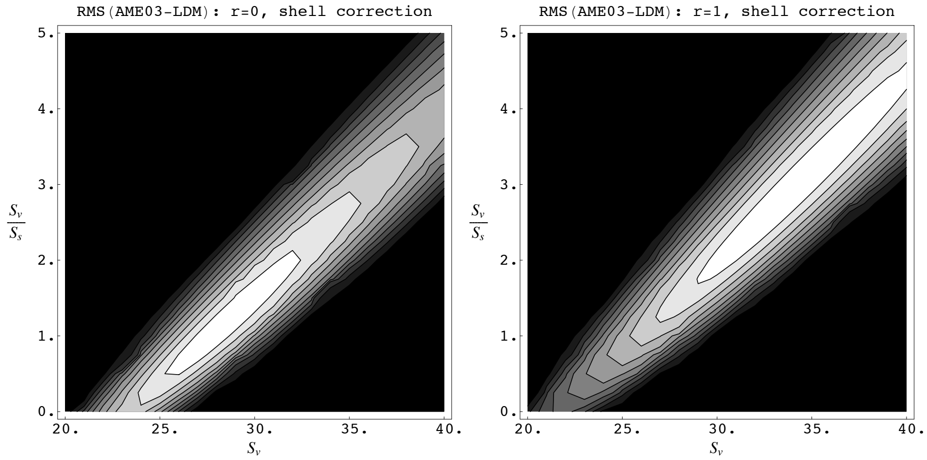

Fig. 2. Correlation plot of $S_{\rm v}$ *versus* $y_{\rm s} = S_{\rm v}/S_{\rm s}$ for $r = 0$ (left) and $r = 1$ (right). The white areas correspond to the combination of $S_{\rm v}$ and $y_{\rm s}$ giving rise to the lowest rms deviation σ, *i.e.* $\sigma = 1.65$ MeV for $r = 0$ and $\sigma = 1.40$ MeV for $r = 1$. Each contour interval corresponds to an increase of the rms deviation by 100 keV.

valley described by the linear relation $S_{\rm v} = a + by_{\rm s}$. The actual values of a and b depend on details of the fitting procedure; *e.g.* $a = 21.5$, 20.7 and 21.2 MeV, and $b = 3.1$, 3.9 and 6.1 MeV in the model of Danielewicz[9] and in the models 1 and 2 of Steiner[10], respectively. In the present approach one finds $a \approx 23$ MeV and $b \approx 4.2$ MeV (see Fig. 2). This correlation can be understood qualitatively from the observation that one fits with a two-parameter function $S_{\rm v}/(1 + y_{\rm s}A^{-1/3})$ and that, in first approximation, for heavy nuclei $A^{-1/3}$ can be replaced by its average value (weighted with $(N-Z)^2/A$), $\langle A^{-1/3}\rangle \approx 0.185$.

If one allows for $r > 0$ the situation becomes even more complicated, as the correlation plot depends quite sensitively on the value of r. As can be seen from Fig. 2 increasing the value of r leads to a valley in the correlation plot that has a similar slope and an even increased shallowness. While the character of the correlation between $S_{\rm v}$ and the ratio $y_{\rm s}$ is rather insensitive to r, the best-fit values for $S_{\rm v}$ and $y_{\rm s}$ do vary substantially with it: $S_{\rm v} = 28.5$, 34.3, 38.2 MeV, and $y_{\rm s} = 1.18$, 2.98, 4.22 with rms deviations of 1.65, 1.40, 1.37 MeV for $r = 0$, 1, 1.5, respectively. In particular, the best-fit value for $y_{\rm s}$ increases by a factor 2.5 by varying r from 0 to 1 (see Fig. 2) which reflects the existence of the strong correlation between $\langle|N-Z|\rangle$ and $\langle A^{1/3}\rangle$.

There is thus an obvious need to determine the value of r independently. In the work of Jänecke *et al.*[17,18] the parameter r is treated as a mass-dependent shell effect, and a strong variation with A is observed. In particular for non-diagonal regions (where neutrons and protons occupy different shells, *e.g.* $50 \le Z \le 82$ and $82 \le N \le 126$) it was found that the value of r is larger, $r \approx 2$–4. In this analysis no surface term was considered, however. We have tried to fit ground-state energies using Eq. (13) with the parameter r free to vary with mass region but did not find a systematic trend, if also S_v and y_s were treated as free parameters. Taking S_v and y_s fixed we do find a weak preference for larger values $r \approx 2.0$ for non-diagonal regions compared to diagonal ones, confirming the finding of Jänecke *et al.*

Since the parameter r cannot be reliably determined from ground-state energies, we have chosen $r = 1$. This corresponds to the case of SU(2) isospin symmetry, valid for a charge-independent nucleon–nucleon interaction, and is also used by Danielewicz[25].

3.4. *Results*

In all fits measured nuclear binding energies are taken from the 2003 atomic mass evaluation[11]. All calculations are done with the extended LDM formula with a Wigner term,

$$a_{\rm v}A - a_{\rm s}A^{2/3} - \frac{S_{\rm v}}{1+y_{\rm s}A^{-1/3}}\frac{4T(T+r)}{A} - a_{\rm c}\frac{Z(Z-1)}{A^{1/3}} + a_{\rm p}\frac{\Delta(N,Z)}{A^{1/2}}, \qquad (13)$$

to which the shell correction (12) is added in some cases. In Fig. 3 we compare the results obtained with the extended LDM with $r = 1$, with and without corrections due to shell structure. It is seen in the top figure (no shell corrections) that the large deviations around doubly-magic nuclei have a diamond-like appearance and this suggests the use of a term linear in $F_{\rm max}$ which indeed provides an excellent parametrization of the shell corrections that are needed. Furthermore, the ellipse-like deviations in mid-shell regions suggest another term which is quadratic in $F_{\rm max}$. The use of these two simple corrections reduces the root-mean-square (rms) deviation from 2.47 to 1.40 MeV while the values of the macroscopic coefficients remain stable. The shell-corrected plot (bottom of Fig. 3) has much reduced deviations for the doubly-magic nuclei and in the mid-shell regions of the heavier nuclei. A large fraction of the remaining rms deviation of 1.40 MeV is due to nuclei lighter than ^{56}Ni where shell effects are large and cannot so easily be parametrized. An interesting feature of the shell-corrected plot is that it reveals additional or *sub-shell* effects: regions of larger deviations are seen around ^{90}Zr and ^{146}Gd. These nuclei are known to have doubly-magic behaviour associated with proton shell closures at $Z = 40$ and $Z = 64$, respectively. Since in the present fit shell closures are taken to be $N, Z = 20, 28, 50, 82, 126$, such sub-shell effects are not included; a more refined algorithm for the choice of magic numbers is needed for this.

Figure 4 shows the coefficient $S_{\rm v}$, the ratio $y_{\rm s} = S_{\rm v}/S_{\rm s}$ and the rms deviation as a function of r, obtained with the LDM mass formula (4) with and without shell corrections. Only the symmetry-energy coefficients $S_{\rm v}$ and $S_{\rm s}$ are sensitive to r and all other coefficients remain approximately constant. The figure illustrates that, although the shell corrections strongly reduce the rms deviation, they do not change the nature of the correlation between $S_{\rm v}$ and $y_{\rm s}$. One may thus expect that the use of shell corrections which are more realistic than the paramerization (12) will not alter our main conclusion here, namely that the volume symmetry energy

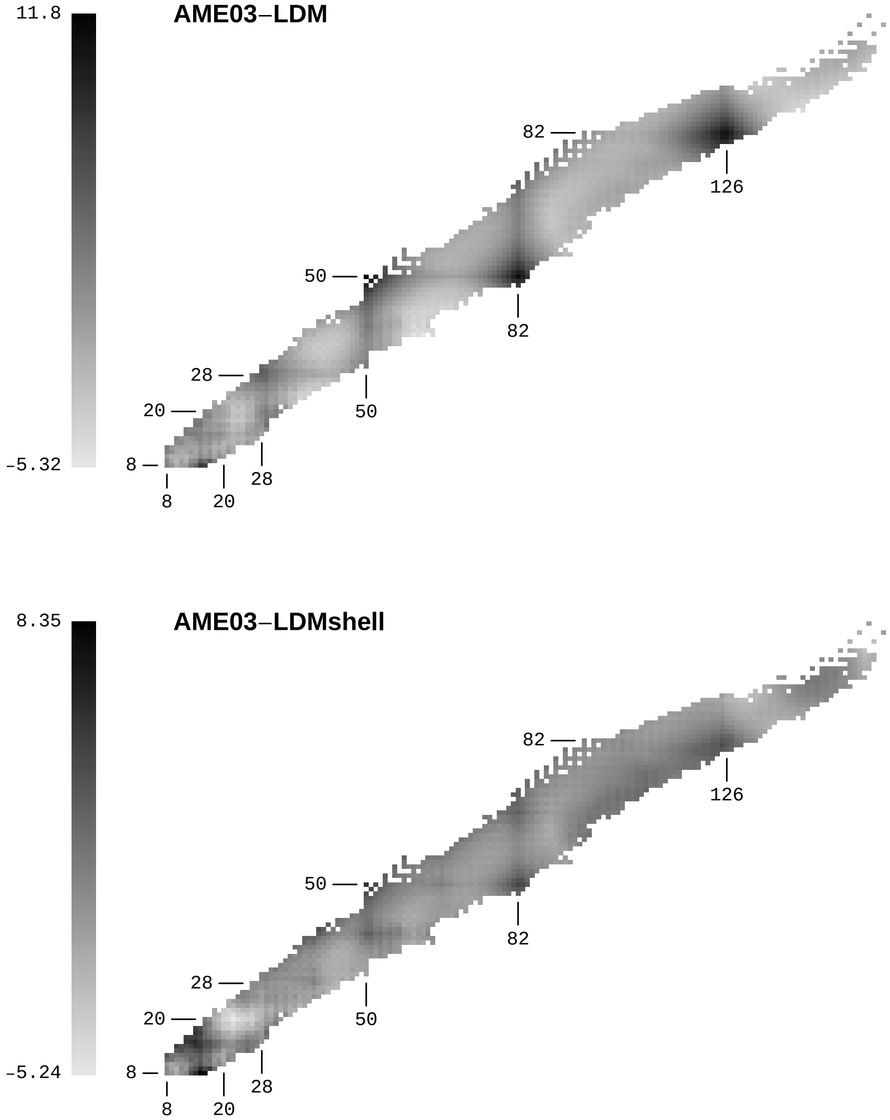

Fig. 3. Differences between measured and calculated binding energies for nuclei with $N, Z \geq 8$. The binding energies are calculated with Eq. (13) which includes surface-symmetry and Wigner energies. The top figure is obtained without corrections due to shell structure while the bottom figure includes the two-parameter term (12). A colour version of the figure, showing more clearly the differences, is available on request.

S_v and the volume-to-surface symmetry energy ratio y_s are strongly correlated and that both vary wildly with r. While shell corrections do not modify the variation of S_v and y_s with r, this is not so for the rms deviation σ which has a less shallow

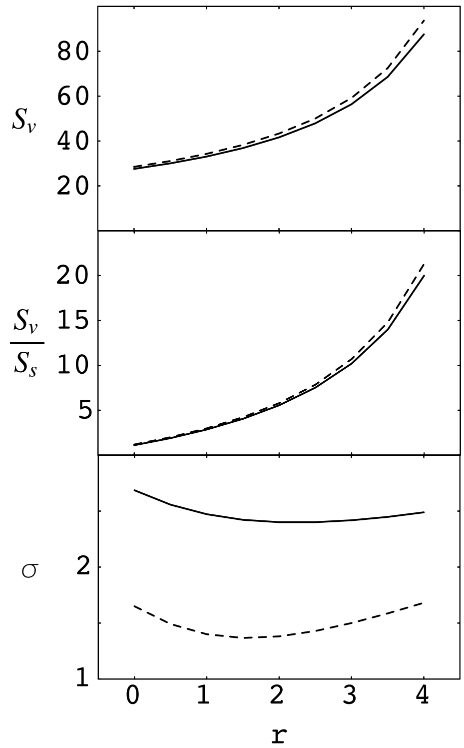

Fig. 4. The coefficient $S_{\rm v}$ (in MeV), the ratio $y_{\rm s} = S_{\rm v}/S_{\rm s}$ and the rms deviation σ (in MeV) as a function of r, obtained with the LDM formula (4) without and with shell corrections (full and dashed lines, respectively).

minimum if Eq. (12) is added to the LDM mass formula. But r remains poorly known and this adds considerable uncertainty to the determination of $S_{\rm v}$ and $y_{\rm s}$.

4. Neutron skin

As was shown in the previous section, a strong correlation exists between the volume symmetry energy of $S_{\rm v}$ and the ratio of the volume-to-surface symmetry energy $y_{\rm s} = S_{\rm v}/S_{\rm s}$. Therefore it appears desirable to use supplementary independent information.

From Eq. (6) it is clear that in the LDM there exists a direct relation between ΔR and $y_{\rm s}$ with a slight dependence on $S_{\rm v}$ through the Coulomb term[9,10]. We note that a similar correlation between $\Delta R = R_{\rm n} - R_{\rm p}$ and (the derivative of) the symmetry energy in nuclear matter was pointed out by Brown[26] and by Furnstahl[27].

One should realize that the extracted experimental information on ΔR is in general the result of a model-dependent analysis of nuclear reactions (elastic proton scattering, anti-protonic atoms, giant-resonance excitation). Indeed, the values deduced for *e.g.* ^{208}Pb , $\Delta R = 0.10$–0.20 fm, vary appreciably with the experimen-

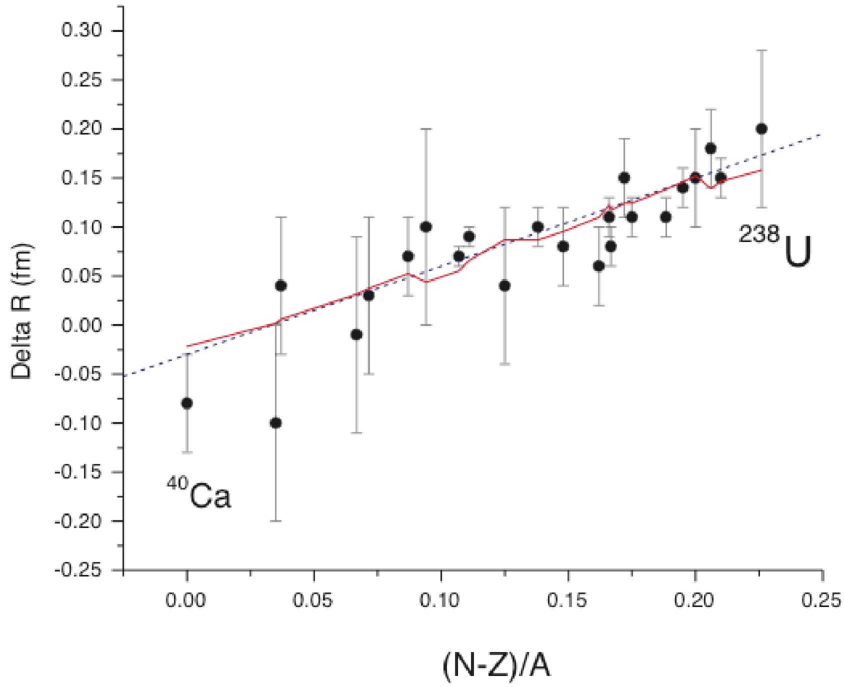

Fig. 5. The neutron-skin data ΔR from anti-protonic atoms for nuclei between ^{40}Ca and ^{238}U. The full line is obtained from the LDM expression (6) with a volume symmetry energy $S_{\rm v} = 30$ MeV and a volume-to-surface ratio $y_{\rm s} = 1.8$. The dashed line is the fit $\Delta R = -0.03 + 0.90(N - Z)/A$ given by Schmidt *et al.*[28].

tal approach and details of the analysis; also, in most cases the associated model uncertainties are difficult to quantify.

In order to minimize the model dependence, we have fitted data obtained with one specific experimental tool only, namely from anti-protonic atoms[28], available for targets between ^{40}Ca and ^{238}U. Therefore we do not use information on unstable light nuclei, which we believe to be more model dependent. Since the dependence of ΔR on $S_{\rm v}$ is weak, we may adopt the value $S_{\rm v} \approx 30$ MeV as it was obtained from ground-state energies, and determine the ratio $y_{\rm s}$ from the neutron-skin data. The resulting fit is shown in Fig. 5 and yields a value of $y_{\rm s} = 1.8 \pm 0.3$. It is seen that in the limited mass region considered the simple two-parameter fit $\Delta R = a + b(N - Z)/A = -0.03 + 0.90(N - Z)/A$ from Schmidt *et al.*[28] works equally well; that parametrization, however, has no obvious physical interpretation, the negative contribution from the a coefficient becomes unphysical for light $N = Z$ nuclei and it does not properly describe the Coulomb term for heavy systems.

Above we have ignored possible shell corrections to Eq. (6). While these are expected to play in role if one would consider a series of isotopes, it is clear from Fig. 5 that the overall slope which is a measure for the value of y_s is determined by the increase of the skin between $A = 40$ and $A = 208$ and is not strongly affected by shell corrections.

5. Nuclear matter

On intuitive grounds one expects a relation between the ratio $S_{\rm v}/S_{\rm s}$ and the ρ dependence of the symmetry energy in nuclear matter. In microscopic mean field models one can at the same time compute isovector properties (like the neutron skin) in finite nuclei and the density dependence of the symmetry energy in nuclear matter. In an LDM approach and in a local-density approximation one may relate the ratio $S_{\rm v}/S_{\rm s}$ to an integral with an integrand which involves the nuclear density $\rho(r)$ and the symmetry energy function $S(\rho)$[9]:

$$y_s = \frac{S_{\rm v}}{S_{\rm s}} \approx \frac{3}{r_0} \int_0^\infty \frac{\rho(r)}{\rho_0} \left(\frac{S_{\rm v}}{S(\rho)} - 1 \right) dr, \tag{14}$$

where ρ_0 is the saturation density and r_0 the radius of the nuclear volume per nucleon; both are related through $\rho_0 = (4\pi r_0^3/3)^{-1}$. Note that $S_{\rm v} = S(\rho_0)$; as a consequence, if the symmetry energy is independent of the density, $S(\rho) = S(\rho_0) = S_{\rm v}$, then $S_{\rm s}$ becomes infinite, as should be.

The application of Eq. (14) becomes even simpler if one assumes a power parametrization for the density dependence of the symmetry energy, $S(\rho) = S_{\rm v} \times (\rho/\rho_0)^\gamma$. Based on the value of $y_{\rm s} = 1.8 \pm 0.3$ from radii, we find $\gamma = 0.5 \pm 0.1$ although it is difficult to give a quantitative estimate of the error due to the Thomas–Fermi approximation. This result is consistent with $0.55 < \gamma < 0.79$ reported by Danielewicz[25]. Nevertheless, a more satisfactory approach would be to combine the data on radii and masses to arrive at a reliable estimate of $y_{\rm s}$. More work in this direction is needed. It is also of interest to compare to the results from (relativistic) mean field calculations. Recently Piekarewicz[29] reported that the use of two sets of parameters, which both describe properties of finite nuclei, can lead to a drastically different density dependence of the symmetry energy: $S_{\rm v} = 36.9$ (32.7) MeV and $\gamma = 0.98$ (0.64) for NL3 and FSUGold, respectively. There have been also attempts to constrain the value of γ using heavy-ion reactions (isospin diffusion); it was found[30,31] that $0.7 < \gamma < 1.1$.

6. Concluding remarks

Recent studies by Danielewicz[9] and Steiner *et al.*[10] have demonstrated the need for the inclusion of surface effects in the LDM mass formula, in particular in the symmetry energy. These authors showed that the bulk and surface symmetry energies are strongly correlated, rendering a reliable determination of these quantities from nuclear ground-state masses difficult. Data on neutron skins must be combined with mass data to arrive at a separation of bulk and surface contributions.

In this paper we examined the influence of two further refinements of this extended LDM, namely nuclear shell structure and the Wigner energy. With use of a simple parametrization of shell effects it was shown that, although a substantially smaller rms deviation is obtained with shell corrections than without, the correlation between the symmetry-energy parameters remains unchanged. So shell corrections do not substantially alter the observations made by Danielewicz[9] and Steiner *et al.*[10] as regards the correlation between the volume and surface symmetry-energy parameters. The conclusion is different, however, if a Wigner correction is added to the LDM formula. We have followed here the usual procedure of parametrizing the Wigner effect through $T(T + r)$ where r is adjusted to mass data. Even if the decrease of the rms deviation due to this additional Wigner term is only about 300 keV, the effect of r on the symmetry-energy parameters is spectacular. As a result, even more than before, the separation of volume and surface symmetry-energy contributions can only be accomplished by use of data on neutron skins of nuclei.

The fact that the Wigner energy strongly influences the symmetry-energy parameters comes, with hindsight, as no surprise: a quantal calculation of the symmetry energy that takes account of the degeneracy structure of nuclei [either two-fold due to SU(2) or four-fold due to SU(4) symmetry] does lead in a natural way to a term linear in T. Symmetry and Wigner energies are thus closely related and one expects a strong correlation between them.

We have not been able to devise a procedure for a reliable determination of the Wigner parameter r from nuclear ground-state binding energies, possibly because $T(T+r)$ is a woefully inadequate parametrization of the Wigner effect. Nevertheless, in spite of this difficulty, one clear qualitative conclusion can be drawn from the present study: the Wigner effect implies a non-zero r value which unavoidably leads a larger value of S_{v}, the symmetry energy of nuclear matter at the saturation point.

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