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IBM-1 calculations towards the neutron-rich nucleus $^{106}$Zr

Stefan Lalkovski$^{1,2,3}$ and P. Van Isacker$^3$

$^1$Faculty of Physics, University of Sofia, Sofia, BG-1164 Bulgaria
$^2$School of Environment and Technology, University of Brighton, Brighton BN2 4JG, UK
$^3$Grand Accélérateur National d’Ions Lourds, CEA/DSM-CNRS/IN2P3, BP 55027, F-14076 Caen Cedex 5, France

(Dated: March 23, 2009)

The neutron-rich $N = 66$ isotonic and $A = 106$ isotopic chains, covering regions with varying types of collectivity, are interpreted in the framework of the interacting boson model. Level energies and electric quadrupole transition probabilities are compared with available experimental information. The calculations for the known nuclei in the two chains are extrapolated towards the neutron-rich nucleus $^{106}$Zr.

PACS numbers:

I. INTRODUCTION

In the last decade the neutron-rich nuclei in the $40 \leq Z \leq 50$ region have attracted both theoretical and experimental attention. They were extensively studied via spontaneous or induced fission reactions. Nuclei from this region of Segré chart exhibit vibrational, transitional, and rotational types of collectivity. The neutron-rich palladium and ruthenium nuclei, for example, show a typical transitional behavior while molybdenum isotopes exhibit a vibrational to near-rotational evolution. Such changes in the degree of collectivity are even stronger in the zirconium isotopic chain. Effects arise precisely at $N = 66$ in the ruthenium isotopes which have 64 neutrons and is the heaviest zirconium nucleus known to date. The shift of the neutron-rich region comes from prompt $\gamma$-ray spectroscopy. The observed levels are grouped into $\Delta J = 2$ sequences, corresponding to the ground-state band, and/or into $\Delta J = 1$ sequences, based on a $2^+_1$ level which usually is interpreted as a quasi-$\gamma$ band. Such structures naturally appear in the framework of the interacting boson model.

In the lower-$Z N = 82$ nuclei, however, have not yet been studied experimentally and the suppression of the shell effects in this region is still an open question. Properties of these nuclei were predicted within a relativistic Hartree-Fock Bogoliubov (HFB) approach. The calculations, performed with different parametrizations, show that the $N = 82$ shell gap persists in the heavy palladium and ruthenium nuclei but that a weakening of this gap is expected in the zirconium isotopes. For example, the two-neutron separation energies from $^{118}$Zr to $^{126}$Zr, obtained via HFB calculations with the SkP interaction, show no discontinuity at $N = 82$ but a rather smooth behavior. As discussed above, nuclear collective behavior is expected to enhance towards the mid-shell as a result of the increase in valence particle number. If, however, the HFB+SkP scenario for the zirconium isotopes turns out to be valid and $N = 82$ vanishes or is weakened in the neutron-rich region, then the determination of valence-particle number is fraught with ambiguity, leading to a different behavior of nuclear collectivity in this isotopic chain. The aim of the present paper, therefore, is to predict the spectroscopic properties concerning the $N = 66$ mid-shell zirconium nucleus $^{106}$Zr, based on the assumption that $N = 82$ remains a magic number in the heavy zirconium isotopes. Comparison with results of future experiments on this nucleus will then reveal whether this hypothesis is borne out or not.

II. MODEL

The lower-$Z$ is not the only available information about neutron-rich nuclei comes from prompt $\gamma$-ray spectroscopy. The observed levels are grouped into $\Delta J = 2$ sequences, corresponding to the ground-state band, and/or into $\Delta J = 1$ sequences, based on a $2^+_1$ level which usually is interpreted as a quasi-$\gamma$ band. Such structures naturally appear in the framework of the interacting boson model.
which has been shown to be successful in the description of nuclear collective properties.

The IBM in its first version, known as IBM-1, is based on the assumption that nuclear collectivity can be expressed in terms of $s$ and $d$ bosons \cite{IBM}. The model Hamiltonian is constructed from a set of 36 operators, bilinear in the boson creation and annihilation operators and generating the U(6) Lie algebra. Dynamical symmetries occur if the Hamiltonian can be written as a combination of invariant (or Casimir) operators of specific subalgebras of U(6) \cite{IBM} and three such cases occur, namely the spherical vibrational limit U(5), the deformed limit SU(3), and $\gamma$-soft limit SO(6). These dynamical symmetries generate energy spectra with states that are labeled by the irreducible representations of the algebras in the respective chains that reduce the dynamical algebra U(6) into the symmetry algebra SO(3) of rotations in three-dimensional space. The different limits thus correspond to nuclei with distinct collective properties. For example, in the SO(6) limit the ground-state and $\gamma$ bands lie in the same representation while in the SU(3) limit they are in different ones, leading to forbidden or weaker inter-band transitions in the latter case.

In the $A \approx 110$ nuclei of interest here strong transitions between the two bands are observed indicating that none of these nuclei can be interpreted in the exact SU(3) limit, but rather a transitional behavior should be expected. This can be achieved in the IBM-1 by the use of the full Hamiltonian which reads \cite{IBM}

$$
\hat{H} = \epsilon_d n_d + \kappa \hat{Q}_d^X \cdot \hat{Q}_d^X + \kappa' \hat{L} \cdot \hat{L} + c_3 \hat{T}_3 \cdot \hat{T}_3 + c_4 \hat{T}_4 \cdot \hat{T}_4,
$$

where $\hat{n}_d \equiv \hat{d}^+ \cdot \hat{d}$ and $\hat{L}_\mu \equiv \sqrt{10}[\hat{d}^+ \times \hat{d}^{(1)}]_\mu$ are the d-boson number and the angular momentum operators, respectively. Furthermore, the quadrupole operator is defined as $\hat{Q}_d^X \equiv [\hat{d}^+ \times \hat{d}^{(2)}]_X + \chi [\hat{d}^+ \times \hat{d}^{(2)}]_\mu$, while the last two terms in the Hamiltonian involve the operators $\hat{T}_{3,4,\mu} \equiv [\hat{d}^+ \times \hat{d}^{(3)}]_\mu$ and $\hat{T}_{4,\mu} \equiv [\hat{d}^+ \times \hat{d}^{(4)}]_\mu$. In the present work there is no need for the rotational term $\hat{L} \cdot \hat{L}$ and we take $\kappa' = 0$ throughout. The total number of bosons $N = n_\pi + n_\eta$ is taken as half the number of valence particles or holes, counted from the nearest closed-shell configuration, following the prescription of Ref. \cite{IBM}. As such, IBM-1 calculations are indirectly related to the underlying shell structure. Finally, in the consistent-Q formalism \cite{IBM} the operator for electric quadrupole transitions is introduced as $\hat{T}_\mu(E2) = e_b \hat{Q}_\mu^X$, where $e_b$ is the boson effective charge and $\hat{Q}_\mu^X$ is the quadrupole operator with the same parameter $\chi$ as in the Hamiltonian \cite{IBM}.

### III. RESULTS AND DISCUSSION

Since the addition or subtraction of a few nucleons may change significantly a nuclear spectrum, two nuclei with different numbers of valence neutrons and protons but with the same total number of valence nucleons—

and hence the same number of bosons—may display different types of collectivity. In adjusting the parameters of the Hamiltonian \cite{IBM} to spectra observed in a given region of the nuclear chart, it is therefore not sufficient to assume them to be constant for all nuclei since that would lead to identical spectra for nuclei with the same $N$. Instead, some dependence of the parameters on the separate neutron and proton boson numbers $N_\pi$ and $N_\eta$ should be imposed. To avoid the complexity of determining the correct functional dependence on $N_\pi$ and $N_\eta$ for each of the Hamiltonian parameters, we have followed the simpler procedure of separately considering chains of isotopes, isotones, or isobars that end with the nucleus of interest, $^{106}$Zr, and fitting each chain with an independent set of parameters. This method avoids the problem of having identical spectra for constant boson number $N$ (such nuclei belong to different chains) and, in addition, since the extrapolation to $^{106}$Zr obtained from the different chains may vary, it gives an idea on the possible error of the prediction.

In the present application to $^{106}$Zr, the $N = 66$ isotonic chain includes $\gamma$-unstable $^{112}$Pd, more triaxial $^{110}$Ru, and near-rotational $^{106}$Mo. The second set of nuclei belongs to the $A = 106$ isobaric chain \cite{IBM} extending from vibrational-like $^{106}$Pd to near-rotational $^{106}$Mo. Unfortunately, the zirconium isotopic chain does not allow the determination of a unique set of model parameters since only yrast data are available for the neutron-rich isotopes $^{106}$Zr, $^{108}$Zr, and $^{110}$Zr. Consequently, only the isotonic and isobaric chains are considered in the fit. Nuclei in a given chain are distinguished by the number of bosons (i.e., particle or hole pairs), counted from the nearest closed-shell configuration which are $Z = 28$ or $Z = 50$ for the protons, and $N = 50$ or $N = 82$ for the neutrons. Figure \ref{fig:fig1} summarizes known nuclei in the neighbourhood of $^{106}$Zr. Nuclei with known excited levels are distinguished (full lines) from those of which only ground-state properties are known (dashed lines). Furthermore, the nuclei used in the fit are highlighted with thick lines.

The model parameters $\epsilon_d$, $\kappa$, $c_3$, and $c_4$, listed in Table \ref{tab:table1} are obtained after a least-squares fit to the experimental level energies. Since its effect on the excitation energies is weak, the parameter $\gamma$ is varied independently to give a fair description of E2 transition rates in $^{106}$Pd within the consistent-Q formalism, and other nuclei are calculated with the same value. Although within a given chain parameters may vary with boson number, any allowed variation does not markedly improve the quality of the fit. Consequently, to keep the number of parameters

| $N$ = 66 | $1171$ | $-24.5$ | $-70.8$ | $-134.9$ | $-0.30$ | $148$ |
| $A$ = 106 | $1053$ | $-25.6$ | $-70.6$ | $-118.3$ | $-0.30$ | $143$ |

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\epsilon_d$</th>
<th>$\kappa$</th>
<th>$c_3$</th>
<th>$c_4$</th>
<th>$\gamma^*$</th>
<th>rms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensionless.</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
respectively. The nucleus

The energy of the

as a function of boson number. The experimental en-

The same trend is observed for the calculated energies

2

isotopic and isobaric chains, the observed energy of the

2

+ band and is shown separately in Table IV. In both the

region 600–650 keV. Furthermore, the two different

fits give approximately the same

value is predicted highest for

112Pd (10 bosons) shows the highest amplitude in

S(J) among the N = 66 isotones. The γ-band stag-

gering is less pronounced in

110Ru (11 bosons) while in

108Mo (12 bosons) it is highly suppressed. From these

systematics trends a γ band with a very low staggering

amplitude is expected in

106Zr, namely

S(J) ≈ 10 keV, which would make

106Zr the nucleus with the lowest stag-

gering amplitude observed in the 40 ≤ Z ≤ 50 region.

Also electric quadrupole transition probabilities have

been considered for the neutron-rich nuclei belonging to

the N = 66 and A = 106 chains. The calculated values

are listed in Tables V and VI along with the few

known experimental values [16]. The boson effective

charge e_b = 0.11 eb is obtained after a least-squares fit

to the

B(E2) values measured in

106Pd and this value is used for all other nuclei. The results show an increase of the

B(E2; 2^+_1 → 0^+_1) value from the near-vibrational

106Pd towards the near-rotational

108Mo. This

B(E2) value is predicted highest for

108Zr and the calculations for the two different chains give a consistent value of

B(E2; 2^+_1 → 0^+_1) ≈ 0.50–0.55 e^2b^2. Furthermore, we note from Table VI the behavior predicted for the

B(E2; 2^+_2 → 0^+_1) value which is small in

106Pd and

106Zr but becomes large for the isotopes in between, a characteristic feature of transitional nuclei.

IV. CONCLUSIONS

The aim of this paper was to illustrate the usefulness of

the interacting boson model in its simplest version, the

IBM-1, for predicting properties of exotic nuclei. The

usual difficulty that arises with such attempts is that

the variations of the model’s parameters with the va-

lence neutron and proton numbers are not known, pre-

FIG. 1: Even-even nuclei in the region close to

106Zr. All nuclei in boxes with full lines have known excited levels while those in boxes with broken lines are only known in their ground state. The nuclei used in the fit are in boxes with thick lines.

to a minimum, they are taken constant for a given chain

and any structural evolution within that chain is due to

the changing boson number N.

The quality of the fit is summarized by the root-mean-
square (rms) deviation between the experimental and cal-

culated level energies. The rms values obtained in the

two independent fits are given in Table II. The experimental

[12, 13, 14, 15, 16] and theoretical energies are listed

in Tables II and III for the

N = 66 and

A = 106 isotones. The

boson effective

charge

and any structural evolution within that chain is due to

the changing boson number N.

The quality of the fit is summarized by the root-mean-
square (rms) deviation between the experimental and cal-

culated level energies. The rms values obtained in the

two independent fits are given in Table II. The experimental

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charge

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the changing boson number N.

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culated level energies. The rms values obtained in the

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A = 106 isotones. The

boson effective

charge

and any structural evolution within that chain is due to

the changing boson number N.

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culated level energies. The rms values obtained in the

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in Tables II and III for the

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A = 106 isotones. The

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charge

and any structural evolution within that chain is due to

the changing boson number N.

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culated level energies. The rms values obtained in the

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A = 106 isotones. The

boson effective

charge

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the changing boson number N.

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culated level energies. The rms values obtained in the

two independent fits are given in Table II. The experimental

[12, 13, 14, 15, 16] and theoretical energies are listed

in Tables II and III for the

N = 66 and

A = 106 isotones. The

boson effective

charge

and any structural evolution within that chain is due to

the changing boson number N.
FIG. 2: Experimental (full symbols connected with full lines) and theoretical (open symbols connected with dashed lines) γ-band staggering \( S(J) \) in the \( N = 66 \) isotones. In \(^{106}\text{Zr}\) both near-identical predictions are shown based on the extrapolation of the \( N = 66 \) isotones (open squares) and of the \( A = 106 \) isobars (open diamonds).

<table>
<thead>
<tr>
<th>TABLE II: Experimental and theoretical excitation energies (in keV) of levels in neutron-rich ( N = 66 ) isotones.</th>
</tr>
</thead>
<tbody>
<tr>
<td>nucleus</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>(^{112}\text{Pd}) ((N = 10))</td>
</tr>
<tr>
<td>2(^+)</td>
</tr>
<tr>
<td>4(^+)</td>
</tr>
<tr>
<td>6(^+)</td>
</tr>
<tr>
<td>8(^+)</td>
</tr>
<tr>
<td>10(^+)</td>
</tr>
<tr>
<td>8(^+)</td>
</tr>
<tr>
<td>9(^+)</td>
</tr>
<tr>
<td>10(^+)</td>
</tr>
<tr>
<td>(^{110}\text{Ru}) ((N = 11))</td>
</tr>
<tr>
<td>2(^+)</td>
</tr>
<tr>
<td>4(^+)</td>
</tr>
<tr>
<td>6(^+)</td>
</tr>
<tr>
<td>8(^+)</td>
</tr>
<tr>
<td>10(^+)</td>
</tr>
<tr>
<td>8(^+)</td>
</tr>
<tr>
<td>9(^+)</td>
</tr>
<tr>
<td>10(^+)</td>
</tr>
<tr>
<td>(^{108}\text{Mo}) ((N = 12))</td>
</tr>
<tr>
<td>2(^+)</td>
</tr>
<tr>
<td>4(^+)</td>
</tr>
<tr>
<td>6(^+)</td>
</tr>
<tr>
<td>8(^+)</td>
</tr>
<tr>
<td>10(^+)</td>
</tr>
<tr>
<td>8(^+)</td>
</tr>
<tr>
<td>9(^+)</td>
</tr>
<tr>
<td>10(^+)</td>
</tr>
<tr>
<td>(^{106}\text{Zr}) ((N = 13))</td>
</tr>
<tr>
<td>2(^+)</td>
</tr>
<tr>
<td>4(^+)</td>
</tr>
<tr>
<td>6(^+)</td>
</tr>
<tr>
<td>8(^+)</td>
</tr>
<tr>
<td>10(^+)</td>
</tr>
<tr>
<td>8(^+)</td>
</tr>
<tr>
<td>9(^+)</td>
</tr>
<tr>
<td>10(^+)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE III: Experimental and theoretical excitation energies (in keV) of levels in neutron-rich ( A = 106 ) isotobars.</th>
</tr>
</thead>
<tbody>
<tr>
<td>nucleus</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>(^{106}\text{Ru}) ((N = 9))</td>
</tr>
<tr>
<td>2(^+)</td>
</tr>
<tr>
<td>4(^+)</td>
</tr>
<tr>
<td>6(^+)</td>
</tr>
<tr>
<td>8(^+)</td>
</tr>
<tr>
<td>10(^+)</td>
</tr>
<tr>
<td>8(^+)</td>
</tr>
<tr>
<td>(^{106}\text{Mo}) ((N = 11))</td>
</tr>
<tr>
<td>2(^+)</td>
</tr>
<tr>
<td>4(^+)</td>
</tr>
<tr>
<td>6(^+)</td>
</tr>
<tr>
<td>8(^+)</td>
</tr>
<tr>
<td>10(^+)</td>
</tr>
<tr>
<td>8(^+)</td>
</tr>
<tr>
<td>9(^+)</td>
</tr>
<tr>
<td>10(^+)</td>
</tr>
<tr>
<td>(^{106}\text{Zr}) ((N = 13))</td>
</tr>
<tr>
<td>2(^+)</td>
</tr>
<tr>
<td>4(^+)</td>
</tr>
<tr>
<td>6(^+)</td>
</tr>
<tr>
<td>8(^+)</td>
</tr>
<tr>
<td>10(^+)</td>
</tr>
<tr>
<td>8(^+)</td>
</tr>
<tr>
<td>9(^+)</td>
</tr>
<tr>
<td>10(^+)</td>
</tr>
</tbody>
</table>

Including a reliable extrapolation to unknown regions of the nuclear chart. Our method proposes to circumvent this problem by studying the structural evolution in three different chains of nuclei, namely isotopic, isotonic, and isobaric ones, which cross at the nucleus of interest at the outskirts of the region of stable nuclei. Predictions are obtained by extrapolating the different chains to the exotic nucleus in question, and, in addition, a comparison of these extrapolations gives an idea of the errors involved.
TABLE IV: Experimental and theoretical excitation energies (in keV) of levels in $^{106}$Pd.

<table>
<thead>
<tr>
<th>nucleus $^{106}$Pd $(N = 7)$</th>
<th>$J^\pi$</th>
<th>$E_1$</th>
<th>$E_2$</th>
<th>$E_3$</th>
<th>$E_4$</th>
<th>$E_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^+_0$</td>
<td>0</td>
<td>1134</td>
<td>1706</td>
<td>2001</td>
<td>2278</td>
<td></td>
</tr>
<tr>
<td>$2^+_2$</td>
<td>512</td>
<td>1128</td>
<td>1562</td>
<td>1909</td>
<td>2242</td>
<td></td>
</tr>
<tr>
<td>$2^+_2$</td>
<td>519</td>
<td>982</td>
<td>1575</td>
<td>1859</td>
<td>2004</td>
<td></td>
</tr>
<tr>
<td>$3^+_3$</td>
<td>1558</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3^+_3$</td>
<td>1483</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$4^+_4$</td>
<td>1229</td>
<td>1932</td>
<td>2077</td>
<td>2213</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$5^+_5$</td>
<td>2366</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$6^+_6$</td>
<td>2077</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$6^+_6$</td>
<td>1896</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE V: Theoretical $B(E2)$ values (in units e$^2$b$^2$) for transitions in neutron-rich $N = 66$ isotones.

<table>
<thead>
<tr>
<th>transition</th>
<th>$^{112}$Pd</th>
<th>$^{110}$Ru</th>
<th>$^{108}$Mo</th>
<th>$^{106}$Zr</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2^+_1 \rightarrow 0^+_0$</td>
<td>0.27</td>
<td>0.35</td>
<td>0.45</td>
<td>0.53</td>
</tr>
<tr>
<td>$4^+_1 \rightarrow 2^+_2$</td>
<td>0.43</td>
<td>0.54</td>
<td>0.65</td>
<td>0.75</td>
</tr>
<tr>
<td>$0^+_2 \rightarrow 2^+_2$</td>
<td>0.22</td>
<td>0.17</td>
<td>0.057</td>
<td>0.003</td>
</tr>
<tr>
<td>$2^+_2 \rightarrow 0^+_0$</td>
<td>0.015</td>
<td>0.024</td>
<td>0.025</td>
<td>0.020</td>
</tr>
</tbody>
</table>

This method was applied to the neutron-rich members of the $N = 66$ isonotic and $A = 106$ isobaric chains of which level energies and electric quadrupole transition probabilities were fitted with IBM-1. The two chains intersect at $^{106}$Zr which allowed the prediction of this nucleus' excitation energies and electric quadrupole transition properties.

As a final remark we emphasize that the IBM-1 is a valence-nucleon model and that the extrapolations as described here crucially depend on the definition of neighboring closed-shell configurations. Thus, the results on $^{106}$Zr indirectly involve the assumption of the persistence of magic numbers in this region of neutron-rich nuclei.

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