The Discrepancy Between tau and e+e- Spectral Functions Revisited and the Consequences for the Muon Magnetic Anomaly

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The discrepancy between τ and e^+e^- spectral functions revisited and the consequences for the muon magnetic anomaly

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We revisit the procedure for comparing the $\pi\pi$ spectral function measured in τ decays to that obtained in e^+e^- annihilation. We re-examine the isospin-breaking corrections using new experimental and theoretical input, and find improved agreement between the $\tau^- \to \pi^-\pi^0\nu_\tau$ branching fraction measurement and its prediction using the isospin-breaking-corrected $e^+e^- \to \pi^+\pi^-$ spectral function, though not resolving all discrepancies. We recompute the lowest order hadronic contributions to the muon $g-2$ using e^+e^- and τ data with the new corrections, and find a reduced difference between the two evaluations. The new tau-based estimate of the muon magnetic anomaly is found to be 1.9 standard deviations lower than the direct measurement.

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PoS(EPS-HEP 2009)373

The precision measurement and predictions of the muon magnetic anomaly a_μ has been an active research field in particle physics in the last decade or so. The experimental world average [1], $a_\mu^{\rm exp} = 11\,659\,208.9 \pm 5.4_{\rm stat} \pm 3.3_{\rm syst}$[1], dominated by the E821 experiment at BNL [2], has reached a precision of 0.53 ppm. The Standard Model (SM) prediction receives contributions from all three sectors, $a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm weak} + a_\mu^{\rm had}$, with $a_\mu^{\rm QED} = 11\,658\,471.810 \pm 0.016$ and $a_\mu^{\rm weak} = 15.4 \pm 0.1_{\rm had} \pm 0.2_{\rm Higgs}$ known to high precision [1]. The hadronic contribution is usually further divided into three parts, $a_\mu^{\rm had} = a_\mu^{\rm had,LO} + a_\mu^{\rm had,HO} + a_\mu^{\rm LBL}$, involving quark and gluon loops in leading-order (LO), higher-order (HO) and light-by-light (LBL) scattering, respectively. They cannot be predicted from first principles. The dominant $a_\mu^{\rm had,LO}$ is calculated with a combination of experimental cross section data involving e^+e^- annihilation to hadrons at low energy, and perturbative QCD at high energy. About 73% of $a_\mu^{\rm had,LO}$ is provided by the $\pi^+\pi^-(\gamma)$ final state and 82% of its total error stems from the same mode. For this reason, there has been effort [3, 4, 5] to use the accurate $\tau^- \to \pi^-\pi^0\nu_\tau, 2\pi^-\pi^+\pi^0\nu_\tau, \pi^-3\pi^0\nu_\tau$ spectral functions [6, 7, 8] correcting for all known isospin breaking (IB) effects to transform the τ into e^+e^- equivalent data for providing a τ-based prediction.

The previous situation [9, 10] is that the e^+e^--based SM prediction is lower than the direct measurement by about 3.3σ. The τ-based prediction is, however, in agreement with the measurement with the errors. This talk reports a recent work[2] [11] updating the τ- and e^+e^--based predictions using the reevaluated IB corrections and including the new high statistics 2π spectral function τ data from Belle [12] and the published CMD2 [13] and new KLOE [14] e^+e^- data. A newly developed software package HVPTools [15] has been used to perform accurate data interpolation and combination for both τ and e^+e^- data.

Source	$\Delta a_\mu^{\rm had,LO}[\pi\pi,\tau](\times 10^{-10})$		$\Delta B_{\pi^-\pi^0}^{\rm CVC}(\times 10^{-2})$	
	GS model	KS model	GS model	KS model
$S_{\rm EW}$	-12.21 ± 0.15		$+0.57 \pm 0.01$	
$G_{\rm EM}$	-1.92 ± 0.90		-0.07 ± 0.17	
FSR	$+4.67 \pm 0.47$		-0.19 ± 0.02	
ρ–ω interference	$+2.80 \pm 0.19$	$+2.80 \pm 0.15$	-0.01 ± 0.01	-0.02 ± 0.01
$m_{\pi^\pm} - m_{\pi^0}$ effect on σ	-7.88		$+0.19$	
$m_{\pi^\pm} - m_{\pi^0}$ effect on Γ_ρ	$+4.09$	$+4.02$	-0.22	
$m_{\rho^\pm} - m_{\rho^0_{\rm bare}}$	$0.20^{+0.27}_{-0.19}$	$0.11^{+0.19}_{-0.11}$	$+0.08 \pm 0.08$	$+0.09 \pm 0.08$
$\pi\pi\gamma$, electrom. decays	-5.91 ± 0.59	-6.39 ± 0.64	$+0.34 \pm 0.03$	$+0.37 \pm 0.04$
Total	-16.07 ± 1.22	-16.70 ± 1.23	$+0.69 \pm 0.19$	$+0.72 \pm 0.19$
	-16.07 ± 1.85		$+0.69 \pm 0.22$	

Table 1: Contributions to $a_\mu^{\rm had,LO}[\pi\pi,\tau]$ ($\times 10^{-10}$) and $B_{\pi^-\pi^0}^{\rm CVC}$ ($\times 10^{-2}$) from the isospin-breaking corrections. $S_{\rm EW}$ and $G_{\rm EM}$ are the short and long distance radiative corrections, respectively. FSR stands for the final state radiative corrections. Corrections shown in two separate columns correspond to the Gounaris-Sakurai (GS) and Kühn-Santamaria (KS) parametrisations, respectively.

The main new results are summarised in Table 1 and Fig. 1. In Table 1, the middle col-

[1]If not stated otherwise, this and the following numbers for a_μ are given in units of 10^{-10}.

[2]The results shown here correspond to the revised version of the paper.

PoS(EPS-HEP 2009)373

umn shows the new IB corrections to $a_\mu^{\rm had,LO}$ calculated using the Gounaris-Sakurai (GS) [16] and Kühn-Santamaria (KS) [17] parametrisations fitted to bare e^+e^- 2π form factor data. The total IB corrections -16.07 ± 1.85 represents a net change of -6.9 units on $a_\mu^{\rm had,LO}$, dominated by the new electromagnetic decay correction [18], compared to the previous corrections [4].

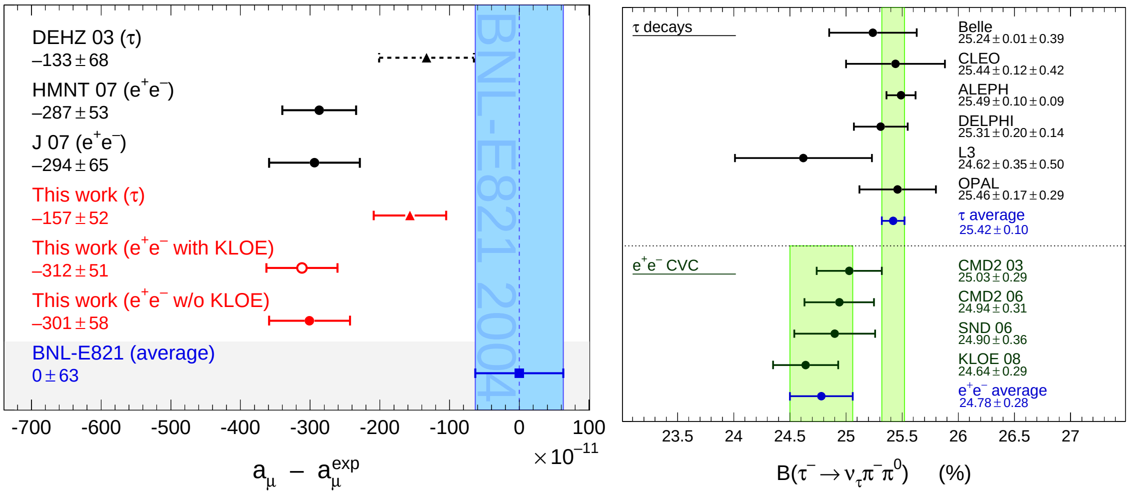

Figure 1: Left: Compilation of recently published results for $a_\mu^{\rm SM}$ (in units of 10^{-11}), subtracted by the central value of the experimental average [1, 2]. The shaded band indicates the experimental error. The SM predictions are taken from DEHZ 03 [5], HMNT 07 [19], J 07 [21], and the present τ- and e^+e^--based predictions using τ and e^+e^- spectral functions. Right: The measured branching fractions for $\tau^- \to \pi^-\pi^0\nu_\tau$ [6, 7, 8, 12, 22, 23] compared to the predictions from the e^+e^- 2π spectral functions, applying the IB corrections. For the e^+e^- results, we have used only the data from the indicated experiments in $0.63 - 0.958\,$GeV and the combined e^+e^- data in the remaining energy domains below m_τ. The long and short error bands correspond to the τ and e^+e^- averages of $(25.42 \pm 0.10)\%$ and $(24.78 \pm 0.28)\%$, respectively.

Applying the new IB corrections and using the new combined tau spectral function including Belle, we obtain $a_\mu^{\rm had,LO}[\pi\pi,\tau] = 515.2 \pm 2.0_{\rm exp} \pm 2.2_B \pm 1.9_{\rm IB}$ where the first error is associated with the experimental uncertainty on the shape of the spectral function, the second error on the normalisation is due to the measurement uncertainty of the averaged branching fraction of $B_{\pi^-\pi^0} = (25.42 \pm 0.10)\%$, and the third error corresponds to the uncertainty of the IB corrections.

The corresponding e^+e^--based result is $a_\mu^{\rm had,LO}[\pi\pi, e^+e^-] = 503.5 \pm 3.8_{\rm exp} (504.6 \pm 4.3_{\rm exp})$ including (excluding) the KLOE data[3]. Therefore, the discrepancy between the τ- and e^+e^--based evaluations in the dominant $\pi^+\pi^-$ channel has reduced from 2.9σ previously to $2.4\sigma (1.9\sigma)$.

Including contributions from other exclusive channels at energy below $1.8\,$GeV as well as the inclusive perturbative QCD calculation at higher energy, one obtains $a_\mu^{\rm had,LO}[\tau] = 705.3 \pm 3.9_{\rm exp} \pm 0.7_{\rm rad} \pm 0.7_{\rm QCD} \pm 2.1_{\rm IB}$ and $a_\mu^{\rm had,LO}[e^+e^-] = 689.8 \pm 4.3_{\rm exp+rad} \pm 0.7_{\rm QCD} (690.9 \pm 5.2_{\rm exp+rad} \pm 0.7_{\rm QCD})$. Including further $a_\mu^{\rm QED}$, $a_\mu^{\rm weak}$, $a_\mu^{\rm had,HO} = -9.79 \pm 0.08_{\rm exp} \pm 0.03_{\rm rad}$ [19] and $a_\mu^{\rm LBL} = 10.5 \pm 2.6$ [20], one gets the total SM predictions $a_\mu^{\rm SM}[\tau] = 11\,659\,193.2 \pm 4.5_{\rm LO} \pm 2.6_{\rm HO+LBL} \pm 0.2_{\rm QED+weak}$ and $a_\mu^{\rm SM}[e^+e^-] = 11\,659\,177.7 \pm 4.4_{\rm LO} \pm 2.6_{\rm HO+LBL} \pm 0.2_{\rm QED+weak} (11\,659\,178.8 \pm 5.2_{\rm LO} \pm 2.6_{\rm HO+LBL} \pm 0.2_{\rm QED+weak})$, which are compared with the direct measurement [1, 2] and other SM predictions [5,

[3] The inclusion of the KLOE data only makes 1.1 units of difference on $a_\mu^{\rm had,LO}[\pi\pi]$, the discrepancy is in fact more pronounced in the comparison of the spectral functions [11].

19, 21] in Fig. 1(left). The new τ-based result for the SM prediction is now 1.9 standard deviations lower than the direct measurement, moving closer to the e^+e^- value.

An alternative comparison (Fig. 1(right)) is performed between the direct measurements of the τ branching fraction $B_{\pi^-\pi^0}$ and the corresponding $B^{\rm CVC}_{\pi^-\pi^0}$ derived from the e^+e^- 2π data correcting for the IB effects (Table 1, right column). The advantage of a such comparison is that the branching fractions are integrated mass spectrum, hence are essentially insensitive to those experimental systematic uncertainties connected with the shape of the τ spectral function. The discrepancy between $B_{\pi^-\pi^0}$ and $B^{\rm CVC}_{\pi^-\pi^0}$ has reduced from 4.5σ previously to $2.2\sigma(1.6\sigma)$ when the KLOE data is included (excluded).

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PoS(EPS-HEP 2009)373