Exotic hadrons and Steiner-tree confinement

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After a brief review on exotic hadrons, some recent results are presented about multiquarks bound by a minimal path that generalizes the linear potential in quarkonium.

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I. INTRODUCTION

The pentaquark episode is probably over, though some indications persist about possible exotic baryons [1, 2]. It started with a speculation guided by chiral dynamics [3], and a search by Nakano [4], against the fashion of that time. The announcement in 2003 of positive results created a wave of investigations. Experimental groups realized they had data on tapes, that nobody had the curiosity to analyze. Theorists also discovered that instead of refining the estimate of the properties of ordinary hadrons, they could use their tools for looking at exotic configurations. Nowadays, the multiquark dynamics is studied by lattice QCD, sum rules, and even AdS/QCD.

For years, the possibility of multiquarks was mainly discussed in the framework of constituent models already fitting ordinary mesons and baryons. There is a non-trivial technical difficulty when the number of quarks increases from $N = 2$ or $N = 3$ to $N > 3$. But the main problem consists of extrapolating the interaction towards a domain where new color coupling can be envisaged. In the past, the most current recipe consisted of a pairwise interaction with color factors. This is justified for the short-range part, but not for the confining part, except when $(N-1)$ of the constituents are closely clustered far from the N^{th} one. More realistic models have been proposed for many years, and they are now supported by lattice QCD. The linear interaction in quarkonium, understood as a flux tube of minimal length, is generalized as a Steiner-tree linking the constituents through the minimal path. I shall present a review of the recent results obtained using this new confining interaction.

II. BINDING MECHANISMS FOR MULTIQUARKS

a. Duality The first serious argument in favor of multiquark hadrons was an indirect one, in the context of duality. For a review, see, e.g., [5]. To get consistency in the description

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of hadronic reactions, a *duality* principle was imposed relating s-channel and t-channel exchanges. In baryon–antibaryon scattering, the partners of ordinary mesons are mesons made of two quarks and two antiquarks, preferentially coupled to baryon–antibaryon pairs. In the late 70s and the 80s, such new mesons were tentatively seen in proton–antiproton experiments, but none of baryonium candidates were confirmed in further experiments using better antiproton beams.

b. Light scalar mesons For light mesons, especially scalars, creating a quark–antiquark pairs does not cost more than providing the existing pair with an orbital excitation [6, 7]. This was the beginning of the saga of scalar mesons, which also includes flavor excitation, hybrids, glueballs and various mixing schemes among them. This subject has been discussed in many contributions to this conference.

c. Chromomagnetism In QED, the Breit–Fermi term $V_{SS} \propto \sum_{i<j} \delta(\boldsymbol{r}_{ij})\, \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j$ accounts for the observed hyperfine splittings (for the positronium and other positronic atoms, the annihilation diagram also contributes). The analog in QCD reads [8] $V_{SS} \propto \sum_{i<j} \delta(\boldsymbol{r}_{ij})\tilde{\lambda}_i.\tilde{\lambda}_j\, \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j$, considered either phenomenological, or given by one-gluon-exchange. There is also a $1/(m_i m_j)$ dependence that reduces the effect for heavy quarks.

At first sight, the color factor $\tilde{\lambda}_i.\tilde{\lambda}_j$ simply induces a factor 1/2 for baryons as compared to mesons, and helps to fit the data. However, when combined with the spin factor, it gives a remarkable coherence in some configurations, as first noted by Jaffe [9]. In the SU(3)$_\mathrm{f}$ limit, the expectation value $\langle \lambda_i.\lambda_j\, \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j \rangle$ is twice larger for $(uuddss)$ with spin $J = 0$ than for the $\Lambda\Lambda$ dissociation threshold. For comparison, remember that the positronium molecule experiences a Coulomb potential $\sum_{i<j} g_{ij}/r_{ij}$ with the same cumulated strength $\sum g_{ij} = -2$ as for its threshold made of two positronium atoms. In pure spin–spin model without color, $V_{SS} \propto \sum \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j$, a $J = 0$ tetraquark would receive a cumulated strength $\sum_{i<j} \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j = -6$, whereas *each* scalar meson of its threshold experiences $\sum_{i<j} \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j = -3$, and thus there would be no obvious excess of attraction.

The di-lambda was first estimated to be deeply bound, since the orbital matrix element $\langle \delta(\boldsymbol{r}_{ij}) \rangle$ was assumed to be the same as for ordinary baryons. This assumption, and that of SU(3)$_\mathrm{f}$ symmetry, turn out to give too optimistic estimates. The various corrections work against the stability of the di-lambda [10–13].

Other configurations receive a coherent attraction of chromomagnetic forces [14], in particular, the 1987-vintage version of the pentaquark [15–17]. But, again, it turns out very difficult to build a realistic wave function that gives enough short-range correlation for all pairs.

d. Hadronic molecules It has been often stressed that the conventional strong interactions, which build the deuteron out of two nucleons, can produce other bound states or resonances. There are many contributions to this conference. In particular, a $D\overline{D}^*$ composite was *predicted*, mainly due to one-pion-exchange. See the review [18] for references to the original papers. So, when the $X(3872)$ was discovered, it was greeted as a success for this approach. Unfortunately, some of the latest measurements suggest a radial excitation of charmonium, and there is no obvious way to combine the two pictures consistently (see the section on mixing).

In the past, we learned to be careful with the molecular interpretation of the hidden-charm states. When higher ψ resonances were found, theorists were puzzled by the anomalies in the relative decay rates into $D\overline{D}$, $D^*\overline{D}$ + c.c. and $D^*\overline{D}^*$, and a molecular interpretation was suggested [19] (see, also, [20]). But the branching ratios were later understood from the node structure of the decaying states [21–23].

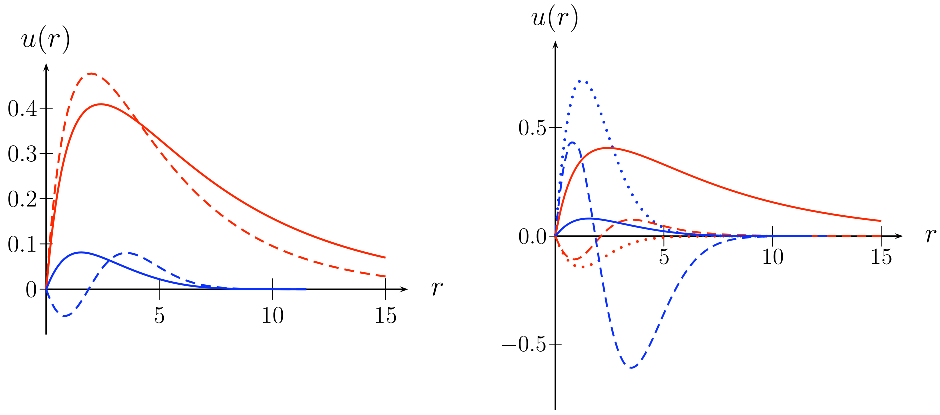

FIG. 1: Left: Mixing of $(D\bar{D})$ and $(c\bar{c})$ in a simple local model. The solid lines represent the reduced radial wave function for the main $(D\bar{D})$ and the small $(c\bar{c})$ components, as obtained from an actual coupled-channel calculation. The dashed lines correspond to the naive mixing scheme of Eq. (1), with neighboring unperturbed states, and θ adjusted to reproduce the same normalization for $(c\bar{c})$. Note that the coupled-channel calculation does not produce any node in the $(c\bar{c})$ sector. Units are $\text{GeV}^{1/2}$ for $u(r)$ and GeV^{-1} for r. Right: Reduced radial wave functions for the $(c\bar{c})$ (blue) and $(D\bar{D})$ (red) components in the same coupled-channel model, for the ground-state (dotted lines), first (dashed lines) and second (solid lines) excitations.

e. Mixing dynamics This is a current trend in the physics of hadrons: if a first model describes satisfactorily some properties of a hadron, but fails for others that are accounted for by a second model, a tantalizing improvement of the picture consists of writing the wave function as

$$\psi = \cos\vartheta\,\psi_1 + \sin\vartheta\,\psi_2\ , \tag{1}$$

and this can be extended as to include more components. This game has been played endlessly for scalar states involving radial excitations, hidden strangeness, glueballs and hybrids. In the case of the $X(3872)$, ψ_1 could be a $D\overline{D}^*$+c.c. molecule, and ψ_2 a charmonium $c\bar{c}(2P)$ with the same quantum numbers and a radial excitation. However, the dynamics of coupled channels dictates that if $\cos\vartheta\,\psi_1$ is the leading term, the small admixture $\sin\vartheta\,\psi_2$ is *not* very much governed by the diagonal interaction in the second channel. Instead it is given by a folding of the leading component and the transition operator. In particular, if the former is nodeless and the latter smooth, a node hardly appears in ψ_2.

A toy model illustrating this property is an S-wave analogue of the $X(3872)$ system, with a $(D\overline{D})$ (the difference between D and D^* is ignored) channel weakly bound by a Yukawa potential, and a $(c\bar{c})$ channel with a standard Coulomb-plus-linear potential, such that before mixing, the threshold is at 3.8 GeV, $(D\overline{D})$ at 3.77 GeV and $\psi(2S)$ at 3.57 GeV. The mixing is crudely mimicked by a local interaction with a Gaussian shape. The mass of the $(D\overline{D})$ is slightly shifted (more details will be given elsewhere), and the wave function acquires a $(c\bar{c})$ component, displayed in Fig. 1, which is *nodeless*.

The current belief is that the eigenstates mix unchanged, dominantly by affinity of neighboring unperturbed energies, once a gate is open between the two channels. This is inspired by the denominator in the first order correction to a wave function within perturbation theory, namely, in a obvious notation,

$$\phi_i = \phi_i^{(0)} + \sum_{j\neq i} \frac{\langle \phi_i^{(0)} | V | \phi_i^{(0)} \rangle}{E_i^{(0)} - E_j^{(0)}} \phi_i^{(0)} + \cdots \tag{2}$$

However, the node structure of the $\phi_i^{(0)}$ sometimes gives more drastic constraints. Another example is S-D mixing in ordinary charmonium dynamics. It is almost ever considered that $\psi(1D)$ preferentially mixes with $\psi(2S)$ which is very close. But when one actually computes the small S-wave admixture in $\psi(1D)$ using an explicit tensor force with suitable regularization, one finds a nodeless radial function for the admixed S-component in the $\psi(3770)$. Note that S-D mixing can be calculated analytically for the muonium $(\mu^+ e^-)$ and the approximations can be tested there. In the above model, the ground-state has two nodeless components, with the $(c\bar{c})$ one dominating. The second state has two nodes, one in the main $(c\bar{c})$ channel, and another in the small $(D\bar{D})$ admixture, as seen in Fig. 1. The third one has no nodes. Many variants are possible, if the parameters are modified.

Back to $X(3872)$, the $\psi(2S)$ to $\psi(1S)$ ratio of radiative decays has to be studied within a *consistent* picture involving probably several Fock-space components, $c\bar{c}$, $c\bar{c}q\bar{q}$, etc. This requires a modeling of the transition operator creating of annihilating a pair of light quarks, and a few calculations.

A consistently-managed mixing scheme can give interesting results. An example is the celebrated Cornell picture of $(c\bar{c})$, supplemented by coupling to real of virtual $D^{(*)}\overline{D}^{(*)}$ (including D_s) decay channels [23]. Before the discovery of the $\eta_c(2s)$, this model predicted a substantial reduction of the $\psi(2s) - \eta_c(2s)$ splitting [24], as compared to simple potential models. The same effect could explain why the $\Upsilon - \eta_b$ splitting is slightly larger than estimated by most authors, if the reduction due the meson–meson channels is than for $J/\psi - \eta_c$. Other interesting treatments of the configuration mixing can be found, e.g., in [25–27].

f. Diquark clustering The concept of diquark is very useful in several branches of particle physics, for instance to analyze the baryon-to-meson ratio in multiparticle production. It has been introduced rather early in spectroscopy, see, e.g., Ref. [28] for a review. However, some of the pioneering works are sometimes ignored in recent rediscoveries of the diquarks. A first problem to which diquarks provide a sufficient solution, is why Regge trajectories (squared mass M^2 vs. spin J) are linear with the *same* slope for baryons as for mesons. For two-body systems, the linear character is reproduced in many pictures, e.g., $H_2 = \sum(m^2 + \boldsymbol{p}_i^2)^{1/2} + \lambda r_{12}$. Thus the equality of slopes comes automatically if the quark–diquark baryons, $[q - (qq)]$, are the partners of the quark–antiquark mesons, $[q - \bar{q}]$.

However, it is *not necessary* to introduce diquarks by hand to obtain the equality of slopes. In the symmetric quark model, the baryon analogue of H_2 reads $H_3 = \sum(m^2 + \boldsymbol{p}_i^2)^{1/2} + V_3$, where V_3 is $\lambda \sum r_{ij}/2$ or the Y-shape potential (discussed below). The Hamiltonian H_3, for the large angular momentum, gives linear Regge trajectories for baryons, with the same slope as mesons. There is a dynamical clustering, or, say, a spontaneous breaking of symmetry when J increases [29], as illustrated in Fig. 2.

The best argument in favor of the quark–diquark model comes perhaps from the problem of missing resonances. In a recent review on baryons [30], very few states are tentatively interpreted with both degrees of freedom (i.e., Jacobi variables $\boldsymbol{x}$ and $\boldsymbol{y}$) excited. On the other hand, many states predicted in the three–quark model are not observed, and this was the subject of contributions to this Conference. The most striking state with double

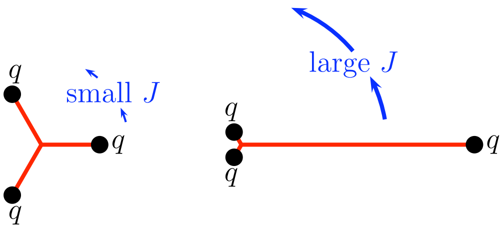

FIG. 2: Schematic picture of a rotating baryon at low angular momentum J and higher J.

excitation in the three-quark model is

$$\Psi = \boldsymbol{x} \times \boldsymbol{y} \, \exp[-a\,(\boldsymbol{x}^2 + \boldsymbol{y}^2)/2] \; , \tag{3}$$

(here in the case of harmonic interaction, but a state with the same symmetry exists for other confining potentials). For experts, it is named the $[20, 1^+]$ multiplet. Its fully antisymmetric orbital function, with $N = 2$ degrees of excitation, is associated with an antisymmetric spin–isospin wave function, and an antisymmetric color wave function. It is absent in the quark–diquark picture. Experimentally, it is not (yet?) seen.

If the diquark model is taken seriously, it can be extrapolated outside the framework of baryon spectroscopy. We have seen that the late baryonium states have been described as a diquark and an antidiquark. More recently, cq or cs diquarks and the associated antidiquarks were introduced to describe the X, Y or Z states seen in the hidden-charm spectrum. See, e.g., [31]. The light pentaquark was also described as $[(qq) - (qq) - \bar{q}]$ [32].

Obviously, the Pandora box syndrome becomes threatening here too. In particular, three diquarks can well build a dibaryon. It should be checked whether in models describing the Y mesons as $(cs) - (\bar{c}\bar{s})$, the configuration $(cs)^3$ is not below the threshold for $(sss) + (ccc)$, since it would be embarrassing to pay the price of a very exotic dibaryon to explain the Y meson. Years ago, a light dibaryon (or "demon deuteron") was shown to be a consequence of the diquark model [33]. [1]

g. Flavor symmetry breaking and chromoelectric binding We adopt here the language of potential models but we believe that the results are much more general. The main advantage of potential models is the possibility of switching on or off some contributions to single out the most effective one for binding.

A remarkable property of the spin-independent interaction among quarks is *flavor independence*, which induces interesting *symmetry breaking* effects.

Remember that symmetry breaking tends to lower the ground-state energy. The simplest example is $h = p^2 + x^2 + \lambda\, x$ in one-dimensional quantum mechanics, with a ground state energy $e(\lambda) = 1 - \lambda^2/2$ which is always below $e(0)$. But this is very general. If

$$H(\lambda) = H_{\text{even}} + \lambda\, H_{\text{odd}} \; , \tag{4}$$

the variational principle applied to $H(0)$ with the even ground state $\Psi(0)$ of H_{even} as trial wave function gives

$$E(\lambda) \leq \langle \Psi(0) | H(\lambda) | \Psi(0) \rangle = E(0) \; . \tag{5}$$

[1] There is an error in [33] about the quantum numbers of the orbital wave function, but this does not remove the issue of dibaryons in diquark models.

One can apply this result to few-body systems with a variety of symmetries for which the labels "odd" and "even" make sense, in particular *particle identity* and *charge conjugation*. But stability is a competition between a configuration with collective binding and another configuration where the system is split into separate decay products. The threshold also benefits from the symmetry breaking, and very often more! In this case, stability deteriorates, though the energy of the compound configuration decreases.

For instance, consider the barely bound (e^+, e^+, e^-, e^-) molecule, or any rescaled version with the electron mass replaced by another mass μ, and move to configurations involving two different masses. Then it is observed, and proved, that:

- for (M^+, m^+, M^-, m^-): binding deteriorates and is lost for $M/m \gtrsim 2.2$ [34],
- for (M^+, M^+, m^-, m^-): the binding is improved.

But the Coulomb character matters little. What is important, is that the potential does not change when the masses are varied. Hence, a similar behavior is observed for any four-body problem with *flavor independence*. The splitting (the potential V is assumed to be symmetric under charge conjugation, and independent of the masses)

$$H(M, M, \bar{m}, \bar{m}) = H(\mu, \mu, \bar{\mu}, \bar{\mu}) + \left[\frac{1}{4M} - \frac{1}{4m}\right] (\boldsymbol{p}_1^2 - \boldsymbol{p}_2^2 + \boldsymbol{p}_3^2 - \boldsymbol{p}_4^2) \;, \tag{6}$$

implies for the ground state

$$E(M, M, \bar{m}, \bar{m}) \leq E(\mu, \mu, \bar{\mu}, \bar{\mu}) \;, \quad 2\,\mu^{-1} = M^{-1} + m^{-1} \;, \tag{7}$$

but, meanwhile, the threshold energy remains constant, as $(M, \bar{m})$ and $(\mu, \bar{\mu})$ have the same reduced mass. So the stability is improved.

Explicit quark-model calculations have been carried out to illustrate how this favorable symmetry breaking works with flavor-independent potentials. The corresponding four-body problem is rather delicate, as most other four-body problems. Remember that after Wheeler's proposal in 1945 (the paper was published somewhat later [35]) that the positronium molecule might be stable, a first numerical investigation by Ore [36] concluded that the system is likely unstable, but the following year, Hylleraas and the same Ore published a beautiful analytic proof of the stability [37].

In current quark models, the main conclusion is that binding a doubly-flavored tetraquark requires a large mass ratio, usually $(bb\bar{q}\bar{q})$ or $(bc\bar{q}\bar{q})$. However, a more sophisticated calculation by Janc and Rosina [38] found $(cc\bar{q}\bar{q})$ barely bound. See, e.g., [39] for a detailed survey of the situation.

III. STEINER-TREE MODEL OF CONFINEMENT

It should be acknowledged, however, that these early constituent-model calculations suffer from a basic ambiguity: how to extrapolate from mesons towards multiquarks. The usual recipe is

$$V = -\frac{3}{16} \sum_{i<j} \tilde{\lambda}_i^{(c)}.\tilde{\lambda}_j^{(c)}\, v(r_{ij}) \;, \tag{8}$$

which is presumably justified for the short-range part, but not for the long-range part, except for very peculiar spatial configurations. In (8), the normalization is such that $v(r)$ is the

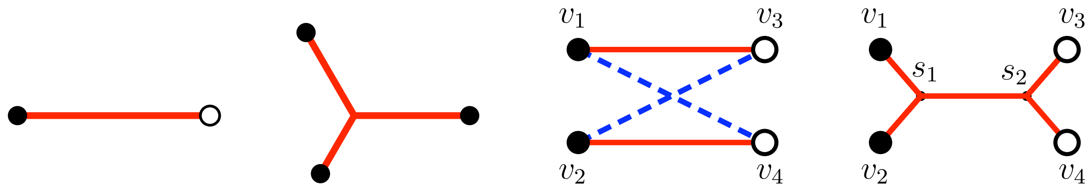

FIG. 3: Confinement of mesons and baryons, and tetraquarks. The minimum over the quark permutations gives the flip–flop potential. For the tetraquarks, the minimum is taken of the flip–flop (left) and Steiner tree (right) configurations

quark–antiquark interaction in ordinary mesons. Strictly speaking, (8) holds for a *pairwise* interaction with color-octet exchange. Clearly color-singlet exchange cannot contribute to confinement, otherwise everything would be confined together, but color-singlet exchange can contribute to short-range terms. Moreover, there are very likely three-body and multi-body forces in baryons and multiquarks.

In the case of baryons, it was suggested very early [40, 41] that the potential generalizing the linear confinement of mesons is

$$V_Y = \sigma \min_J \sum_{i=1}^{3} r_{iJ} \ . \tag{9}$$

This was often rediscovered in the context of models (adiabatic bag, flux tubes), or in studies dealing with the strong-coupling regime of QCD [42]. Estimating the baryon energies and properties with the potential (9) is a very interesting 3-body problem. However, the results differ little from these obtained using the color-additive rule, which for baryons reduces to the "1/2" rule, namely

$$V_3 = \frac{\sigma}{2}\,(r_{12} + r_{23} + r_{31}) \ . \tag{10}$$

This Y-shape interaction has been generalized to tetraquarks. At first, this looked as an astute guess, but it was later endorsed by detailed lattice QCD [43], including the interplay between flip–flop and connected Steiner tree. See, also, [44] for a study within AdS/QCD. The confining potential reads

$$U = \min\{d_{13} + d_{24}, d_{14} + d_{23}, V_4\} \ , \quad V_4 = \min_{s_1,s_2}\left(\, \|v_1 s_1\| + \|v_2 s_1\| + \|s_1 s_2\| + \|s_2 v_3\| + \|s_2 v_4\|\,\right) \ , \tag{11}$$

corresponding schematically to the flux tubes in Fig. 3.

A first study of the tetraquark spectrum with this potential concluded to the "absence of exotics" [45], but a re-analysis by Vijande et al. [46] with a better wave function, indicated that this potential, if alone, and free of constraints due to the Pauli principle, gives stability for the equal-mass case ($qq\bar{q}\bar{q}$), and improved stability for the flavor asymmetric ($QQ\bar{q}\bar{q}$). It remains to analyze how this stability survives antisymmetrization, short-range terms in the potential, relativistic effects and spin-dependent corrections. This is however, very encouraging.

Two developments came. First, a better understanding (at least by physicists) of the Steiner trees, with efficient algorithms [47] to compute them, and novel inequalities [48]. For instance, in a Steiner tree, a set of point can be replaced by its associated torroïdal domain, For instance, in the baryon Y-problem, the continuous minimization over the location of the Steiner point s can be replaced by a *discrete maximization*: the desired Y-shape length, $\|sv_1\| + \|sv_2\| + \|sv_3\|$ in Fig. 4 is the largest of the two distances $\|sw_3\|$ and $\|st_3\|$, where the points w_3 and t_3, named the Melznak points or torroïdal domain of $\{v_1, v_2\}$, form an equilateral triangle with v_1 and v_2.

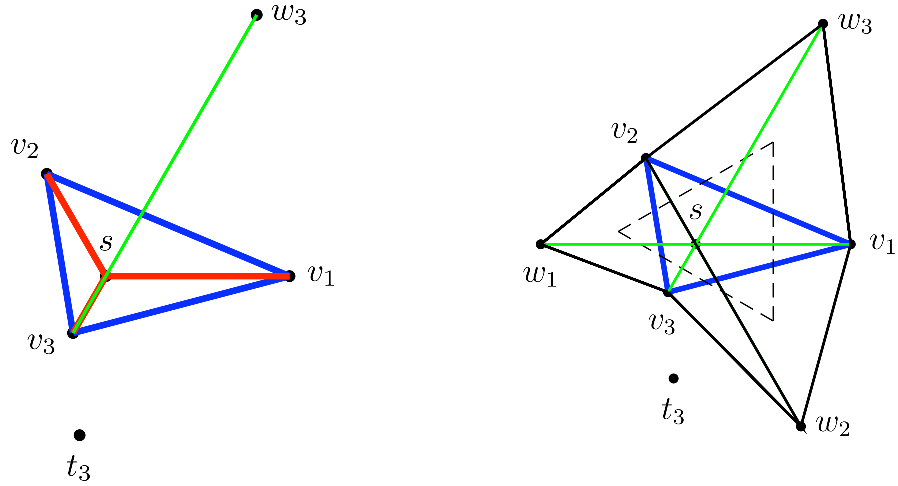

FIG. 4: The continuous minimization over the location of s is replaced by the discrete maximum $\max\{\|sw_3\|, \|st_3\|\}$, where w_3 and t_3 make an equilateral triangle with the quarks v_1 and v_2. Here, $V_3 = \|sv_1\| + \|sv_2\| + \|sv_3\| = \|v_3 w_3\|$. The symmetry restoration is reminiscent of the construction of Napoleon's theorem.

This property, and the interesting *symmetry restoration* (even for an asymmetric triangle, the flux tubes from the junction to the quarks make 120° angles in Fig. 3) are related to the theorem by Napoleon (see Fig. 4): if one starts from an asymmetric triangle $v_1v_2v_3$ and builds an external equilateral triangle such as $v_1v_2w_3$ along each side, the centers of these auxiliary triangles form an equilateral triangle.

The analogue for a planar tetraquark is

$$V_S = \|s_1v_1\| + \|s_1v_2\| + \|s_2v_3\| + \|s_2v_4\| = \|w_{12}w_{34}\| \ , \tag{12}$$

and is illustrated in Fig. 5: the length of Steiner tree linking the quarks v_1 and v_2 to the antiquarks v_3 and v_4 *via* the junctions s_1 and s_2 is he distance between the two Melznak points w_{12} and w_{34}.

In space, one gets the problem pictured in [48]: the auxiliary points w_{12} and w_{34} are located on a circle of axis v_1v_2 (resp. v_3v_4), and estimating the minimal Steiner tree can be shown to be equivalent to finding the *maximal* distance between the two circles [48]. This is a standard problem in computer-assisted geometry, as applied, e.g., in the cartoon industry.

Then you can derive inequalities on the Hamiltonian, and recover rigorously in some limiting cases the stability established by numerical methods.

The disappointing observation, however, is that the dynamics is dominated by the simple *flip–flop* term, and the most interesting connected Steiner tree plays a relatively minor role.

The exercise can be repeated for the pentaquark, using the linear model with minimal cumulated length,

$$V_P = \min(V_{\rm ff}, V_{\rm St}) \ , \quad V_{\rm ff} = \min_i \left[r_{1i} + V_Y(\boldsymbol{r}_j, \boldsymbol{r}_k, \boldsymbol{r}_\ell)\right] \ , \quad V_{\rm St} = \text{connected Steiner tree} \ , \tag{13}$$

the latter, $V_{\rm St}$, being shown in Fig. 6 (left).

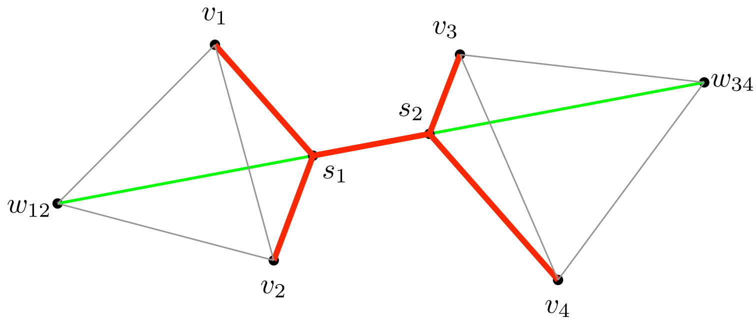

FIG. 5: The length of the Steiner tree is the distance between the vertices making an external triangle in the quark and in the antiquark sector.

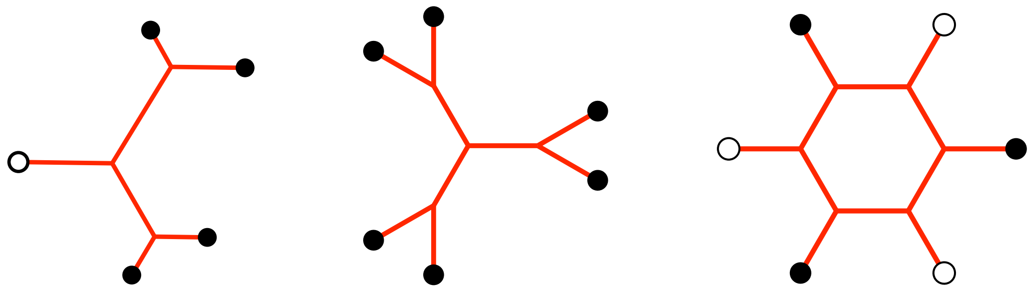

FIG. 6: Connected contributions to the confining interaction of a pentaquark, a dibaryon and a baryon–antibaryon system.

A simple variational calculation gives stability at least for $(\bar{q}qqqq)$, and $(\overline{Q}qqqq)$ and $(\bar{q}qqqQ)$ where $m(Q) \gg m(q)$ [49]. The other mass configurations remain to be studied.

This proliferation of stable states in the minimal-length model becomes embarrassing. Very likely, the dibaryon will also be found stable. If one believes into this confinement, this means that the role of the neglected effects should be investigated with care, in particular, the short-range part of the interaction (Coulomb-like forces) and the antisymmetrization of identical quarks. If the constraint of antisymmetrization turns out the main obstacle to multiquark stability, then exotic hadrons have to be searched in configurations with quarks of different flavors.

IV. CONCLUSIONS

The problem of multiquark binding is now addressed very seriously with QCD sum rules, Lattice QCD and even AdS-QCD. These ambitious but delicate approaches have first confirmed some results that were previously obtained empirically, such as the Steiner-tree structure of the linear term of the quark interaction in the static limit.

The constituent quark model remains a valuable tool of investigation, to detect the most interesting configurations and to analyze the role of the different pieces of the dynamics. In the case of mesons or baryons, the constituent models have been refined over the years, to include relativistic effects, coupling to the continuum, etc. The case of multiquark is of course much more delicate, with the mixing of confined channels and hadron–hadron components probably more crucial to build a reliable wave function.

On the experimental side, it is hoped that the future collider experiments will devote a reasonable amount of time to search for exotics with heavy flavor. As shown by B factories, there is a very good potential of discoveries in hadron physics within experiments primarily designed for studying other aspects of particle physics.

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