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**Abstract.** Some particular facets of the numerous works by Marcos Moshinsky and Yurii Fedorovich Smirnov are presented in these notes. The accent is put on some of the common interests of Yurii and Marcos in physics, theoretical chemistry, and mathematical physics. These notes also contain some more personal memories of Yurii Smirnov.

## 1. Introduction

Yurii Fedorovich Smirnov passed away in 2008 and Marcos Moshinsky in 2009. They were two famous physicists with common interests in nuclear physics, atomic and molecular physics, and mathematical physics. More generally, both of them were at the origin of significant achievements in symmetry methods in physics. They actively participated in several Symmetries in Science Symposia in Bregenz. These two giants had parallel centers of interest in the sense that they developed separately some complementary works in nuclear physics (that led in particular to the concept of Moshinsky-Smirnov coefficients), dealt with some related problems in atomic, molecular and mathematical physics, and, finally, combined efforts to produce a beautiful and very useful book [1] on the applications of the harmonic oscillator system to various areas of physics and chemistry.

It is not the purpose of these notes to extensively list and analyse the numerous papers by Marcos and Yurii. I shall focus on some particular facets of their works. I had the opportunity to meet Marcos and Yurii several times in Bregenz and on several other occasions, and to discuss with them about Wigner-Racah algebras for finite groups, Lie groups and quantum groups. I had also a chance to collaborate with Yurii Smirnov. Therefore, I shall devote the main part of these notes to some specific domains of importance to Yurii and Marcos and to some more personal reminiscences on Yurii.

## 2. Marcos Moshinsky

Marcos Moshinsky was a Mexican physicist. He was born in Kiev (Ukraine) in 1921. He arrived as a refugee in Mexico when he was three years old and obtained Mexican citizenship in 1942. He received a Bachelor's degree in physics from the *Universidad Nacional Autónoma de México* (U.N.A.M.) and a Ph.D. degree in theoretical physics, under the guidance of Eugene P. Wigner, from Princeton University. Marcos was also the recipient a post-doctoral fellowship at the *Institut Henri Poincaré* in Paris. Afterwards, he returned to Mexico and pursued a brillant career at the U.N.A.M. in Mexico City.

Professor Marcos Moshinsky had important responsabilities as the President of the *Sociedad Mexicana de Física*, as a member of *El Colegio Nacional*, and as a member of the editorial board of several international scientific reviews. He produced and/or co-produced more than 200 scientific papers and four books among which the most well-known are the one written in collaboration with Thomas A. Brody on transformation brackets for nuclear shell-model calculations [2] and the one with Yurii F. Smirnov on the applications of the harmonic oscillator in various fields of physics and quantum chemistry [1]. He received several prizes, namely the *Premio Nacional de Ciencias y Artes* in 1968, *Premio Luis Elizondo* in 1971, *Premio U.N.A.M. de Ciencias Exactas* in 1985, *Premio Príncipe de Asturias de Investigación Científica y Técnica* in 1988, and the prestigious UNESCO Science Prize in 1997 for his work in nuclear physics. He also received the Wigner medal in 1998.

## 3. Marcos and Yurii

A first seminal paper by Marcos concerned the transient dynamics of particle wavefunctions, a phenomenon that gives rise to diffraction in time [3]. However, most of his scientific work dealt with collective models of the nucleus, canonical transformations in quantum mechanics, and group theoretical methods in physics, with a special emphasis on symplectic symmetry in nuclear, atomic and molecular physics. These themes were of interest to Yurii too. Following the pioneering work of Talmi (who prepared his Ms.S. thesis with Guilio Racah, his Doctorate thesis with Wolfgang Pauli and who was a post-doctoral fellow with Eugene P. Wigner) [4], both Marcos and Yurii were interested in the description of pairs of nucleons in a harmonic-oscillator potential. In 1959, Moshinsky developed a formalism to connect wavefunctions in two different coordinate systems for two particles (with identical masses) in a harmonic-oscillator potential [5]. In this formalism, any two-particle wavefunction $|n_1\ell_1, n_2\ell_2, \lambda\mu\rangle$, expressed in coordinates with respect to the origin of the harmonic-oscillator potential, is a linear combination of wavefunctions $|n\ell, NL, \lambda\mu\rangle$, expressed in relative and centre-of-mass coordinates of the two particles. The so-called transformation brackets $\langle n\ell, NL, \lambda|n_1\ell_1, n_2\ell_2, \lambda\rangle$ make it possible to pass from one coordinate system to the other. Moshinsky gave an explicit expression of these coefficients in the case $n_1 = n_2 = 0$ and derived recurrence relations that can be used to obtain the coefficients for $n_1 \neq 0$ and $n_2 \neq 0$ from those for $n_1 = n_2 = 0$ [5]. Along this vein, Brody and Moshinsky published extensive tables of transformation brackets [2]. At the end of the fifties, Smirnov worked out a similar problem, viz. the calculation of the Talmi coefficients for unequal mass nucleons, and gave solution for the case $n_1 \neq 0$ and $n_2 \neq 0$ [6, 7]. (Indeed, the transformation brackets and the Talmi coefficients are connected via a double Clebsch-Gordan transformation.) The coefficients $\langle n\ell, NL, \lambda|n_1\ell_1, n_2\ell_2, \lambda\rangle$, called *transformation brackets* by Moshinsky and *total Talmi coefficients* by Smirnov, are now referred to as Moshinsky-Smirnov coefficients. Both the Moshinsky-Smirnov coefficients and the Talmi coefficients were revisited at the end of the seventies in terms of generating functions in the framework of the approaches of Julian S. Schwinger and Valentine Bargmann to the harmonic-oscillator bases [8]. (The work by Mehdi Hage Hassan [8], who prepared his State Doctorate thesis at the *Institut de Physique Nucléaire de Lyon* and conducted his career in Beyrouth, constitutes a very deep and original approach to the Talmi coefficients and Moshinsky-Smirnov coefficients.) It should be noted that the transformation brackets or Moshinsky-Smirnov coefficients are also of importance for atoms and molecules as shown by Marcos and Yurii in their book [1] written during the time Yurii was a visiting professor at the *Instituto de Física* of the *Universidad Nacional Autónoma de México*.

A second area of common interest to both Marcos and Yurii concerns the many-body problem considered from the point of view of unitary and symplectic groups and the use of nonlinear and nonbijective canonical transformations. In this vein, Moshinsky and some of his collaborators introduced the concept of an ambiguity group, a group required for taking into account the nonbijectivity of certain canonical transformations [9]. Indeed, this concept is closely related to the one of Lie algebra under constraint [10], which in turn is connected to nonbijective transformations like Hopf fibrations on spheres and Hopf fibrations on hyperbolids [11]. Among the nonbijective transformations, one may mention the $\mathbf{R}^4 \to \mathbf{R}^3$ Kustaanheimo-Stiefel transformation and the $\mathbf{R}^2 \to \mathbf{R}^2$ Levi-Civita transformation as well as their various extensions [11, 12]. In particular, the Kustaanheimo-Stiefel transformation allows one to pass from a four-dimensional harmonic oscillator subjected to a constraint to the three-dimensional hydrogen atom (see for instance [13]). This subject was of interest to Yurii and he revisited the hydrogen-oscillator connection with Corrado Campigotto [14].

The harmonic oscillator is a central ingredient in numerous studies by Smirnov and Moshinsky. Many applications of the nonrelativistic and relativistic harmonic oscillators to modern physics (from molecules, atoms and nuclei to quarks) were pedagogically exposed in the book by Marcos and Yurii [1] with a special attention paid to the $n$-body problem (in the Hartree-Fock approximation), the nuclear collective motion and the interacting boson model.

But their common interests were not limited to transformation brackets and harmonic oscillators. Let us briefly mention that both of them were also interested in group theoretical methods and symmetry methods in physics and also contributed to several fields of mathematical physics including, for instance, the state labelling problem, special functions, and generating functions (see, for example, [15, 16]).

## 4. Yurii Fedorovich Smirnov

Yurii Fedorovich Smirnov was a Russian physicist. He was born in the city of Il'inskoe (Yaroslavl' region, Russia) in 1935. He graduated from Moscow State University. Subsequently, he completed his Doctorate thesis at the same university under the guidance of Yurii M. Shirokov and benefited from fruitful contacts with other distinguished physicists including Yakov A. Smorodinsky. He pursued his career in the (Skobeltsyn) Institute of Nuclear Physics and in the Physics Department of (Lomonosov) Moscow State University with many stays abroad. The last fifteen years of his life were shared between Moscow and Mexico City where he was a visiting professor and later a professor at the *Instituto de Física* and at the *Instituto de Ciencias Nucleares* of the U.N.A.M. (he spent almost 11 years in Mexico). He received prestigious awards: the K.D. Sinel'nikov Prize of the Ukrainian Academy of Sciences in 1982 and the M.V. Lomonosov Prize in 2002. He was also a member of the Academy of Sciences of Mexico.

Yurii Smirnov authored and/or co-authored eleven books and more than 250 scientific articles. He also translated several scientific books into Russian. He translated, for example, a book on the harmonic oscillator written by Marcos Moshinsky in 1969, precisely the book that was a starting point for their common book on the same subject, published in 1996 [1]. He was a member of the editorial board of several journals and a councillor of the scientific councils of the Skobeltsyn Institute of Nuclear Physics and of the Chemistry Department of Moscow State University, as well as of the Institute for Theoretical and Experimental Physics (ITEP) in Moscow.

## 5. Some personal reminiscences on Yurii

My first contact with the work of Yurii Smirnov dates back to 1978 when my colleague J. Patera showed me, on the occasion of a NATO Advanced Study Institute organised in Canada by J.C. Donini, a beautiful book written by D.T. Sviridov and Yu.F. Smirnov [17]. This book dealt with the spectroscopy of $d^N$ ions in inhomogeneous electric fields (part of a disciplinary domain known as crystal- and ligand-field theory in condensed matter physics and explored via the theory of level splitting from a theoretical point of view). In 1979, B.I. Zhilinskiĭ, while visiting Dijon and

Lyon in France in the context of an exchange programme between the USSR and France, gave me another interesting book, on $f^N$ ions in crystalline fields, written by D.T. Sviridov, Yu.F. Smirnov and V.N. Tolstoy [18]. At that time, the references for mathematical aspects of crystal- and ligand-field theory were based on works by Y. Tanabe, S. Sugano and H. Kamimura from Japan [19], J.S. Griffith from England [20], and Tang Au-chin and his collaborators from China [21] (see also some contributions by the present author [22]). The two above-mentioned books by Smirnov and his colleagues shed some new light on the mathematical analysis of spectroscopic and magnetic properties of partly filled shell ions in molecular and crystal surroundings. In particular, special emphasis was put on the derivation of the Wigner-Racah algebra of a finite group of molecular and crystallographic interest from that of the group $SO(3) \sim SU(2)/Z_2$.

My second (indirect) contact with Yurii is related to an invitation to participate in the fifth workshop on *Symmetry Methods in Physics* in Obninsk in July 1991. Unfortunately, I did not get my visa on time reducing my participation to a paper in the proceedings of the workshop edited by Yu.F. Smirnov and R.M. Asherova [23].

In the beginning of the 1990's, I had a chance to discover another facet of Yurii's work. In 1989, a Russian speaking student from Switzerland, C. Campigotto, spent one year in the group of Prof. Smirnov. He started working on the Kustaanheimo-Stiefel transformation, an $\mathbf{R}^4 \to \mathbf{R}^3$ transformation associated with the Hopf fibration $S^3 \to S^2$ with compact fiber $S^1$. (Such a transformation makes it possible to connect the Kepler-Coulomb system in $\mathbf{R}^3$ to the isotropic harmonic oscillator in $\mathbf{R}^4$.) Then, Campigotto (well-prepared by Smirnov and his team, especially Andrey M. Shirokov and Valeriy N. Tolstoy) came to Lyon to prepare a Doctorate thesis (partly published in [24]). He defended his thesis in 1993 with George S. Pogosyan (representing Yu.F. Smirnov) as a member of the jury.

A fourth opportunity to work with Yurii stemmed from our mutual interest in quantum groups and in nuclear and atomic spectroscopy. I meet him for the first time in Dubna in 1992. We then started a collaboration (partly with R.M. Asherova) on $q$- and $qp$-boson calculus in the framework of Hopf algebras associated with the Lie algebras $su(2)$ and $su(1,1)$ [25]. In addition, we pursued a group-theoretical study of the Coulomb energy averaged over the $n\ell^N-$ atomic states with a definite spin [26]. We also had fruitful exchanges in nuclear physics. Indeed, Prof. Smirnov and his colleagues D. Bonatsos (from Greece), S.B. Drenska, P.P. Raychev and R.P. Roussev (all from Bulgaria) developed a model based on a one-parameter deformation of $SU(2)$ for dealing with rotational bands of deformed nuclei and rotational spectra of molecules [27] (see also [28]). Along the same line, a student of mine, R. Barbier, developed in his thesis a two-parameter deformation of $SU(2)$ with application to superdeformed nuclei in mass region $A \sim 130 - 150$ and $A \sim 190$ (partly published in [29]). It was a real pleasure to receive Yurii in Lyon on the occasion of the defence of the Barbier thesis in 1995. Indeed, from 1992 to 1995, Yurii made four stays in Lyon (one with his wife Rita and one with his daughter Tatyana) and we jointly participated in several meetings, one in Clausthal in Germany (organised by H.-D. Doebner, V.K. Dobrev and A.G. Ushveridze) and two in Bregenz in Austria (organised by B. Gruber and M. Ramek).

I cannot do justice to all of the fields in which Yurii was recognized as a superb researcher. It is enough to say that he contributed to many domains of mathematical physics (e.g., finite groups embedded in compact or locally compact groups, Lie groups and Lie algebras, quantum groups, special functions) and theoretical physics (e.g., nuclear, atomic and molecular physics, crystal- and ligand-field theory). Let me mention, among other fields, that he achieved alone and with collaborators significant advances in the theory of clustering of shell-model (nuclear) systems [30], in projection operator techniques for simple Lie groups [31], in the theory of heavy ion collisions [32], and in the so-called $J$-matrix formalism for quantum scattering theory (see [33] and references therein). The $J$-matrix formalism requires the solution of three-term recurrence relations (or second-order difference equations); along this line, Yurii and some of his collaborators published several works (see for instance [34]). As another major contribution, at the end of the sixties he proposed in collaboration with Vladimir G. Neudatchin a method, the so-called (e,2e) method (an analog of the (p,2p) method used in nuclear physics), for the experimental investigation of the electronic structure of atoms, molecules and solids; this method was successfully tested in many laboratories around the world (see [35] and references therein).

Yurii was also an exceptional teacher. It was very pleasant, profitable and inspiring to be taught by him. I personally greatly benefited from discussions with Yurii Smirnov.

### Closing

Yurii and Marcos had many students who are now famous physicists. They interacted with many collaborators in their countries and abroad, and had an influence on many scientists. Marcos Moshinsky and Yurii Fedorovich Smirnov will remain examples for many of us. We shall not forget them.

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