# Correlation between nuclear symmetry energy and the core-crust transition in neutron stars

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Camille Ducoin[1], Jérôme Margueron [2], Constança Providência[1]
[1] *CFC, Department of Physics, University of Coimbra, P3004 - 516, Coimbra, Portugal*
[2]*Institut de Physique Nucléaire, Université Paris-Sud, IN2P3-CNRS, F-91406 Orsay Cedex, France*

It has been pointed out that the slope of the nuclear symmetry energy at saturation density ($L$) is a crucial quantity to determine the mass and width of neutron-star crusts. This letter intends to clarify the relation between $L$ and the core-crust transition. We confirm that the transition density is soundly correlated with $L$ despite differences in the nuclear models, and we propose a clear understanding of this correlation based on a generalized liquid drop model (GLDM). Using a large number of nuclear models, we evaluate the dispersion affecting the correlation between the transition pressure $P_t$ and $L$. Furthermore, from a detailed analysis it is shown that this correlation is weak due to a cancellation between different terms. We point out that the correlation between the isovector coefficients $K_{\rm sym}$ and $L$ plays a crucial role in this discussion.

Stimulated by the development of exotic nuclear physics, the efforts to determine the nuclear equation of state (EOS) have focused in the last few years on the density dependence of the symmetry energy $S(\rho)$ [1]. In particular, the symmetry-energy slope at saturation density, represented by the quantity $L$, has raised a great deal of interest [1–5]: while the different nuclear models widely disagree on the value of this basic quantity, increasing experimental data [6–8] are expected to bring more and more stringent constraints, leading to a radical progress in our knowledge of the EOS of neutron-rich matter. This impacts strongly on the physics of compact stars. In this letter, we will discuss the link between $L$ and the transition from the liquid core to the solid crust of a neutron star. It has been claimed that a precise determination of $L$ would give a tight indication of the density $\rho_t$ and pressure $P_t$ at the transition point [1], and consequently the mass and extension of the crust which play a crucial role in the interpretation of pulsar observations [9]. However, the role of $L$ in the determination of the core-crust transition needs to be checked against model dependence and clarified, as mentioned in Ref. [10]. In the present work, we use a variety of nuclear models to address this issue. We verify and explain the strong correlation between $L$ and $\rho_t$. However, we show that when independent models are considered there is no real correlation between $L$ and the pressure at the transition point. This behavior results from a competition between opposite effects which destroy the correlation. This serious limitation has to be taken into consideration when drawing astrophysical consequences from the experimental determination of $L$.

Catalyzed matter in compact stars satisfies the $\beta$-equilibrium condition which favors very neutron-rich matter: the proton fraction is reduced to a few percent in the region of the core-crust transition. Their structure crucially depends on the symmetry energy, for a wide density range. The density dependence of the symmetry energy , $S(\rho)$, is deduced from the energy density functional obtained in the framework of mean field nuclear models. Besides, it can be expressed as a development around the saturation density $\rho_0$, whose coefficients correspond to the isovector parameters of a Generalized Liquid-Drop Model (GLDM):

$$S(\rho) \;=\; \sum_{n\geq 0} c_{{\rm IV},n}\frac{x^n}{n!}\,, \qquad (1)$$

where $x = (\rho - \rho_0)/(3\rho_0)$. Here and in the sequel, the index "IV" ("IS") attributed to the coefficients of the GLDM stands for "isovector" ("isoscalar"). The first coefficients have received traditional denominations: $c_{\rm IV,0} = J \equiv S(\rho_0)$, $c_{\rm IV,1} = L$, $c_{\rm IV,2} = K_{\rm sym}$, etc. In the framework of the parabolic approximation, the energy per particle for asymmetric matter is given by $E(\rho, y) = E(\rho, 0) + S(\rho)y^2$, where $y = (\rho_n - \rho_p)/\rho$. For convenience, we will use in the following either the isospin-asymmetry $y$ or the proton fraction $Y_p = (1 - y)/2$. This approximation allows to emphasize the role of the GLDM coefficients, so we will use it to analyze our results, although the calculations have been performed using the complete density functional of each model. In the parabolic-GLDM framework, the energy per particle reads:

$$E(\rho, y) \;=\; \sum_{n\geq 0}\left(c_{{\rm IS},n} + c_{{\rm IV},n}y^2\right)\frac{x^n}{n!}\,. \qquad (2)$$

In the isoscalar channel, we have $c_{\rm IS,0} = E_0 \equiv E(\rho_0)$, $c_{\rm IS,1} = 0$, $c_{\rm IS,2} = K_\infty$, etc.

We will show results obtained from a set of non-relativistic and relativistic effective interactions, together with results from a microscopic Brueckner-Hartree-Fock (BHF) calculation using the interaction Av18 [11] with Urbana three-body forces [12]. As non-relativistic effective models, we take Skyrme type interactions from different groups (SV, SGII, RATP, SkMP, Gs, Rs, SkI2, SkI3, SkI4, SkI5, SkI6, Sly10, Sly230a, Sly230b, Sly4, SkO, NRAPR, LNS, BSk14, BSk16, BSk17); the respective references can be found in [13–15]. Besides, we consider two different types of relativistic effective nuclear models: (i) non-linear Walecka models with constant couplings (NL3 [17], TM1 [18], GM1, GM3 [19], FSU, NL$\omega\rho$ [20], NL$\rho\delta$ [21]); (ii) hadronic models with density dependent coupling constants (TW [22], DD-ME1, DD-ME2 [23], DDH$\delta$ [24]). Let us remark that the EOS features present more variability within the relativistic models than within the Skyrme ones [13].

The inner crust of a neutron star is usually modelized as a lattice of very neutron-rich nuclei, immersed in a gas of electrons and dripped neutrons. As the density increases, the difference between the nuclei and surrounding neutron gas decreases, until the stellar matter becomes homogeneous: this is the transition to the liquid core. In order to determine the transition point, one should in principle compare the free energy of homogeneous matter to that of any inhomogeneous configuration [25]. However, it has been verified that the transition density obtained in this way can be very well approximated by the entrance into the dynamic spinodal region, under the constraint of $\beta$ equilibrium [10]. The spinodal is the density region where the homogeneous matter is unstable against density fluctuations, due to the nuclear liquid-gas phase transition affecting the bulk EOS. In the case of finite size density fluctuations, the Coulomb and surface terms reduce the instability: the dynamic spinodal region is then smaller than the thermodynamic one obtained when only the bulk term is considered. The difference between the two regions is in principle model dependent, via the nuclear surface term. However, we found that this effect is too small to play a role in the present discussion. Thus, for simplicity, we will focus on the transition density, $\rho_t$, proton fraction $Y_{p,t}$ and pressure $P_t$ taken at the crossing point between the $\beta$ equilibrium EOS of stellar matter and the thermodynamic spinodal, keeping in mind that they represent shifted values of the actual density, proton fraction and pressure at the core-crust transition.

It has been noticed in previous works that the transition density decreases as $L$ increases [1, 5]; this correlation has been verified with many different models, see Fig. 1a). We have found that this behavior can be understood through the energy-density curvature of pure neutron matter (NM), denoted $C_{\rm NM}$. More specifically, we have considered the curvature $C_{\rm NM}$ taken at the density of the upper spinodal border in symmetric matter, $\rho_s$. This quantity, denoted $C_{\rm NM,s} = C_{\rm NM}(\rho_s) = \frac{d^2(\rho E_{\rm NM})}{d\rho^2}(\rho_s)$, where $E_{\rm NM}$ is the energy per particle in neutron matter, is indeed correlated with the transition density $\rho_t$, as shown in Fig. 2a). This result allows a qualitative interpretation. The spinodal region corresponds to the region of ($\rho$, $Y_p$) where the energy density has a negative curvature. For very asymmetric matter such as $\beta$-equilibrium matter, $C_{\rm NM,s}$ gives a good indication to localize the position of the spinodal border: the larger $C_{\rm NM,s}$ is, the farther should be the spinodal contour from the point ($\rho = \rho_s, Y_p = 0$); and this corresponds to a lower $\rho_t$. Besides, $C_{\rm NM,s}$ is strongly correlated with $L$: see Fig 2b). This relation appears clearly when $C_{\rm NM,s}$ is expressed in the parabolic approximation, in terms of

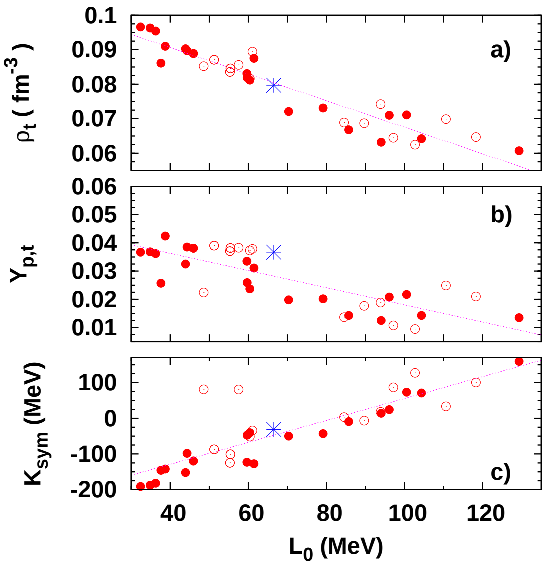


FIG. 1: (Color online) Correlation between $L$ and a) the transition density $\rho_t$, b) the transition proton fraction $Y_{p,t}$, c) $K_{\rm sym}$. The full (empty) symbols are for Skyrme forces (relativistic models) and the asterisk for BHF.

the isovector coefficients $c_{{\rm IV},n}$:

$$C_{\rm NM,s} = \frac{2}{3\rho_0}L + \frac{1}{3\rho_0}\sum_{n\geq 2} c_{{\rm IV},n}\frac{x_{\rm s}^{n-2}}{(n-2)!}\left[\frac{n+1}{n-1}x_{\rm s} + \frac{1}{3}\right].$$

Note that the isoscalar terms of the expansion are exactly zero at $\rho = \rho_s$ by definition of $\rho_s$. Since $x$ is negative, the influence of the higher order terms $n \geq 2$ is weakened. Furthermore, for all the models considered, we have $\rho_s \simeq (2/3)\rho_0$: this makes the contribution of the term $n = 2$ in Eq. (3) close to zero. As a result, $C_{\rm NM,s}$ depends very weakly on $K_{\rm sym}$, and is essentially determined by $L$. In summary, the correlation observed between $L$ and $\rho_t$ can be understood as the consequence of the link existing between $L$, $C_{\rm NM,s}$ and $\rho_t$.

Defining the transition point as the crossing between the spinodal border and the $\beta$ equilibrium EOS of stellar matter, we also have to take into account the model-dependence of $Y_{p,t}$, the proton fraction at the transition, which is expected to decrease with increasing $L$. Indeed, a smaller symmetry energy corresponds to a lower proton fraction. At subsaturation densities, assuming a consensual value of $J$ (about 32 MeV), a larger $L$ means a smaller symmetry energy and, consequently, a smaller $Y_{pt}$, as is shown in Fig. 1b). The dispersion of data in this figure reflects the model dependence of $J$. The correlation between $K_{\rm sym}$ and $L$ is also shown in Fig. 1c) and will be useful for the following analysis.

At saturation density the pressure in neutron matter is strongly correlated with $L$ [2, 3]. Indeed, in the parabolic-

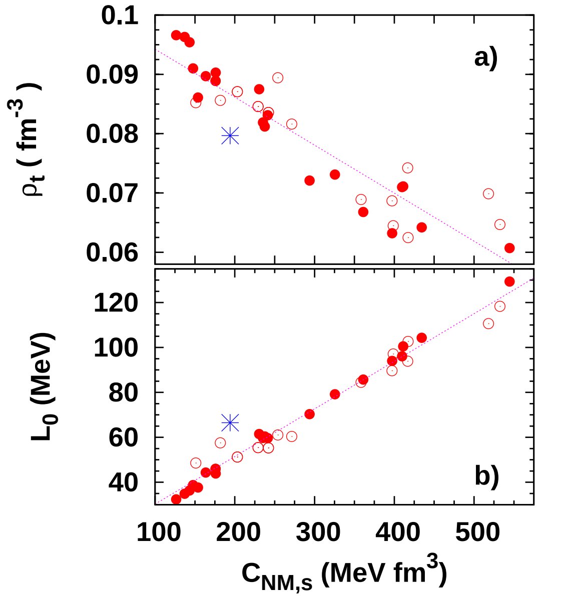


FIG. 2: (Color online) Correlations of the energy-density curvature of neutron matter $C_{\rm NM,s}$ with a) the transition density $\rho_t$, and b) the symmetry energy slope $L$. The full (empty) symbols are for Skyrme forces (relativistic models) and the asterisk for BHF.

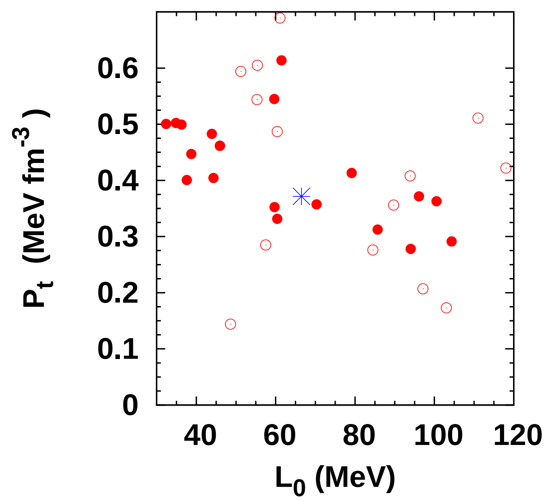


FIG. 3: (Color online) The pressure at the transition point versus $L$. The full (empty) symbols are for Skyrme forces (relativistic models) and the asterisk for BHF.

GLDM, the pressure reads

$$P(\rho,y) \;=\; \frac{\rho^2}{3\rho_0}\left[Ly^2+\sum_{n\geq 2}\left(c_{{\rm IS},n}+c_{{\rm IV},n}y^2\right)\frac{x^{n-1}}{(n-1)!}\right]. \tag{3}$$

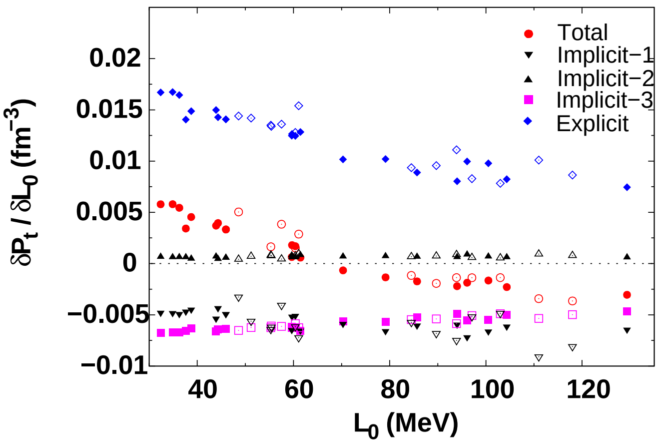


FIG. 4: (Color online) Expected variation of $P_t$ with $L$, comparing the different contributions: variation due to the explicit term (diamonds), implicit-1 (down-triangles), implicit-2 (up-triangles), and a first order approximation of implicit-3 (squares). The resulting total pressure variation is very close to zero (circles). The full (empty) symbols are for Skyrme forces (relativistic models).

From this expression we conclude that the pressure of neutron matter, $y = 1$, at saturation density, $x = 0$, is simply $P_{\rm NM}(\rho_0) = L\rho_0/3$. The situation is however different for the transition pressure $P_t$, $x < 0$, where the implicit dependence of the point $(\rho_t, y_t)$ and of the coefficients $c_{{\rm IV},n}$ on $L$ may affect the correlation pattern between the pressure and $L$. In Fig. 3 we represent $P_t$ versus $L$ calculated consistently for each of the models considered.

If we consider only the sub-group formed by the Skyrme models, it could be noticed a very slight decreasing correlation of $P_t$ for $L$ >60 MeV. However, considering all the models in Fig. 3, there is a large dispersion for the values of $P_t$ and we conclude that the transition pressure is almost independent of the value of $L$.

Let us now analyze the result presented in Fig. 3 using the GLDM for the pressure as shown in Eq. (3). The variation of the pressure (3) can be decomposed into different contributions: the one induced by the explicit dependence of the pressure with respect to $L$, $\frac{\delta P_t}{\delta L}|_e = \rho^2 y^2/(3\rho_0)$ (hereafter called explicit), and three terms coming from its implicit dependence: $\frac{\delta \rho_t}{\delta L}\left[\frac{\partial P}{\partial \rho}\right]|_{i1}$ (implicit-1), $\frac{\delta y_t}{\delta L}\left[\frac{\partial P}{\partial y}\right]|_{i2}$ (implicit-2), and the final one summing the implicit dependence of the isovector coefficients over $L$, that we will call in the following implicit-3,

$$\left[\frac{\delta P}{\delta L}\right]|_{i3} = \frac{\rho^2 y^2}{3\rho_0}\sum_{n\geq 2}\frac{\delta c_{{\rm IV},n}}{\delta L}\frac{x^{n-1}}{(n-1)!}\,. \tag{4}$$

A qualitative understanding of the correlation pattern for the term implicit-3 is obtained considering only the first term in the sum such that

$$\left[\frac{\delta P}{\delta L}\right]|_{i3} \approx \frac{\rho^2 y^2}{3\rho_0}\frac{\delta K_{\rm sym}}{\delta L}\, x\,. \qquad (5)$$

The implicit dependence of $\rho_t$, $y_t$ and $K_{\rm sym}$ is extracted from linear fits in Figs. 1a)-c), giving the values $\delta\rho_t/\delta L = (-3.84 \pm 0.24) \times 10^{-4}$ MeV$^{-1}$ fm$^{-3}$ and $\delta y_t/\delta L = (6.08 \pm 0.82) \times 10^{-4}$ MeV$^{-1}$, and $\delta K_{\rm sym}/\delta L = 3.07 \pm 0.33$. We show in Fig. 4 the different contributions to the variation of $P_t$. The contribution due to the explicit term is clearly large and positive, as expected. This term alone would predict an increase of the transition pressure with $L$, which is not observed. However, this term is balanced by the sum of the two terms implicit-1 and implicit-3 which are negative and of similar magnitudes. The term implicit-2 brings only a negligible contribution. The sum of the different contributions, represented by the circles in Fig. 4, is thus very close to zero. It is important to notice that a) if the term implicit-3 had not been considered the prediction that $P_t$ increases with $L$ would persist, since the term implicit-1 is not sufficiently large to overcome the explicit term; b) the term implicit-3 was approximated by the contribution of the leading term in the sum (4) and the effect of the higher order terms has not been included, and could be investigated. We conclude that there is no clear correlation between $L$ and $P_t$, due to the non-trivial competition between several explicit and implicit contributions which differ among models.

In the present letter we have explained why there is a good correlation between the crust-core transition density and the symmetry energy slope, $L$; and we predict that this behavior should not depend on the relation between $L$ and $K_{\rm sym}$. On the contrary, no correlation of the transition pressure with $L$ was obtained. We have highlightened the competing contributions to the variation of $P_t$ with $L$, which weaken the link between $L$ and $P_t$; this explains that the dispersion among models destroys any clear correlation. This means that an experimental determination of $L$ alone will not be sufficient for a good estimation of the crust mass and moment of inertia of a compact star. In fact, the range of variation of $P_t$ obtained within the present letter with a large set of nuclear models lies within the interval indicated in [9], $0.20 < P_t < 0.65$ MeV/fm$^3$, but completely out of the interval obtained in [1] from the expected values of $L$, which was determined from isospin diffusion and supposing the existence of a correlation between $P_t$ and $L$. The large dispersion of the predicted transition pressure obtained when independent models are considered needs to be reduced. A more accurate knowledge of the isoscalar EOS could improve the situation. As for the isovector EOS, the striking correlation between $K_{\rm sym}$ and $L$ appears to be an important feature, which should be further investigated.

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