GALOPR, a beam transport program, with space-charge and bunching

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Abstract

GALOPR is a first order beam transport code including three dimensional space-charge forces and the beam bunching process. It deals with usual optical devices (bending magnets, lenses, solenoids, drifts, bunchers) and can take into account any special optical device represented by its transfer matrix with spacecharge the "(Mueller inflector" was recently introduced as one of these devices). The beam can be continuous or undergoing a bunching or rebunching process. The beam line parameters can be optimized in order to fit at will the 6 x 6 transfer matrix and the 6 x 6 covariance matrix for a maximum beam intensity. The results are presented with useful data tables and graphical displays.

1) Introduction

The computer code GALOPR (GANIL beam Lines Optics including Radiofrequency bunchers) has been developed at GANIL from the code PHEINJ (Ref. 1, 2) which was written at CERN to study the focusing and bunching characteristics of the Low Energy Beam Transport system (LEBT) for the actual proton linac injector, including space charge forces.

Firstly, a version without buncher but allowing to choose the ion mass number and charge state was used at GANIL to calculate the transfer matrix along the 3 cyclotrons; the space charge forces were taken into account in the dipoles.

Since 1985, we have been developing the GALOPR version. Its universality makes it possible to calculate with linear approximation, the optics of a beam line either existing or being designed, for continuous or bunched beams, or beams in the process of being bunched, including space charge forces. Any buncher or rebuncher and any kind of element with an analytically known transfer matrix (drifts, quadrupoles, dipoles, solenoids) or a numerical transfer matrix are taken into account (Ref. 3).

The beam is considered as an hyperellipsoid in the 6 dimensional phase space. All forces, including space charge forces are linear with respect to the reference particle which in the present version is energy-fixed.

We have used GALOPR to study the axial injection beam line into the two injector cyclotrons for the GANIL modifications to come (0.A.E and 0.A.I) including a "Mueller inflector" in the 20 kV injection line and a "Pabot-Belmont inflector" in the 100 kV injection line.

2) beam parameters

Each particle at a point S of the beam line is located in the 6-dimensional phase space by its coordinates : x, x' = dx/ds, y, y' = dy/ds, z and z' = dz/ds, where s is the curvilinear coordinate. The fifth coordinate z is proportional to the difference between the time of arrival of a particle at point S and the corresponding time t0 for the reference particle.

\[ z = v_r(t - t_0) \quad \text{(1)} \]

where \( v_r \) is the velocity of the reference particle.

The beam is entirely defined by its intensity and the second order moments of the particle distribution in the 6-D phase space; we will sometimes call these "moments" "rms" values". They can be written, for a centered distribution :

\[ \rho(\mathbf{u}, \mathbf{v}) d^2 \mathbf{u} d^2 \mathbf{v} = \rho(\mathbf{u}, \mathbf{v}) d^2 \mathbf{u} d^2 \mathbf{v} \quad \text{with} \quad \mathbf{u}, \mathbf{v} \in \{x, x', y, y', z, z'\} \]

where \( d\mathbf{u} \) and \( d\mathbf{v} \) are the volume elements and the volume in the 6-D space.

If the 6 coordinates of each of the \( N \) particles of the distribution are known, the "rms" values become merely :

\[ \frac{1}{N} \sum_{i=1}^{N} u_i v_i \quad \text{(3)} \]

and define the covariance matrix \( \Sigma \) of the beam.

The type of distribution is assumed to be hyperellipsoidal, i.e. the surfaces of equal density are homothetic to the hyperellipsoid defined by \( \mathbf{F} \).

If \( \mathbf{F} \) represents the envelope of the hyperellipsoid for the coordinate \( u \), then the ratio \( k = \mathbf{F}/u^2 = \mathbf{F}/u \) is characteristic of the distribution. For an uniform distribution ( \( o = \text{constant} \) ) in a 6-D space, \( k = \sqrt{n+2} \). A continuous beam, is represented by an infinitely long cylinder ( \( n = 2 \), \( k = 2 \) ), and a bunched beam by an ellipsoid ( \( n = 3 \), \( k = \sqrt{3} \) ), both being uniformly charged.

The rms emittance in each of the 3 phase planes \( (x, x'), (y, y') \) and \( (z, z') \) is given by :

\[ \epsilon_{ij}^2 = u_i^2 v_j^2 \quad \text{(4)} \]

and is connected to the marginal emittance by :

\[ \epsilon_j = k^2 \epsilon_{ij} \quad \text{(5)} \]

As demonstrated in Ref. 4, the evolution of the rms values depends mainly on the linear component of the space charge force, in the case of linear external forces. Therefore, all types of distributions having the same rms values can be treated by such linear programs; the tuning of the physical parameters of the line is then the same for any distribution.

The above-mentioned (cylinder and ellipsoid) have space charge forces varying linearly with the coordinates and will be chosen in the following treatment as models.

3) Space charge force calculation

The components of the radial electric field inside an infinitely long cylinder with elliptical section are given by :

\[ E_x = \frac{\rho}{a + b} x \quad E_y = \frac{\rho}{a + b} y \quad E_z = 0 \quad \text{(6)} \]

and inside an ellipsoid by :

\[ E_x = \frac{\rho}{c_0} J_x x \quad E_y = \frac{\rho}{c_0} J_y y \quad E_z = \frac{\rho}{c_0} J_z z \quad \text{(7)} \]

where \( a, b \) and \( c \) are the axes, if these volumes are in their principal axes, and where :

\[ c_0 = \text{dielectric constant of vacuum} \]

\( \sigma = \text{space charge density} \)

\[ J_x = 2 \int_0^\infty \int_0^\infty (a^2 + x)(b^2 + y)(c^2 + z) \quad \text{(8)} \]

and analogous for \( J_y \) and \( J_z \) are dimensionless elliptic integrals numerically computed by the Gauss' method.
Choosing \( s = \sqrt{v_{rt}} \) as the independent variable, the equations of motion can be written for both models:

\[
\frac{d^2x}{ds^2} = -\frac{e}{2\epsilon_0 (W/E)} F_x = V_x v
\]

and same for \( y \) and \( z \) with \( W = \) energy per nucleon of the reference particle,
\( \epsilon = \) charge to mass ratio.

Introducing the rms values, we then get:

\[
K_x^e = \frac{\rho}{2\epsilon_0 (W/E)} \frac{-1}{x+y}
\]

for the cylinder (same for \( y \))

\[
K_x^e = \frac{\rho}{\epsilon_0 (W/E)} \frac{j_x}{j-y}
\]

for the ellipsoid (same for \( y, z \))

### 4) Transfer matrices

The beam line is composed of thick and thin elements: the space charge forces act only in the thick ones, which are subdivided into steps, the length of which is chosen in such a way that it has only a small effect on the transverse dimensions.

We use 4 types of transfer matrices:

#### 4.1. Thin elements

The transfer matrix for a pole-face rotation \( \theta \) at one end of a dipole, and for a rotation of the transverse coordinate system around the curvilinear coordinate are given in Ref. 5.

Any element (or combination of elements) previously calculated and given in a numerical form (for instance, the Pabot - Belmont inflector) can be introduced in the program as a \( 6 \times 6 \) matrix.

#### 4.2. Thick elements with decoupled phase planes

If the reference frame of the ellipsoid and of the motion are the same, then the space charge forces \( (K_U) \) can be added to the external ones \( (Q_U) \) in each phase plane, leading to the Hill-type equation:

\[
\frac{d^2u}{ds^2} + (Q_U - K_U) u = 0
\]

The following table gives the constants \( Q_U \) corresponding to the external forces of the different elements:

<table>
<thead>
<tr>
<th>Element</th>
<th>( Q_x )</th>
<th>( Q_y )</th>
<th>( Q_z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drift Space</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Quadrupole</td>
<td>( G/BR )</td>
<td>( -Q_x )</td>
<td>0</td>
</tr>
<tr>
<td>Solenoid(*)</td>
<td>( \frac{1}{2} \frac{G}{3}/BR )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Rotunber</td>
<td>( 2\pi B \sqrt{E_0} )</td>
<td>( Q_x )</td>
<td>( Q_x )</td>
</tr>
</tbody>
</table>

\( BR = \) magnetic rigidity
\( B \) = distance between 2 consecutive bunches adjacent
\( G = \) quadrupole gradient \((G > 0)\) is focusing in the \( x \) direction
\( R = \) maximum field in the solenoid
\( V = \) maximum voltage times transit time factor on the axis
\( Lg = \) gap length for a single-gap element
\( L = \) solenoid length

(*) A rotation by an angle \( \theta \) \( Lg \) must be added afterwards to get the transfer matrix in the initial reference frame.

In each plane, the \( 2 \times 2 \) transfer matrix contains sine and cosine terms if \( Q_U = K_U > 0 \) and cosh and sinh terms if \( Q_U = K_U < 0 \).

#### 4.3. Thick elements with coupled phase planes

We consider the most complicated case of a dipole with the bunch tilted with respect to the axes. We first calculate the eigenvalues of the covariance matrix in the real space and the eigenvectors matrix \((V)\) which transforms the \((x, y, z)\) frame into the \((X, Y, Z)\) frame of the ellipsoid: the "rms" values \( \sqrt{\sigma^2} \) of the ellipsoid are the above eigenvalues.

According to the previous paragraph, the space charge forces \( K_x, K_y \) and \( K_z \) can be calculated.

The new 6-D covariance matrix may be obtained in two ways:

1st. method

The 6-D covariance matrix is calculated in the \((x, y, z)\) frame; then, a new one is obtained by the action of a thin lens with a strength equivalent to the effect of the space charge force over one step.

This new matrix is projected on the original reference frame, and is finally transferred through the next element step.

2nd. method

The forces \( K_x X, K_y Y \) and \( K_z Z \) are projected on the original axes, giving:

\[
\begin{bmatrix}
V_x & K_x & [V] \\
V_y & K_y & [V] \\
V_z & K_z \\
\end{bmatrix}
\]

These constants \( K_{Ux} \) must be added to the external forces implied in the classical system:

\[
\frac{d^2x}{ds^2} - \frac{n}{\rho^2} x - \frac{1}{\rho} \frac{dx}{ds}
\]

\[
\frac{d^2y}{ds^2} + \frac{n}{\rho^2} y = 0
\]

\[
\frac{d^2z}{ds^2} = \frac{1}{\rho} \frac{dx}{ds}
\]

with \( n \) = field index
\( R = \) radius of curvature of the reference frame.

And the resulting system is solved by classically going to a Runge-Kutta method applied to a six first-order differential equation system.

The final transfer matrix is composed of the 6 vectors, solutions of the system. It is however necessary to perform a matrix transformation at both ends in order to take into account the difference between entrance and output reference frames.

This second method is more time consuming but it is more physical to add all the forces before solving the equations.

#### 4.4. Special elements

The Müller inflector can be given as an example since it can be represented by an analytical matrix from the origin to any points inside it. Care of the space charge effect is taken by adding a perturbation matrix (Ref. 6). In GALACPR, we are only using the total transfer matrix (Ref. 6) for a \( 90^\circ \) deflection angle; since an inflector is necessarily attached to the central region of a compact cyclotron, two additional factors must be taken into account: the magnetic field variation along the trajectory and a rotation to match the reference frame of the accelerated motion.
5. Transition from a continuous to a bunched beam.

All the elements studied above have a linear action on the beam. When a continuous beam, with a negligible energy spread traverses a buncher, the energy modulation resulting from the sinusoidal voltage generates a longitudinal emittance. In our model, it is of importance to include in this emittance only the particles that will be captured further while taking into account the effect of all the particles (including those which will be lost) for the space charge force calculations, we proceed in 3 steps:

5.1. Generation of the longitudinal emittance at the buncher

Before the buncher, the emittance is a single line along the phase axis (zero energy spread). The relative momentum spread is given by:

$$ z' = \frac{\delta p}{p} = \frac{e V B T}{2 (W/E) \sin (2 \pi z \eta)} $$

(15)

$$ \eta = \phi_c / \pi $$ where $\phi_c$ is the current upstream.

The assumption is that, at the buncher, the trapped particles lie inside the hatched region of figure 1; the separation between the trapped and rejected particles is called the "cut-off" phase $\phi_c$.

Since, at this point, the beam is continuous, the trapped intensity is proportional to $\phi_c$:

$$ \text{trap} = \text{current} \times \phi_c / \pi $$

When several bunchers are present, the modulation is no longer a simple sine and it is much easier to make a numerical treatment to calculate the modulation and therefore to calculate the rms values, which is the case in our computer code where a double drift harmonic buncher is considered.

5.2. Introduction of the space charge

We have to fulfill the following requirement: there must be continuity of the space charge forces during the transition from a 4-D to a 6-D phase space.

The chosen model is a combination of the actions of a double drift harmonic buncher, the space charge force effects in an element whose transfer matrix is given in a numerical form, and the quadrupole gradients (and rarely the drift lengths); they are varied in order to:

- fulfill given conditions on some elements either of the covariance matrix or of the transfer matrix at the matching point,
- limit the beam dimensions locally or along the whole line.

In the longitudinal plane, the matching is possible by varying $\phi_c$, $V_B T$ of the first buncher and the voltage of a rebuncher.

Due to the various coupling between the transverse and longitudinal planes, all the parameters, conditions and limits are included in the same optimization which is carried out by a least-squares method.

7. Conclusion

The code SALOPR is now running onto the IBM 3080 (VM/CMS system) of the C2 IN2P3 (IN2P3 computer centre in Lyon) and into the 32 bits MOCOMP at GANIL: an off-line code allows to draw the beam envelopes onto a "BENSON" plotter.

Two further improvements to this code are planned by introducing:

- a thin line taking into account the space charge effects in an element whose transfer matrix is given in a numerical form,
- elements in which the central particle is accelerated.

References:

(1) R. Bru and M. Weiss, CERN/MPS/LIN 72-4
(3) B. Bru, SALOPR : Un programme de transport de faisceau incluant la charge d'espace et le groupement, GANIL 88/16/TF/01
(4) F. Sacherer, CERN/S/DL/70-12
(5) K.L. Brown, D.C. Carey, Ch. Iselin and F. Rothacker, CERN 80-04
(6) R. Beck, B. Bru and Ch. Ricaud, L'inflecteur hyperboloidal de R.W. Muller, GANIL A86-02 a et b.