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Abstract

ECRIPAC is a new ion-electron accelerator concept using ECR, GYRAC and PLEIADE effects, which is being studied in a collaboration between CEA/DRFMC (Grenoble), LNS (Saclay) and GANIL (Caen).

A 6D Vlasov code using the "particle method" was developped to simulate the electron heating process, the plasma compression and the energy transfer from electrons to ions.

Two versions are available : the first one allows a precise description of the plasma during small time intervals, at different stages of the process. The second corresponds to an adiabatic approximation of electron motions, where electron superparticles are replaced by circles obeing special equations coupled with collective effects. External electromagnetic fields depending on x, y, z and t are accurately computed, while collective fields are calculated using the "cloud in cell" scheme.

1. INTRODUCTION

The collective accelerator project ECRIPAC, described in detail in [1], is composed of 3 parts, which have been separately tested with success in other contexts :

- a multicharged ECR source (NEOMAFIOS or NANOGAN [4]) able to produce a plasma of highly charged ion distribution with a typical density of 5.10^{10} cm^{-3} measured at 30 cm on the z axis.
- a GYRAC-C section where the electrons are accelerated by gyromagnetic autoresonant effect and the plasma cylinder compressed into a thin disk using an increasing magnetic field.
- a PLEIADE section where a part of transverse electron energy is converted into longitudinal one in the space decreasing magnetic field, while most of the charged ions are dragged.

Such an accelerator can theorically produce either ion bunches up to 400 MeV/amu or electrons of 4 MeV energy with very short pulses.

Compared to Electron Ring Accelerators, ECRIPAC presents some originality : due to the use of an ECR source, ions are present at the beginning of the process and the ion-electron ratio can be easily controlled. The fact that the ECR source produces multiply charged ions allows, if necessary, a good neutrality up to the end of compression, the most charged ions being accelerated during PLEIADE phase. Electron bunch is a disk instead of a ring, which seems to be better for some stability aspects, and gives a good electric field for the ion acceleration.

From the point of vue of simulation, two aspects were considered : the behaviour of single particles in real fields as compared to the GYRAC simplified theory, and the influence of collective fields.

One major difficulty was to find a way of limiting CPU time, specially for the compression phase during which electrons operate several million rotations.

2. SUMMARY OF THE SIMPLIFIED THEORY

As explained in [1] and [2], the relativistic factor of a single electron during gyromagnetic autoresonance verifies :

$$(1) \qquad \gamma(t) = \frac{B(t)}{B(0)} \qquad 0 \le t \le 10\ \mu s$$

where $B(0) = \frac{m_0\ \omega_{HF}}{e} \cong 0.0875$ T is the magnetic field at the beginning of the heating process, ω_{HF} being the pulsation of the rotating electric field.

During the compression phase, γ satisfies :

$$(2) \qquad \gamma(t) = \left[1 + (\gamma_1^2 - 1)\frac{B(t)}{\gamma_1 B(0)}\right]^{\frac{1}{2}} \qquad t \le 100 \text{ ms}$$

where $\gamma_1 = 2$ is the relativistic factor at the end of HF phase, and γ increases up to $\gamma_s = 9$ for a B growing up to $B_s = 5$T.
The rotation radius and the transverse impulsion of the single electron verify the following adiabatic laws :

$$(3) \qquad r^2 B = \text{cste} \qquad (4) \qquad \frac{P_\perp^2}{B} = \text{cste}$$

It is possible to show that the center of rotation of the electron verifies also (3), so that the same compression occurs for the envelope of the disk.

During the PLEIADE stage, relations (3) and (4) are satisfied ,with an expansion instead of a compression, even taking into account the energy transfer from electrons to ions. With a space decreasing magnetic field, the condition for one ion to be accelerated by the electron bunch is :

$$(5) \qquad \left|\frac{\nabla B}{B}\right| \le \frac{2e}{m_p c^2}\frac{Z}{A}E, \qquad (6) \qquad E = \frac{e}{2\varepsilon_0}\frac{N_e}{S}$$

where m_p is the proton mass, Z and A the charge and the mass number of the considered ion, E the peak electric field created by the electron disk of section S, and N_e the number of electrons per pulse.

Assuming good stability and focusing conditions, and depending on the profile of B, an immediate or progressive shake out occurs for the least charged ions, which leads to the following energy equations for the remaining ions :

$$(7)\quad \frac{1}{A}\frac{dW_i}{dz} = -m_0c^2\frac{(\gamma_s-1)\frac{1}{B_s}\frac{dB}{dz}}{\gamma_f\frac{m_0}{m_p}+A\frac{N_i(z)}{N_e}} \quad , \; W_i(0)=0$$

$$(8)\quad N_i(z) = \int_{Z^*(z)}^{\infty} p(Z)\,dZ$$

$$(9)\quad Z^*(z) = \sup_{0\le s\le z}\left\{-\frac{m_pc^2A}{2Fe}\frac{1}{B}\frac{dB}{dZ}(s)\right\}$$

where p(Z) is the initial ion charge state distribution and N_i the remaining number of ions of energy W_i at the position z.

3. VL1 PROGRAM

This code solves a system of time dependant Vlasov equations in 6D phase space, for electrons and multiply charged ions. External real fields E and B depending on x,y,z,t are calculated with great precision, while collective effects are computed by solving the following equations :

$$(10)\quad -\Delta V = \frac{\rho}{\varepsilon_0} \qquad (11)\quad -\vec{\Delta}\vec{B} = \mu_0\,\overrightarrow{rot}\,\vec{j}$$

ρ and $\vec{j}$ are evaluated on a 3D grid of 27000 nodes using a CIC projection, (10) and (11) being solved with the Conjugate Gradient method.

We established that relation (1) was verified in real 3D fields for HF amplitudes between 30000 and 3.10^6 V/m, with oscillations of γ compatible with the results of [2].

Various magnetic mirror profiles were tested, choosing different values of B(0). In order to "absorb" the initial electron velocities, this initial B(0) is chosen between 50 and 100 Gauss under the theorical value.

In the 3D context, we observed that some electrons could be axially lost during the first microsecond. Depending on HF amplitude and mirror profile, losses were found to be between 10 and 60 %. This is due to the pulsation of electron radii at the beginning of gyromanetic autoresonance. (this 3D effect has been independantly observed by M. Umnov [5]).

One way to reduce the loss could be to choose a decreasing HF amplitude approaching the following formula :

$$(12)\quad E_{HF}(t) = \frac{c}{\omega_{HF}}\frac{B\dot{B}}{\sqrt{B^2-B(0)^2}}$$

A first conclusion for the HF stage of GYRAC is that it is important to allow some latitude in the choice of the initial mirror shape and the HF amplitude, for the construction of an ECRIPAC prototype.

With VL1 and without collective effects, we verified that equations (2), (3) and (4) were fully satisfied during the compression phase, up to 5 Tesla.

4. VL2 PROGRAM

We noticed with VL1 that at least 10000 particles were necessary to take into account collective fields.

The compression phase being very long, it was necessary to allow a time step greater than the rotation period of electrons. A new version VL2 was developed, where ions remain standard particles, but where electrons are replaced by "circle particles".

For each circle, we need evolution equations for the radius r_e, the transverse impulsion $P_\perp$, and the position (x_c,y_c,z_c) and impulsion (p_{cx},p_{cy},p_{cz}) of the center of rotation.

Deriving relations (3) and (4), we find :

$$(13)\quad \frac{dr_e}{dt} = -\frac{1}{2}\frac{\dot{B}}{B}r_e \qquad (14)\quad \frac{dP_\perp}{dt} = \frac{1}{2}\frac{\dot{B}}{B}P_\perp$$

Noting $\vec{X} = {}^t(x_c,y_c)$ and $\vec{P_x} = {}^t(p_{cx},p_{cx})$, deriving equation (3) applied to the center of rotation of the electron and neglecting the very slow azimutal rotation of the plasma, we obtain :

$$(15)\quad \vec{P_x} = -\frac{1}{2}\frac{\dot{B}}{B}m\vec{X}$$

Deriving once more, we find an expression of the radial compression force applied to the center, which can now be added to the collective force :

$$\frac{d\vec{P_x}}{dt} = -\frac{\dot{m}}{2}\frac{\dot{B}}{B}\vec{X} - \frac{m}{2}\left(\frac{\ddot{B}}{B}-\frac{\dot{B}^2}{B^2}\right)\vec{X} + \frac{m}{4}\frac{\dot{B}^2}{B^2}\vec{X} + \overrightarrow{F_{xy(coll)}}$$

with $\dot{m} = -\frac{1}{2}\frac{1}{c^2}\frac{P_\perp^2}{m}\frac{\dot{B}}{B}$ and $\ddot{B}=0$, we obtain :

$$(16)\quad \frac{d\vec{P_x}}{dt} = \frac{1}{2}m\frac{\dot{B}^2}{B^2}\vec{X} + \overrightarrow{F_{xy(coll)}}$$

The replacement of the electron trajectory by a succession of circle motions consists in giving a negative velocity (15) at each time step, while applying the "repulsive" compression (16), combined with collective forces.

Although these forces are applied to each center, they must be evaluated as an average on each circle (in practice 8 points) because they act physically on the electrons. (Otherwise, we observe a very nice but numerical "cristallisation" into 3 or 5 groups of circles).

The evolution of axial compression is expressed by the following equations :

$$(17)\quad \frac{dP_{cz}}{dt} = \frac{eP_\perp}{m}(\overline{B_r} + \overline{B_{r(coll)}}) - e\overline{E_{z(coll)}}$$

$$(18)\quad \frac{d z_c}{d t} = \frac{P_{cz}}{m}$$

where $\overline{B_{r(coll)}}$ and $\overline{E_{z(coll)}}$ are the average radial and axial collective fields, while $\overline{B_r}$ is the average of the external radial magnetic field on the electron circle (approximatively equal to the radial external field at the distance r_e of the z axis).

Finally, (13) to (18) form a family of first order differential equations treated with Runge-Kutta or leap-frog method and incorporated easily in the Vlasov code.

VL2 program was at first tested without collective fields and we verified that the results were those given by VL1 for the single particle compression.

With collective fields, we observed that a good compression occurs for a number of electrons up to 10^{11}, with a magnetic field increase of 80 Gauss/μs. In this case, ions operate small rotations very slowly, and remain trapped in the compressed electron bunch.

For greater densities, we detected too much numerical noise in the electric collective field, which led to a meaningless explosion of ions and electrons. A greater number of particles is necessary in this case.

We can show that the "circle particle" approach may also be used for the PLEIADE stage, with minor modifications (essentially replacement of dB/dt by dB/dz.dz/dt). The axial acceleration being very rapid, either VL1 or VL2 can be used in this case.

With the two programs, we verified that the shake-out condition (5) is valid.

If the magnetic gradient is important, all ions are ejected from the electron bunch, which is accelerated following approximately the differential equation :

$$(19)\quad \frac{d z}{d t} = c\sqrt{1 - \frac{B(z(t),t)}{B_s}}$$

For smooth gradients verifying (5), ions are correctly dragged. However, the complete stability study of PLEIADE is not yet achieved.

5. CONCLUSION

The numerical simulations of ECRIPAC are in good accordance with the theory.

A more precise simulation is contemplated with a new version of the code, written in FORTRAN 90. This will allow the use of massively parallel computers, with a greater number of particles and a better evaluation of collective fields.

During 1991, technical studies have shown that ECRIPAC machine could be designed without major difficulties.

We hope that a prototype will be soon planned.

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