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We investigate whether isospin mixing can be determined in a model-independent way from the relative strength of E1 transitions in mirror nuclei. The specific examples considered are the $A = 31$ and $A = 35$ mirror pairs, where a serious discrepancy between the strengths of $7/2^- \rightarrow 5/2^+$ transitions in the respective mirror nuclei has been observed. A theoretical analysis of the problem suggests that it ought to be possible to disentangle the isospin mixing in the initial and final states given sufficient information on experimental matrix elements. With this in mind, we have obtained a lifetime for the relevant $7/2^-$ state in ^{31}S using the Doppler-shift attenuation method (DSAM). We have then collated the available information on matrix elements in order to examine the level of isospin mixing for both $A = 31$ and $A = 35$ mirror pairs.

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I. INTRODUCTION

The concept of isobaric spin (isospin) was introduced into nuclear physics to represent the fact that the nuclear force is to first order charge independent [1]. Were charge independence obeyed in the strictest sense, then isobaric multiplets would be degenerate in energy, and all bound nuclear states would have a definite and pure isospin. Moreover, there would be an exact correspondence between the wavefunction of states in an isobaric multiplet. In fact, these degeneracies are lifted by the action of isospin non-conserving interactions, the most important of which is the Coulomb force. The resulting separation of the members of an isobaric multiplet is termed the Coulomb displacement energy (CDE). Taking into account the substantial mass differences arising from the CDE, there are also discrepancies between the excitation energies of states in mirror nuclei as a function of angular momentum at the level of 100 keV [2]; such deviations reflect detailed nuclear structure effects such as the difference in the alignment of proton-proton and neutron-neutron pairs [2], and the electromagnetic spin-orbit interaction [3, 4]. A more general question is the extent to which the Coulomb interaction can induce the breakdown of isospin purity; an issue reviewed by Soper [5]. These impurities can be manifested in a range of experimental scenarios including isospin-forbidden particle decays from highly-excited, particle-unbound states, perturbation of electromagnetic matrix elements and nuclear beta decay.

Isospin mixing and its effect on beta-decay matrix elements is of considerable interest in the context of tests of the Conserved Vector Current hypothesis [6]. Such tests concern the beta decay of Fermi superallowed emitters for which the $\log ft$ value should have a fixed value if the CVC hypothesis is correct. Small isospin breaking effects lead to weak Gamow-Teller decay branches in competition with the dominant superallowed branch and the influence of such branches must be accounted for. Conventionally, the relevant isospin mixing of the ground states is evaluated using shell-model calculations; typical values being $\sim 0.5\%$ for mass 50 (fp-shell nuclei) rising to 1% or higher for $A \approx 70$ (fpg shell) [7]. Approaches which could extract isospin mixing from experimental data in a model-independent manner are, therefore, of considerable interest.

An open question is to what extent isospin mixing may be inferred from the impact it may have on electromagnetic transition rates. Warburton and Weneser reviewed this issue nearly forty years ago [8] and discussed isospin mixing in the context of a number of selection rules expected for both conjugate and self-conjugate nuclei. Their first rule concerned the fact that, between states with $T_z \neq 0$, electromagnetic transitions must obey the selection rule $\Delta T = 0, \pm 1$. A γ decay from a $T = 2$ to $T = 0$ state, therefore, could only arise due to isospin mixing of $T = 1$ components in initial or final states, or the isotensor component of the Coulomb interaction. In practice, the relevant $T = 2$ states lie at very high exci-

tation energy, and the relevant γ-ray transitions are high in energy. This poses an experimental challenge. Moreover, the fact that such states are particle unbound further complicates the analysis. Warburton and Weneser also pointed out a number of selection rules applying to self-conjugate ($N = Z$) nuclei [8]. For example, since E1 transitions are purely isovector in nature, they are strictly forbidden between states of the same isopin in $T_z = 0$ nuclei [9]. This behaviour has been examined by Farnea *et al.* [10] in the case of the $5^- \rightarrow 4^+$ transition in ^{64}Ge, where this transition is found to be dominated by its M2 component. The E1 component of this transition has the very weak strength of $\sim 2.5 \times 10^{-7}$ W.u. Calculations suggest that the level of isospin mixing needed to account for the observed E1 transition strength is $\alpha^2 = 2.50\%^{+1.0\%}_{-0.7\%}$ [10]. A second rule relating to self-conjugate nuclei, advanced by Warburton and Weneser is the weakness of $\Delta T = 0$, M1 transitions in such nuclei. A good system for searching for the role of isospin mixing, therefore, is an odd-odd $N = Z$ nucleus, since $T = 0$ and $T = 1$ states lie close together near the ground state, and accidental degeneracies are likely. Lisetskiy *et al.* [11] have made a detailed analysis of γ decays in the odd-odd $N = Z$ nucleus ^{54}Co. In particular, they considered the decays of a doublet of 4^+ states with $T = 0$ and $T = 1$, respectively, to a $T = 0$, 3^+ state. Analysis of the E2/M1 multipole mixing ratios for these decays allowed the isospin mixing to be quantified at $\sim 0.2\%$ [11].

A further selection rule advanced by Warburton and Weneser is that corresponding E1 transitions in conjugate nuclei (i.e. the mirror pair of nuclei, one with A protons and B neutrons, the other with B protons and A neutrons) should have equal strength [8]. They showed that this rule was essentially satisfied for E1 transitions in the conjugate nuclei ^{11}C and ^{11}B, while there was a factor of two difference in the E1 transition strengths for the decay of the $1/2^+$ excited states in ^{13}C and ^{13}N. There is a very large shift in the energy of $1/2^+$ state in ^{13}N attributable to the Thomas-Ehrman effect, which relates to the greater radial extent of the proton wave-function [12]. Such an effect is especially pronounced for the $s_{1/2}$ orbital [12]. The non-equivalence of the wave-functions for the $1/2^+$ states could therefore explain the difference in E1 transition strengths. A still more dramatic example was the case of the decay of the $1/2^+$ levels at 8.312 MeV in ^{15}N and 7.550 MeV in ^{15}O. In this case, the E1 transition strengths differed by a factor of 400, but here the situation is complex as the state in ^{15}O is unbound and so mixing effects may be very large [8]. Although data on E1 transitions was limited at the time Warburton and Weneser wrote their review [8], it appeared to show that for cases where both the decaying states under consideration were bound, the respective E1 transitions had nearly equal strength, even where important nuclear structure phenomena like the Thomas-Ehrman shift is at work. Warburton and Weneser commented that it would be very interesting to test this rule for heavier nuclei, where isospin mixing would be expected to be larger [8];

such data did not exist at that time, and little deviation from this rule has shown up since. It was, therefore, of great interest when Ekman *et al.* [3] in their study of the $T = 1/2$ mirror nuclei ^{35}Ar and ^{35}Cl highlighted isospin mixing as the possible origin of the marked difference in the decay branching of the first $7/2^-$ state in the respective nuclei. In ^{35}Ar, an E1 transition formed a strong decay branch from the $7/2^-$ state to the $5/2^+$ state, while in the well-studied stable nucleus ^{35}Cl, the analogous transition was almost completely absent. In both nuclei, the respective $7/2^-$ states were well bound. Ekman *et al.* [3] suggested that the cancellation of the E1 matrix element in ^{35}Cl arose from isospin mixing, in this case, between the dominant $T = 1/2$ and a weak $T = 3/2$ component. They were unable, however, to quantify this suggestion since absolute transition strengths were not available for the relevant transitions in ^{35}Ar. Similar behaviour to that reported for $A = 35$ was also found in the $T = 1/2$ mirror nuclei ^{31}S and ^{31}P [4, 13]. Again, the decay pattern of the first $7/2^-$ state was found to change dramatically between the mirror nuclei. In this case, a 2195-keV E1 transition clearly present in ^{31}P was found to have no counterpart in the decay scheme of ^{31}S. This is the reverse of the situation in the $A = 35$ mirror pair where the E1 was found to disappear in the $T_z = 1/2$ member of the pair. Again, the levels concerned were all particle-bound and so the effects could not be related to the effects of the loosely-bound proton. In the $A = 31$ example, as for the $A = 35$ mirror nuclei, the relevant transition strengths were unavailable prior to the present work. The motivation of the present work, therefore, was to obtain transition strengths to quantify this phenomenon as well as to examine how such information could be used to extract the isospin mixing, if present.

II. ISOSPIN MIXING AND ELECTRIC DIPOLE TRANSITIONS IN MIRROR NUCLEI

Let us consider, from a theoretical perspective, whether E1 transitions between analogue states of mirror nuclei can be used to extract information on isospin mixing. This discussion is formulated in general terms without explicit reference to specific isobaric systems. As will be shown, the main result of this analysis is that, provided sufficient B(E1) values are known experimentally, a model-independent estimate of isospin mixing can be obtained which does not rely on a calculation of E1 matrix elements. Having set this challenge to experiment, we will review whether sufficient information is available in either the $A = 31$ or $A = 35$ systems to obtain a model-independent determination of isospin mixing.

We begin the discussion with reference to Fig. 1 which shows a generic ensemble of isobaric analogue states in $T_z = \pm 3/2$ and $\pm 1/2$ nuclei near the $N = Z$ line. We assume that the B(E1) values between initial and final low-lying states J_i and J_f in the nuclei with $T_z = +1/2$ and $T_z = -1/2$ are known experimentally. These states have
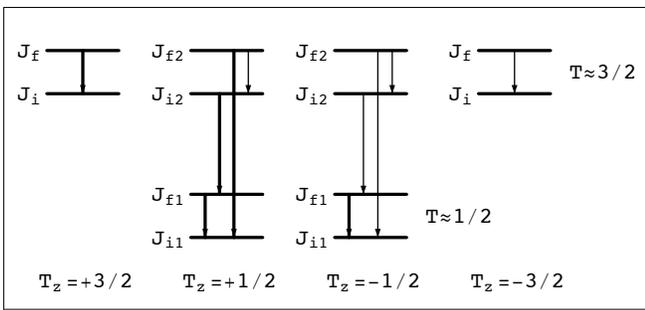
FIG. 1: Schematic figure of isobaric analogue states in $T_z = \pm 3/2$ and $\pm 1/2$ nuclei near the $N = Z$ line. The arrows indicate the 10 possible E1 transitions from where information on the isospin mixing can be extracted. The thick arrows correspond to the E1 transitions known in the $A = 31$ and $A = 35$ nuclei (see text).

a dominant $T = 1/2$ component, but contain small admixtures of higher-lying states with $T = 3/2$. The problem at hand is the following: What additional experimental information is required to determine the isospin mixing, which, for the present purposes, may be different for the two states involved? In Fig. 1, one such set of higher-lying states is shown; they necessarily have the same angular momenta J_i and J_f, as their low-lying siblings, but carry predominantly isospin $T = 3/2$. Of course, these higher-lying states may in turn contain $T = 5/2$ admixtures, but as long as these are small they do not affect the subsequent argument and they can be neglected. The figure shows the simplest situation when only one higher-lying state for each angular momentum J_i and J_f mixes with the $T = 1/2$ states. If there are several such higher-lying states, each contribution must be considered separately and gives rise to additional isospin admixtures.

An observation of central importance to what follows is that the electric dipole operator, to a very good approximation, has isovector character, that is, it transforms as a pure vector under rotations in isospin space. Its isoscalar part is totally absent from an E1 transition internal to the nucleus. This implies that, under the assumption of isospin symmetry, there exist definite relations between transitions between isobaric analogue states. In particular, the B(E1) values of transitions between corresponding states in mirror nuclei are identical and, more generally, those between isobaric analogue states are proportional with proportionality factors that are related to isospin Clebsch-Gordan coefficients. If isospin were an exact symmetry, all E1 transition strengths could be expressed in terms of only four matrix elements reduced in

angular momentum and in isospin which we denote as

$$M_{2T_f, 2T_i} \equiv \langle J_f; T_f |||T^{(1)}(\mathrm{E1})|||J_i; T_i \rangle. \qquad (1)$$

The triple bars indicate that the dependence on the initial and final angular momentum projections M_i and M_f and on the isospin projection T_z has been factored out. This dependence is contained in the Clebsch-Gordan coefficients $\langle J_f M_f\, 1\mu | J_i M_i \rangle$ and $\langle T_f T_z\, 10 | T_i T_z \rangle$ (or, depending on the convention, corresponding $3j$ symbols), where we use the fact that the E1 transition operator has isovector character as indicated with its superscript '(1)'. The initial and final angular momenta J_i and J_f are the same for all four reduced matrix elements, but the isospins T_i and T_f can be $1/2$ or $3/2$. The explicit expression for an arbitrary B(E1) value in terms of the triply barred reduced matrix elements is

$$
\begin{aligned}
B(\mathrm{E1}&; J_i T_i T_z \to J_f T_f T_z) \\
&= \frac{1}{2J_i + 1} \langle J_f; T_f T_z || T_0^{(1)}(\mathrm{E1}) || J_i; T_i T_z \rangle^2, \qquad (2)
\end{aligned}
$$

with

$$
\begin{aligned}
\langle J_f; & T_f T_z || T_0^{(1)}(\mathrm{E1}) || J_i; T_i T_z \rangle \\
&= (-)^{T_f - T_z} \begin{pmatrix} T_f & 1 & T_i \\ -T_z & 0 & T_z \end{pmatrix} M_{2T_f, 2T_i}, \qquad (3)
\end{aligned}
$$

where the symbol between brackets is a $3j$ symbol. The conclusion is that, if isospin is an exact symmetry, the B(E1) values of the 10 transitions indicated in Fig. 1 can be expressed in terms of four quantities M_{11}, M_{13}, M_{31}, and M_{33}. Note, in particular, that M_{31} is different from M_{13}.

If one allows for isospin mixing between the low- and high-lying J_i and J_f states, a further two unknowns are introduced which can be denoted as mixing angles ϕ_i and ϕ_f. The true low- and high-lying initial states $|J_{1i}\rangle$ and $|J_{2i}\rangle$ can be expressed as follows in terms of the isospin eigenstates:

$$
\begin{aligned}
|J_{1i}\rangle &= \cos\phi_i |J_i; T = 1/2\rangle + \sin\phi_i |J_i; T = 3/2\rangle, \\
|J_{2i}\rangle &= -\sin\phi_i |J_i; T = 1/2\rangle + \cos\phi_i |J_i; T = 3/2\rangle, (4)
\end{aligned}
$$

and similarly for the final states $|J_{1f}\rangle$ and $|J_{2f}\rangle$ in terms of the mixing angle ϕ_f. As a consequence, the B(E1) values of the 10 transitions are modified and now depend on the four matrix elements M_{kl} as well as on the two mixing angles ϕ_i and ϕ_f. We find that the B(E1) values in the $T_z = +1/2$ nucleus are given by

$$
B(\mathrm{E1}; J_{1i} \to J_{1f}) = \frac{1}{6(2J_i + 1)} \left(M_{11} \cos\phi_f \cos\phi_i - \frac{M_{33}}{\sqrt{10}} \sin\phi_f \sin\phi_i - M_{13} \cos\phi_f \sin\phi_i + M_{31} \sin\phi_f \cos\phi_i \right)^2,
$$

$$B(\text{E1}; J_{1\text{i}} \to J_{2\text{f}}) \;=\; \frac{1}{6(2J_\text{i}+1)} \left(-M_{11}\sin\phi_\text{f}\cos\phi_\text{i} - \frac{M_{33}}{\sqrt{10}}\cos\phi_\text{f}\sin\phi_\text{i} + M_{13}\sin\phi_\text{f}\sin\phi_\text{i} + M_{31}\cos\phi_\text{f}\cos\phi_\text{i} \right)^2,$$

$$B(\text{E1}; J_{2\text{i}} \to J_{1\text{f}}) \;=\; \frac{1}{6(2J_\text{i}+1)} \left(M_{11}\cos\phi_\text{f}\sin\phi_\text{i} + \frac{M_{33}}{\sqrt{10}}\sin\phi_\text{f}\cos\phi_\text{i} + M_{13}\cos\phi_\text{f}\cos\phi_\text{i} + M_{31}\sin\phi_\text{f}\sin\phi_\text{i} \right)^2,$$

$$B(\text{E1}; J_{2\text{i}} \to J_{2\text{f}}) \;=\; \frac{1}{6(2J_\text{i}+1)} \left(M_{11}\sin\phi_\text{f}\sin\phi_\text{i} - \frac{M_{33}}{\sqrt{10}}\cos\phi_\text{f}\cos\phi_\text{i} + M_{13}\sin\phi_\text{f}\cos\phi_\text{i} - M_{31}\cos\phi_\text{f}\sin\phi_\text{i} \right)^2. \quad (5)$$

In the $T_z = -1/2$ nucleus, the same expressions apply but with the replacements $M_{kl} \mapsto -M_{kl}$ if $k = l$ and $M_{kl} \mapsto M_{kl}$ if $k \neq l$. Finally, in the $T_z = \pm 3/2$ nuclei the $B(\text{E1})$ values are given by

$$B(\text{E1}; J_\text{i} \to J_\text{f}) \frac{3}{20(2J_\text{i}+1)} (M_{33})^2. \quad (6)$$

If at least six (appropriate) $B(\text{E1})$ values are known, these equations determine the matrix elements M_{kl} and the mixing angles ϕ_i and ϕ_f. If five $B(\text{E1})$ values are known, they determine a relation between ϕ_i and ϕ_f.

To illustrate our method, we assume that the five transitions which are indicated with thick arrows in Fig. 1 have known $B(\text{E1})$ values. We can then eliminate the four unknown matrix elements M_{kl} and we obtain the following equation in ϕ_i and ϕ_f:

$$3\big[M(-1/2,1,1) - M(+1/2,1,1)$$
$$+M(+1/2,1,1)\cos 2\phi_\text{f} + M(+1/2,1,1)\cos 2\phi_\text{i}$$
$$-M(+1/2,1,2)\sin 2\phi_\text{f} - M(+1/2,2,1)\sin 2\phi_\text{i}\big]$$
$$= 4M(+3/2,1,1)\sin\phi_\text{f}\sin\phi_\text{i}, \quad (7)$$

where $M(T_z,k,l)$ is a short-hand notation for the square-root of a measured $B(\text{E1})$ value in a T_z nucleus according to

$$M(T_z,k,l) \equiv \pm\sqrt{B(\text{E1}; J_{\text{i}k} \to J_{\text{f}l})}. \quad (8)$$

Equation (7) defines a relation between the mixing angles ϕ_i and ϕ_f which is *independent of all theoretical matrix elements M_{kl}*.

Note that the measured transition strengths do not provide information on the sign of the quantities $M(T_z,k,l)$. If five measured $B(\text{E1})$ values are known, this leads in principle to $2^5 = 32$ different choices. The problem can be simplified as follows. First, we have that $M(-1/2,1,1) \approx -M(+1/2,1,1)$, so we may assume that these matrix elements have opposite sign. This also follows from Eq. (7) for $\phi_\text{i} = \phi_\text{f} = 0$ (no isospin mixing). Furthermore, without loss of generality, we may choose the sign of one matrix element, and we take $M(+1/2,1,1)$ and $-M(-1/2,1,1)$ positive. We still are left with $2^3 = 8$ possible choices of the signs of $M(1/2,1,2)$, $M(+1/2,2,1)$ and $M(+3/2,1,1)$. A convenient way to run through all eight possibilities is the following. We can always adopt a convention in Eq. (4) such that $\cos\phi_\text{i}$, $\sin\phi_\text{i}$, $\cos\phi_\text{f}$, and $\sin\phi_\text{f}$ are positive which corresponds to the domains $0 \leq \phi_\text{i} \leq \pi/2$ and

$0 \leq \phi_\text{f} \leq \pi/2$. A change of sign of $M(+1/2,1,2)$ in Eq. (7) is equivalent to the substitution $\phi_\text{f} \mapsto \pi-\phi_\text{f}$. Similarly, changing the sign of $M(+1/2,2,1)$ is equivalent to $\phi_\text{i} \mapsto \pi - \phi_\text{i}$. Finally, $M(+3/2,1,1) \mapsto -M(+3/2,1,1)$ corresponds to either $\phi_\text{i} \mapsto \phi_\text{i} + \pi$ or $\phi_\text{f} \mapsto \phi_\text{f} + \pi$. We, therefore, conclude that the entire set of solutions is scanned if $0 \leq \phi_\text{i} \leq \pi$ and $0 \leq \phi_\text{f} \leq 2\pi$ or if $0 \leq \phi_\text{i} \leq 2\pi$ and $0 \leq \phi_\text{f} \leq \pi$. Beyond these boundaries, solutions will repeat themselves. If, as is expected, $|\sin\phi_\text{i}|$ and $|\sin\phi_\text{f}|$ are small, it is more convenient to scan the domain $-\pi/2 \leq \phi_\text{i} \leq \pi/2$ and $-\pi/2 \leq \phi_\text{f} \leq 3\pi/2$. In this convention there are two physical regions, either $(\phi_\text{i},\phi_\text{f}) \approx (0,0)$ or $(\phi_\text{i},\phi_\text{f}) \approx (0,\pi)$.

From an examination of the theoretical background to this problem, it is clear that at least five $B(\text{E1})$ values are required to assess the magnitude of the isospin mixing and six $B(\text{E1})$ values to fix the mixing angles without ambiguity. The present work focusses on the $A = 31$ and $A = 35$ cases. In the former example, a $B(\text{E1})$ value is known for the $7/2^- \to 5/2^+$ transition in ^{31}P from a previous lifetime measurement [19], but lifetimes were only known for a few low-lying states in ^{31}S. The motivation of the present experimental work, therefore, was to obtain lifetimes for levels in ^{31}S in order to extract $B(\text{E1})$ values for the transitions of interest.

III. LIFETIME MEASUREMENTS FOR THE A=31 MIRROR NUCLEI

Lifetimes of excited states in ^{31}S have been obtained in the present work using the Doppler Shift Attenuation Method (DSAM). An earlier study of the mirror nuclei, ^{31}S and ^{31}P used the ^{20}Ne+^{12}C reaction [4]. While it would have been desirable to repeat this reaction, which was known to give a good population of the $7/2^-$ state of interest, it was found that it was difficult to get a reliable adhesion between a carbon foil and a thick target backing. It was, therefore, decided to change to the ^{16}O+^{16}O reaction and make use of metal oxide on metal targets. The Tandem accelerator from the ATLAS facility at Argonne National Laboratory accelerated a ^{16}O beam to an energy of 29 MeV. The beam bombarded a 530 $\mu g/cm^2$-thick target of nickel monoxide on a backing of 3.5 mg/cm^2 of nickel. The resulting gamma radiation was detected using the Gammasphere array [15] consisting of 100 high-purity germanium detectors. In this array, there are 17 different angular ring positions

which could be used to obtain DSAM lineshapes. The γ-ray coincidence data were sorted into a series of matrices with γ rays detected in all detectors on one axis and γ rays detected in a specific ring on the other.

The Monte Carlo DSAM code "lineshape" was employed to fit the observed lineshapes and determine lifetimes [16]. The slowing-down process in the target and backing was modelled using the shell-corrected stopping powers of Northcliffe and Schilling [17]. In a DSAM analysis, the general procedure is to fit the lineshape in angle-sorted spectra gated by transitions lying above the transition of interest in order to remove the effect of side-feeding. The low population of the $T_z=1/2$ nucleus ^{31}S, however, meant that it was not possible to gate on transitions above the level of interest when obtaining lineshape spectra for the decay of the $7/2^-$ level. It was, therefore, necessary to gate on transitions below and include the effects of side-feeding. The feeding of the $7/2^-$ level in ^{31}S comes principally from a 1926-keV γ ray de-exciting a $9/2^-$ state and a 2383-keV γ ray de-exciting an $11/2^-$ level. In the case of ^{31}P, the nucleus is populated to much higher spin and the feeding pattern is complex. Ionescu-Bujor *et al.*[14] have recently reported lifetimes for some of these high-lying high spin states which feed the $7/2^-$ level. In some cases, these exceed 1 ps. Fortunately, these states do not appear to be populated in ^{31}S in our study and the 1926 and 2383 keV transitions decay from states with relatively short lifetimes. Since the population of levels above appeared negligible, we fit the lineshapes corresponding to the 1926- and 2383- keV transitions using lineshape spectra produced by gating below, and obtained effective lifetimes of 270(50) and 200(40) fs for the $9/2^-$ and $11/2^-$ levels, respectively. These are in reasonable conformity with the lifetimes of the mirror states in ^{31}P which have lifetimes of 55(17) fs and 120(50) fs, respectively. Lineshape spectra for the 1166-keV $7/2- \rightarrow 5/2^+$ transition were obtained by summing spectra gated by the 1249- and 2036-keV transitions. Figure 2 shows the lineshape of the 1166-keV transition at 70°, 90° and 110°. Fitting these lineshapes for the 1166-keV transition, incorporating side-feeding from the two discrete transitions, led to a lifetime for the $7/2^-$ state in ^{31}S of 0.98(20) ps.

To examine the reliability of the methodology of gating below the transition of interest, a lifetime was obtained for the corresponding $7/2^-$ state in ^{31}P which decays by a 1136-keV γ ray for which the lifetime had been previously measured to be 0.59(3) ps [19]. First, a gate was set on the 2394-keV transition above the level of interest and lineshapes corresponding to the 1136-keV transition were fit. A lifetime of 0.64(13) ps was obtained. Then, by using the sum of gated spectra of the 2029- and 1266-keV transitions below the level of interest, the line shape of 1136-keV transition was fitted with a consideration of the estimated effective lifetimes contributed by the 2071-, 2365- and 2394-keV transitions. In this case, the lifetime extracted was 0.71(8) ps, consistent with that obtained through gating above.
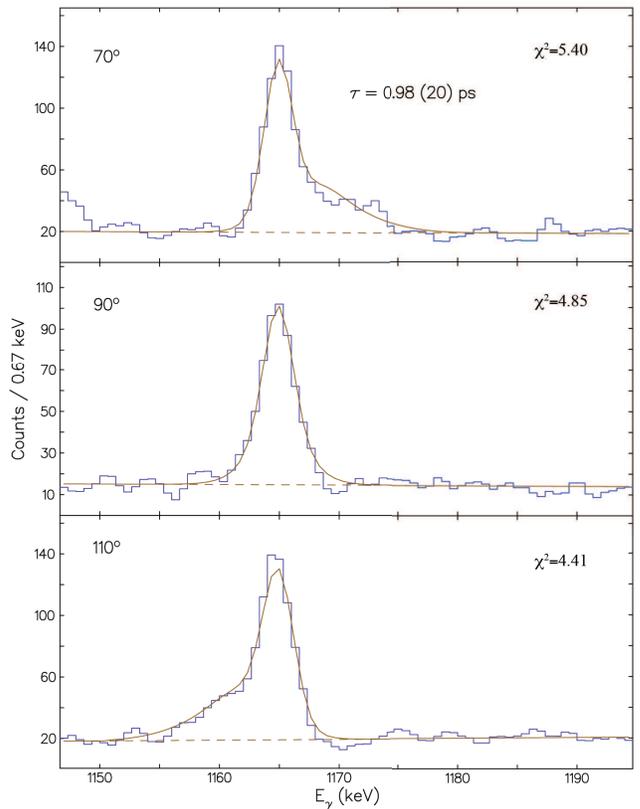

FIG. 2: (color online) Lineshapes for the 1166-keV gamma ray in ^{31}S shown for 70° (top), 90° (middle) and 110° (bottom). The data is shown as blue histograms, with the fit obtained by the lineshape program in brown. The spectra were obtained by gating on the 1249- and 2036 keV transitions below the γ ray of interest.

IV. B(E1) TRANSITION STRENGTHS

Having obtained a lifetime for the $7/2^-$ level in ^{31}S, we are now able to calculate B(E1) transition rates for the transitions de-exciting both $7/2^-$ states in ^{31}P *and* ^{31}S (see table I). We are only able to set an upper limit on the B(E1) strength for the unobserved 2215-keV γ ray. It should be noted that this ignores possible M2 admixtures which become more likely as the E1 strength is attenuated. In any case, the B(E1) strength for the $7/2^- \rightarrow 5/2^+$ transition in ^{31}P exceeds that of the analagous transition in ^{31}S by at least a factor of forty.

In order to put the present observations in context, let us return to the case of the A=35 mirror nuclei, where seemingly the opposite behavior to the A=31 case is observed: a prominent $7/2^- \rightarrow 5/2^+$ E1 transition is observed in the T_z=-1/2 nucleus, ^{35}Ar, whereas this branch is seemingly very weak in the mirror nucleus, ^{35}Cl. In fact, the measured lifetime for the $7/2^-$ state in ^{35}Cl indicates that the latter transition has the extremely weak transition strength of $1.4(3) \times 10^{-8}$ W.u.

Nucleus	E_γ (keV)	$I_i \to I_f$	B(E1) W.u.
^{31}S	1166	$7/2^- \to 5/2_2^+$	$6.4(1.3) \times 10^{-4}$ W.u.
	2215	$7/2^- \to 5/2_1^+$	$<1.9 \times 10^{-6}$ W.u.
^{31}P	1136	$7/2^- \to 5/2_2^+$	$4.5(8) \times 10^{-4}$ W.u.
	2195	$7/2^- \to 5/2_1^+$	$8.2(6) \times 10^{-5}$ W.u.

TABLE I: Table of B(E1) transition rates obtained for ^{31}S from lifetimes measured in the present work and ^{31}P from the literature. The 2215-keV transition is presently unobserved, an upper limit on its branching ratio of 2 % is obtained from previous work [4]. In all cases, the transitions are assumed to have a negligble M2 component.

This transition also exhibits a significant M2 component (B(M2)=6.3(37) $\times$ 10^{-3} W.u.) [18]), which is perhaps not surprising given the extremely small E1 matrix element. The lifetime of the $7/2^-$ level in ^{35}Ar is presently unknown. The approach taken by Ekman et $al.$ was to assume that the M2 transition from the $7/2^-$ state has the same transition strength in each case (B(M2) = 0.28 W.u.) [3]. Following this approach, B(E1) for the 1446 keV transition in ^{35}Ar is 3×10^{-5} which is 2000 times larger than the analogous transition in ^{35}Cl. Ekman et $al.$ assume that isospin mixing is taking place and that there are contributions to the matrix elements diagonal and non-diagonal in T which have similar magnitude (1.5×10^{-5} W.u.), and which cancel in the case of ^{35}Cl and sum in the case of ^{35}Ar. We note that this analysis is not as detailed as that presented in the present work. Moreover, while the analysis by Ekman et $al.$ appears reasonable qualitatively, it does lead to the conclusion that the two components diagonal and non-diagonal in T would have to differ by less than 0.1 %, which would be an astonishing coincidence. Clearly, a possible weakness in this analysis may be the assumption that the M2 transitions in the mirror nuclei have the same strength. Unlike E1 transitions, M2 transitions have both an isoscalar and an isovector part. Warburton and Weneser suggest a "quasi-rule" for such transitions that so long as they are relatively strong, they should be of near-equal transition strength in conjugate nuclei [8]. However, the M2 transitions in this case are not strong. Prosser and Harris calculated M2 transition rates for the A=35 mirror nuclei and predicted that the $7/2^- \to 3/2^+$ transition in ^{35}Ar should have B(M2)=0.0032 W.u., compared to B(M2)=0.185 W.u. in ^{35}Cl [18]. If we use this predicted B(M2) strength for ^{35}Ar, in conjunction with the M2 branching ratios as measured by Ekman et $al.$ then we conclude that the 1446-keV transition would have B(E1)=6.7(20) $\times$ 10^{-7} W.u. This reanalysis suggests that the two E1 transitions differ in transition strength by a factor of 50 rather than a factor of 2000 as suggested by Ekman et $al..$ Both E1 transitions are seen to be extremely weak cf. B(E1) $\approx 2.5 \times 10^{-7}$ W.u for the $5^- \to 4^+$ transition in ^{64}Ge [10]. Moreover, the variation in transition strength between the mirror nu-

clei is now of similar order to that observed in the A=31 mirror pair, albeit it is the T_z=1/2 member which has the weaker B(E1) strength in A=35. Clearly, it would be very worthwhile to measure the lifetime of the $7/2^-$ state in ^{35}Ar to verify the predicted B(M2) value. On the basis of the predicted B(M2) value, this lifetime should be around 350 ps.

As discussed above, in order to begin to determine the level of isospin mixing, we need additional matrix elements in order to determine the isospin-mixing, we now require at least three more matrix elements corresponding to T=3/2 $\to$ T=1/2, T=1/2 $\to$ T=3/2 and T=3/2 $\to$ T=3/2 transitions. For completeness, we review the available data for both the $A = 31$ and $A = 35$ cases.

For the T=3/2 $\to$ T=1/2 component, we can make use of a recent series of detailed (p,γ) studies on ^{30}Si and ^{34}S [22]. These measurements have identified the $1f_{7/2}$ isobaric analogue states in both ^{31}P and ^{35}Cl [22]. Their gamma widths are (1.63(25) eV) and 1.37(20) eV), respectively. These values may be combined with accurate branching ratios for the $1f_{7/2}$ resonance in ^{31}P [21] and ^{35}Cl [18], leading to B(E1) values for the $7/2^-$, T=3/2 $\to 5/2_1^+$, T=1/2 transitions of $2.0(3) \times 10^{-4}$ W.u. in ^{31}P and $3.3(7) \times 10^{-5}$ W.u. in ^{35}Cl.

While the $1f_{7/2}$ analogue states appear unique, which leads to a relatively simple extraction of the relevant matrix elements, the situation appears significantly more complex when we attempt to obtain the T=1/2 $\to$ T=3/2 component. In this case, we need to examine $5/2^+$, T=3/2 $\to$ $7/2^-$, T=1/2 transitions, but this is not straightforward since in both ^{31}P and ^{35}Cl, there are a number of known (p,γ) resonances with J^π=$5/2^+$. The question naturally arises, then, as to which resonances to consider. To first order, we would expect the most significant isospin mixing to take place between analogue states. Analogue states should be connected by strong M1 transitions. In ^{31}P, the first $5/2^+$, $T = 3/2$ resonance is split into two components at 8.032 and 8.105 MeV. These components have a negliglible gamma branch to the T=1/2, $7/2^-$ state [23]. There is a second $5/2^+$ resonance split into a further five fine structure components between 9.009 and 9.131 MeV [24]. These levels have a more significant branch to T=1/2 $7/2^-$ state. If we sum all of these component gamma branches, we obtain B(E1) $\approx 3.4 \times 10^{-4}$ W.u. [24]. While the lower pair of resonances have a very weak M1 decay to the $5/2^+$ state at 2234 keV, the two $5/2^+$ resonances at 9.067 and 9.116 MeV have strong M1 branches to the $5/2^+$ state. The upper set of resonances, therefore, appear the most relevant for determining the required E1 transition strength. To summarise, then, we have considered only the upper set of resonances around 9 MeV as the relevant analogue to the T=1/2, $5/2^+$ state and used the sum of the E1 branches from this set of states in our analysis. The T=1/2 $\to$ T=3/2 and T=3/2 $\to$ T=1/2 matrix elements in ^{31}P, therefore, appear to be of similar order i.e. B(E1) $\sim 10^{-4}$ W.u. Indeed, these are typical values for isovector E1 transitions.

Following a similar procedure, if we sum the measured B(E1) strengths for $5/2^+$ resonances in the ^{34}S(p,γ) reaction which correspond to states at 8.216, 8.893 and 9.081 MeV in ^{35}Cl, we obtain $\approx 4.3 \times 10^{-3}$ W.u. [25]. Each of these resonances gives a similar strength to the total B(E1) probability. The 8.893-MeV resonance has a strong M1 branch to the $5/2^+$,T=1/2 state. In this case, therefore, the T=1/2 $\rightarrow$ T=3/2 matrix element appears to considerably exceed the T=3/2 $\rightarrow$ T=1/2 matrix element.

To obtain matrix elements for the T=3/2 $\rightarrow$ T=3/2 component, we need to turn to the T_z=3/2 nuclei, ^{35}S and ^{31}Si since in the T_z=1/2 nuclei, it would be difficult to observe such isoscalar transitions in competition with transitions to the T=1/2 states. In the case of ^{35}S, we have an upper limit only for the strength of the $5/2^+$, T=3/2 $\rightarrow 7/2^-$, T=3/2 transition, with B(E1) $< 1.5 \times 10^{-3}$ W.u. The $5/2^+$ state involved is the analogue of the upper of the set of $5/2^+$ resonances in ^{35}Cl, which we considered when obtaining the T=1/2 $\rightarrow$ T=3/2 matrix element.

For ^{31}Si, the situation is more complicated since there is a measured B(E1) value for the $5/2^+$, T=3/2 $\rightarrow 7/2^-$, T=3/2 transition of $6.0(13) \times 10^{-4}$ W.u. The $5/2^+$ state involved here, though, corresponds to the lower set of T=3/2, $5/2^+$ resonances in ^{31}P, which had negligible E1 branches to the T=1/2, $7/2^-$ state, as well as weak M1 transitions to the lowest $5/2^+$, T=1/2 state. This inconsistency cautions against the use of this set of values in any calculation. The extent of our knowledge of relevant B(E1) strengths in the A=31 and 35 systems is summarised in figure 3.

Using the matrix elements for the $A = 31$ and $A = 35$ cases shown in Fig. 3, we can solve relation (7) graphically. It is important to remember that, in the $A = 31$ isobars, the results of the analysis will be rendered uncertain since we do not have information on the relevant B(E1) value in ^{31}Si. In the $A = 35$ isobars, on the other hand, we do not have an experimental matrix element for the $T = 1/2 \rightarrow T = 1/2$ component in ^{35}Ar, for which we have been forced to rely on scaling from a predicted B(M2) value [18].

Figure 4 illustrates the allowed values of the isospin mixing angles ϕ_i and ϕ_f in the $A = 31$ nuclei. Since five B(E1) values are known experimentally, only a relation between the mixing angles can be established which corresponds to a curve in the domain $-\pi/2 \leq \phi_i \leq \pi/2$ and $-\pi/2 \leq \phi_f \leq 3\pi/2$. In addition, only an upper limit is known for the $B(E1; 7/2^- \rightarrow 5/2^+)$ value in ^{31}S, and hence consistency with relation (7) is imposed for the range $0 \leq B(E1; 7/2^- \rightarrow 5/2^+) \leq 1.9 \times 10^{-6}$ W.u. This leads to the two closely-spaced curves in Fig. 4 which meander through the entire allowed domain; the small region between one curve and its immediately adjacent one defines the mixing angles consistent with (7). We see from Fig. 4 that none of allowed solutions goes through the regions with $|\sin\phi_{i,f}| < 0.1$, which correspond with isospin mixing smaller than 1% in both states.

We conclude that no coherent picture is obtained from the $A = 31$ data as regards E1 transitions and isospin mixing, which is probably due to our inconsistent use of analogue transitions, as pointed out above.

The situation is more encouraging in the $A = 35$ nuclei. Again, many different values of (ϕ_i, ϕ_f) are consistent with relation (7), but we may focus our attention on the regions $(\phi_i, \phi_f) \approx (0, 0)$ or $(\phi_i, \phi_f) \approx (0, \pi)$. A band of allowed (ϕ_i, ϕ_f) values going through the former region is indicated in Fig. 5. The middle line is the solution of relation (7) with the (largely) experimental matrix elements $M(T_z, k, l)$ in the $A = 35$ isobars. The outer lines are consistent with this solution to within 1σ deviation where the errors on all $B(E1)$ values have been taken into account. The sensitivity to the errors on the different $B(E1)$ values varies strongly. For example, the solution is largely insensitive to the $B(E1; 5/2^+, 1/2 \rightarrow 7/2^-, 1/2)$ value in ^{35}S, and the currently known upper limit suffices for the present purpose, that is, the error on this $B(E1)$ value does not contribute significantly to the error in the (ϕ_i, ϕ_f) plot. For reducing the latter error, a better precision is required for the $7/2^-, 1/2 \rightarrow 5/2^+, 1/2$ transition in ^{35}Ar, and the $7/2^-, 3/2 \rightarrow 5/2^+, 1/2$ and $5/2^+, 3/2 \rightarrow 7/2^-, 1/2$ transitions in ^{35}Cl. Moreover, an additional $B(E1)$ value is needed for pinning down the mixing angles unambiguously.

V. CONCLUSIONS

In conclusion, we have examined theoretically whether isospin mixing in bound nuclear levels can be obtained from consideration of E1 transition strengths in analogue systems. Five $B(E1)$ values are required to determine a relation between the mixing angles of the initial and final states, while an additional $B(E1)$ value would be required to determine the mixing of each state individually. We have obtained a $B(E1)$ value for the $7/2^-, 1/2 \rightarrow 5/2^+, 1/2$ transition in ^{31}S and collated known $B(E1)$ values for the $A = 31$ and $A = 35$ mirror pairs. A solution is obtained in the $A = 35$ case, which is consistent with less than 1% isospin mixing for both levels, using a $B(E1)$ value for the $7/2^-, 1/2 \rightarrow 5/2^+, 1/2$ transition in ^{35}Ar obtained from scaling to a calculated $B(M2)$ value for a transition from the same $7/2^-$ level. Further work to determine this matrix element experimentally would be very valuable in confirming these initial conclusions. Moreover, measurements of additional matrix elements in both systems, which would likely involve challenging measurements perhaps involving radioactive beams, are clearly desirable to extract the isospin mixing of the individual states.
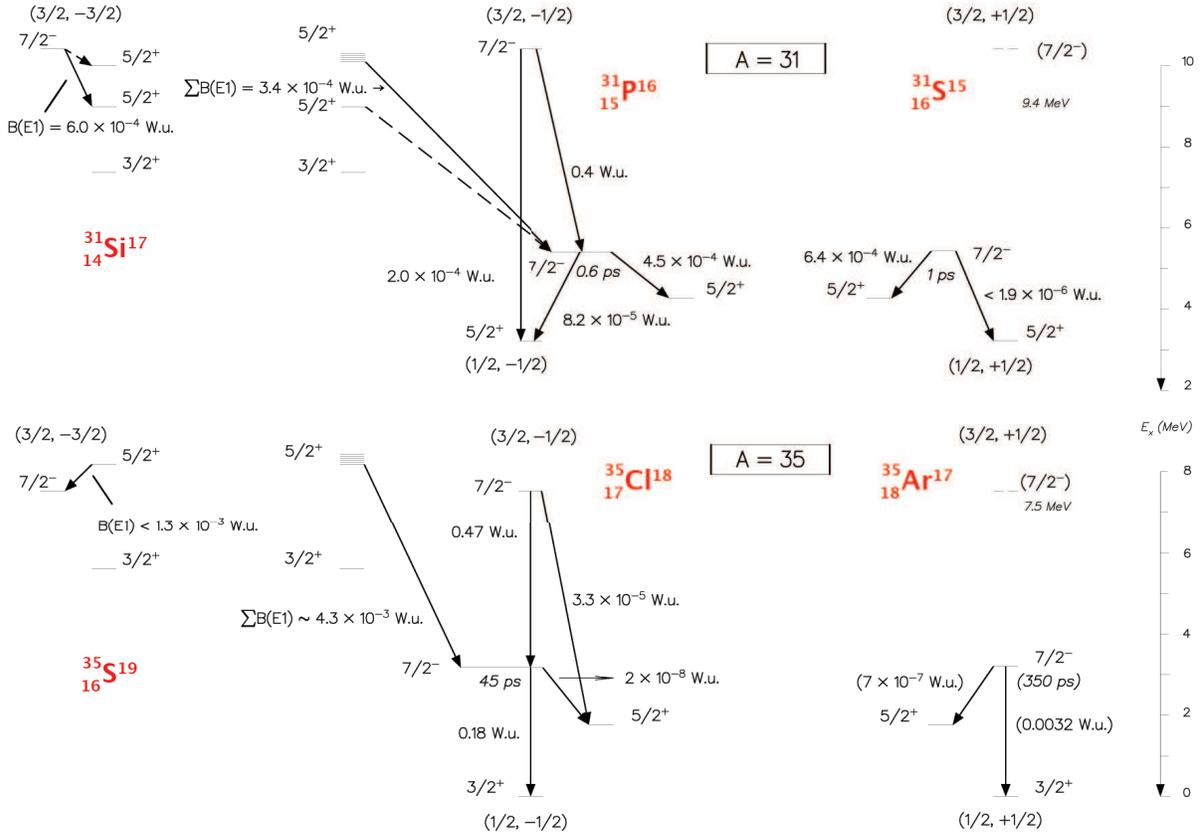
FIG. 3: (color online) Simplified level schemes showing the states of interest in the A=31 and A=35 systems. The states are labelled with their isospin using the convention (T,T$_z$). Transition strengths are shown where known. The manner in which these strengths were arrived at is described in the text, in particular, those for the decay of the T=1/2, 7/2$^-$ state in ^{35}Ar, which are calculated not measured.

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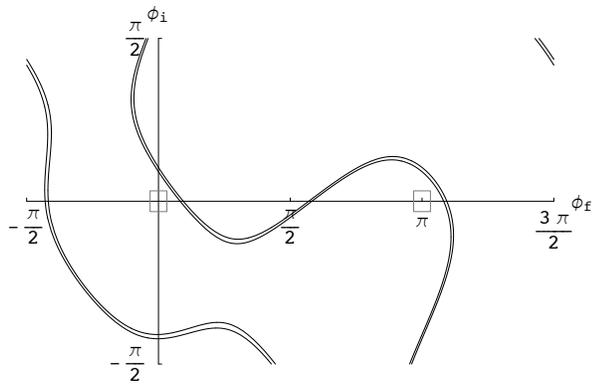
FIG. 4: Correlation plot between the isospin mixing angles ϕ_i and ϕ_f consistent with the relation (7) with experimental information as available in the $A = 31$ nuclei. The solution of the relation (7) meanders through the entire region $-\pi/2 \leq \phi_i \leq \pi/2$ and $-\pi/2 \leq \phi_f \leq 3\pi/2$. The two closely-spaced lines correspond to the solution for the upper limit of the $B(\text{E}1; 7/2^- \to 5/2^+)$ value in ^{31}S and for a zero value for this quantity, respectively. The region of mixing ($|\sin\phi_{i,f}| < 0.1$) is indicated with the grey rectangles.
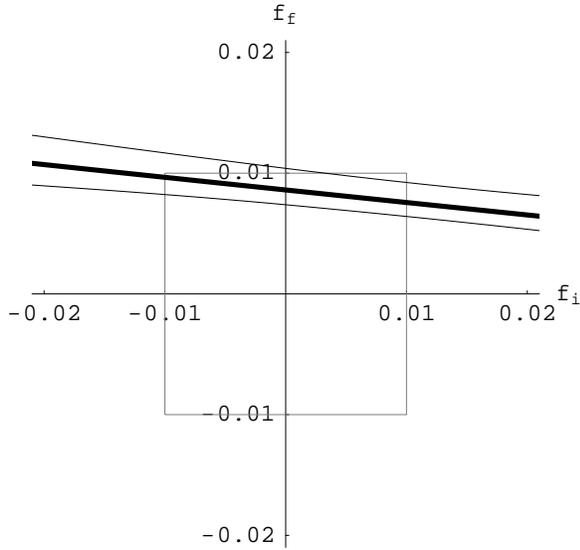

FIG. 5: Correlation plot between the isospin mixing angles ϕ_i and ϕ_f consistent with relation (7) and the experimental information available for the $A = 35$ nuclei. The thick line is the solution of relation (7) and the outer lines are consistent with it to within 1σ deviation. The region of small mixing ($|\sin\phi_{i,f}| < 0.01$) is indicated with the grey rectangle.

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