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# DESIGN ISSUES OF RADIOACTIVE ION BEAM FACILITIES

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Abstract

There is an increasing interest in Radioactive Ion Beams throughout the world. These ions open new domains of research for nuclear physics, nuclear astrophysics and atomic physics. Two methods are used for the production of these beams: fragmentation of a primary, high energy heavy ion beam passing through a thin target or nuclei production in a thick target bombarded either by a heavy ion beam, a proton beam or by neutrons. When radioactive species are produced in a thick target, they must be extracted, ionised, separated, identified and finally accelerated. This requires a radioactive ion source, a mass separator and a post accelerator. This paper reviews these two methods, their respective domains and the specific problems related to the control and the acceleration of radioactive ion beams.

#### 1 INTRODUCTION

The demand for Radioactive Ion Beams (RIB) is rapidly increasing throughout the world. Looking the nuclei chart, one can see that the number of radioactive species amounts to several thousands, limited by the so-called proton and neutron driplines beyond which the nuclei are no longer bound. The RIB's are of interest not only for nuclear physics but also for nuclear astrophysics, atomic physics, material and surface studies, biology and medicine.

Nuclear models can be precisely tested with doubly magic nuclei and with nuclei near the stability limits. The recent discovery of the heaviest doubly magic nucleus  $^{100}\mathrm{Sn}$  [1, 2] is a good illustration of the interest of these techniques. Nuclear reactions with secondary beams of light neutron-rich nuclei like  $^{11}\mathrm{Li}$  and  $^{11}\mathrm{Be}$  led to the discovery of the halo structure of their neutron distribution [3, 4]. A similar structure of the very loosely bound valence proton of the proton-rich  $^{8}\mathrm{B}$  was also recently discovered [5].

During nucleosynthesis (in supernovae), the temperatures (0.2-1.5  $19^9$ K) and the densities ( $10^3$ - $10^6$  g/cm<sup>3</sup>) are such that nuclear reactions proceed on time scales of typically sec-min. Thus any nuclei produced with comparable half-lives will play a significant role in the whole nucleosynthesis process [6].

Many applications of RIB's are used in solid state physics [7]. Programs like defects and impurities in metals and semiconductors, implantation, diffusion in highly non-miscible systems, surface and interface studies by deposition of a monolayer of radioactive atoms have been developed and use detection techniques such as Mossbauer Spectroscopy, Perturbed Angular Correlations, Channeling, Beta NMR and so on.

RIB's also open new fields of research in medicine and biology. For instance, a selected isotope can be implanted and used as a tracer which will allow one to follow the evolution of a protein.

Atomic physicists are also asking for RIB, most often at very low energy, associated with ion traps.

Table 1 summarises the use of RIB for physics as well as the associated energy ranges and the typical research tools.

Physics	Typical Energy/A	Typical tool
Atomic physics	eV	Ion trap
Biology, medicine	1-50 keV	γ spectrometer
Materials science	20-200 keV	Radiotracer
Nuclear astrophysics	0.2-2 MeV	Recoil separator
Coulomb excitation	1-5 MeV	4π detector
Fusion/Fission reactions	5-100 MeV	Spectrometers

Table 1: Some typical RIB's uses and energy ranges.

### 2 RIB PRODUCTION METHODS

Short lived radioactive atoms are not naturally available on earth. They have to be produced on purpose by nuclear reactions involving many different mechanisms. The choice of the mechanism is important and can lead to very different production yields.

Two different production methods can be distinguished according to the thickness of the production target.

The **thin target** process uses the fragmentation of energetic heavy-ion projectiles. The emerging fragments have nearly the same speed as the incident ions so, their energy is quite large. This production method is convenient for nuclear experiments requiring a projectile energy >30 MeV/A. It allows the study of isotopes having half-lives down to the ms range or less. The process is not sensitive to the chemical properties of the radioactive secondary beam.

The **thick target** process consists of stopping the primary beam in the target. The primary beam can be either a high energy proton beam, a medium to high energy heavy ion beam or even the high thermal neutron flux produced by a nuclear reactor.

Many different nuclear reactions can occur in the target: projectile or target fragmentation, fusion-evaporation, fission or spallation, resulting in the production of a wide range of nuclei. The desired exotic species must be extracted from the target, ionised and accelerated to give a RIB. This process is time consuming, it will usually not be convenient for atoms with half lives lower than a few tens of ms. The evaporation process is also sensitive to the chemical interaction between the exotic atom and the target. The kinetic energy of the secondary beam depends on the choice of the post-accelerator. Energies usually range from a few keV up to a few 10 MeV/A.

The two methods are obviously complementary. They can also coexist at the same place. This is the case at

GANIL (Caen-France) where a thin target apparatus SISSI has been delivering exotic beams from 30 to 95 MeV/A since1994, and the thick target and post-accelerator complex SRIRAL will produce exotic beams up to 25 MeV/A in late 1998.

Table 2 summarises the main characteristics of the two methods.

	Thin target	Thick target
Primary beam	Heavy ions	Heavy ions
	E≥30 MeV/A	Protons
		Neutrons
Radioactive ion beam		
Energy	≥ 30 MeV/A	eV to GeV/A
Lifetime	Down to < ms	≥ 50 ms
Chemical	No	yes
interaction		

Table 2: Thin and thick target overall comparisons.

## 3 THIN TARGET RIB PRODUCTION

The production of RIB by projectile fragmentation through a thin target has been intensively used for physics since the advent of medium-energy heavy-ion beams [8-12]. The production target was usually placed at the object point of a recoil spectrometer. This allowed: i) the separation of the secondary fragments from the primary beam and ii) the selection and separation of the required isotope from other species. Furthermore, this process has been used to produce energetic exotic beams that were able to be focused, analysed and directed to the physicist's target.

#### 3.1 Production laws

When an energetic heavy ion beam passes through a thin target, some of the incident ions interact with the target nuclei resulting, by nucleon exchange mechanisms, in the production of fragments of the incident nuclei. The fragments, usually fully ionised, leave the target with a speed near that of the incident beam. They are also straggled by coulomb interaction with the target electrons. According to [13], a qualitative analysis of the fragmentation mechanisms shows that a beam with the atomic number Z<sub>beam</sub> produces fragments of Z≤Z<sub>beam</sub> and, for each Z, an isotopic width approximately proportional to Zbeam. The production yield of a given isotope is consequently inversely proportional to  $(Z_{beam})^2$ . The production yield increases rapidly with the energy of the incident beam : Y≈Ei<sup>7/2</sup>, but it is often necessary to use an energy degrader for the separation of the numerous elements having the same magnetic rigidity (Bρ). This energy degrader introduces some additional angular straggling and energy dispersion resulting, after analysis, in an intensity reduction proportional to  $(E_f/E_i)^2$ .

Finally, after analysis by a fragment separator having respectively the angular and momentum acceptances :  $\theta_s$ ,  $(\partial p/p)_s$ , the total production yield available for physics will be

$$Y_s \approx K\sigma_{\text{Prod}} t_{\text{opt}} \left( \frac{A_f (A_p - 1)}{A_p - A_f} \right)^{\frac{3}{2}} \theta_s \varphi_s \left( \frac{\delta p}{p} \right)_s E_i^{\frac{3}{2}}$$

In this expression,  $t_{opt}$  stands for the optimal thickness of the target (and depends on the primary and secondary beam characteristics),  $A_p$  and  $A_f$  are the primary and secondary mass numbers respectively and K is a constant.

## 3.2 Design issues

The production target receives a non-negligible part of the incident energy. The beam spot on the target must be as small as possible to limit the increase in the emittance due to straggling. One can avoid melting the target by continuously rotating it.

The target, and its environment, are very active after operation due to the high neutron flux emitted. A remote handling system must therefore be provided for the maintenance.

The target must be followed by a two-stage fragment separator. The first stage allows one to select fragments which have the required rigidity, an energy degrader is usually placed before the second stage [9]. Owing to the fact that the fragments are fully ionised, the energy of the ions selected by the first part of the separator is proportional to  $Z^2/A$ . Since, in a given material  $dE/dx \approx AZ^2/E$  (Bethe's formula), the total energy loss in the degrader of thickness e can be written:  $dE/E=K(A^3/Z^2)e$ . So that, by decreasing the magnetic field by a value  $dB/B\approx 1/2dE/E$ , one is able to select a nuclei of a given  $A^3/Z^2$ .

One must also be aware of the fact that the magnetic rigidity of neutron rich fragments is higher than that of the primary beam. As an exemple, at GANIL, <sup>11</sup>Li nuclei obtained from a fully ionised <sup>18</sup>O primary beam have a rigidity 63% higher.

The last concern are the beam diagnostics which must be able to perform measurements on sub-picoampere beams. Such diagnostics, using for instance the ionisation of the residual gas or of a support gas by the beam and the amplification of the signal by microchannel plates have been operational at GANIL for some time now[11].

# 4 THICK TARGET RIB PRODUCTION (ISOL METHOD)

The pioneering effort for the RIB's production from a thick target was undoubtedly performed at ISOLDE which has been delivering at CERN, low energy exotic beams for nearly 28 years [28]. It is more recently that these exotic beams were injected into a post accelerator (Louvain-La-Neuve) to obtain energies usefull for nuclear physics experiments. The post-accelerator can be either an electrostatic machine, a linear accelerator or a cyclotron. The production method, more often known as ISOL (for Isotopic Separation On-Line) involves a large range of physical phenomena. The primary beam can be either protons ranging from 30 MeV up to several GeV, medium to high energy deuterons, alpha particles and

heavy ions, high energy or thermal neutrons. The exotic nuclei are produced by nuclear reactions such as (p,n),  $(p,\alpha)$  with low energy protons, spallation and target fragmentation with high energy protons, projectile or target fragmentation, fusion/evaporation, electromagnetic dissociation, with heavy ions.

The mechanism of interest with neutrons is mainly the fission of the target (235U). The reaction cross-sections have been studied by many authors [14-18] but, the production yields still have large uncertainties.

The existing facilities, and most of the projects under development, are known as first generation machines. The thermal power of the primary beam is limited to a few kW. Second generation machines like ISOSPIN [19] where the primary beam power will reach 100 kW are now proposed.

The intensity of the radioactive beam, which may be obtained at the source output, is usually described by the following equation:

$$I = \sigma. \phi. N. \varepsilon_1. \varepsilon_2. \varepsilon_3$$

where  $\sigma$  stands for the production cross section,  $\phi$  is the primary beam intensity, N the target thickness,  $\varepsilon_1$  the product release from the target and the transfer efficiency up to the source,  $\varepsilon_2$  the source efficiency and  $\varepsilon_3$  the delay transfer efficiency due to the radioactive decay losses.

The production cross section  $\sigma$  can vary over a very large range as shown in Fig 1 [20]. The usefull target thickness N has also large variations as it depends on the range of the primary particles in the target material.

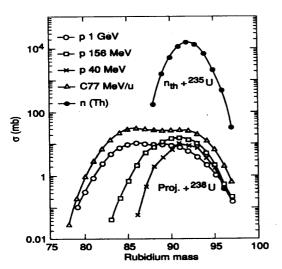


Fig 1. Production cross section of Rubidium isotopes

# 4.1 Target designs

The choice of the target material and its design is of great importance when designing a RIB factory. The best solution depends of the nature and energy of the primary beam as well as the radioactive beam expected.

Common features of the targets are the following: i) they must be able to sustain the primary beam thermal power, ii) they must have the best microscopic and macroscopic structure to release rapidly the radioactive species.

Point i) is quite obvious, one must nevertheless take into account the distribution of the power which, for the heavy ions has a very narrow peak (Bragg peak) producing high power densities.

SPIRAL will use a graphite conical target (fig 2) which allows a better distribution of the power [21].

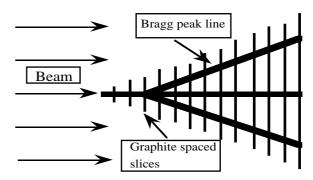


Fig 2. Conical graphite target proposed at SPIRAL.

Liquid metal targets are currently used at ISOLDE (CERN) [22-24]. The principle of a uranium liquid metal target is under evaluation at Argonne for the ATLAS project [25]. An original target concept is proposed at Argonne to sustain the 100 kW beam power of the second generation machines. As illustrated in Fig 3, the proposed method separates the primary target from the production one [26,27].

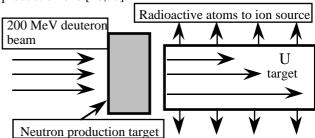


Fig. 3. Second generation targets principle

In this case, one can say that the neutrons act as a first-step secondary beam.

Point ii) is more tricky. The diffusion of atoms out of the target material is described by Fick's second diffusion law, it is favourised by the use of thin foils, wires or powder kept at a high temperature [28-30]. For a powder target with spherical grains of average radius R, the probability that an atom leaves the grain at a time t is [39,28]:

$$p(t) = \frac{6D}{R^2} \sum_{n=1}^{\infty} \exp \left[ \frac{-n^2 \pi^2 Dt}{R^2} \right]$$

D being the so called diffusion constant, which varies as a function of the nature of the target and the product and the temperature. For atoms having a lifetime  $T_m$ , shorter than the delay time, a reasonable estimation of  $\varepsilon_3$ , the delay transfer efficiency is [29]:

$$\varepsilon_3 = \frac{3\sqrt{DT_m}}{R}$$

The temperature plays an important role as shown by the Frenkel equation which describes the residence time  $\tau$  of the atoms on a surface [31]:

$$\tau = \tau_0 \exp[H_{ad}/kT]$$

( $Ha_d$  is the heat of adsorption of the atom on the surface, k the Boltzmann's constant, T the temperature and  $\tau_0$  a constant  $\approx 10^{-13}$ s).

## 4.2 Ion sources

Many type of ion sources are used for the RIB's production: Plasma Sputter, Negative or Positive Surface Ionisation, Electron Beam Plasma, Electron-Cyclotron Resonance [29-31]. All of them except for the ECR's produce mono-charged ions. The target and source assembly must be as compact as possible in order to avoid long delays in the transfer tube. For the production of ions of condensable material, the whole assembly must be kept at high temperature in order to avoid atoms sticking to the surfaces. Important considerations are the simplicity, reliability and cost of the assembly as its lifetime is limited and as it must be remotely handled after use being very active. Permanent magnets are usefull components to create the magnetic fields as they do not require power supplies and connections, but it was shown at SPIRAL that their magnetisation can be rapidly destroyed by the high flux of energetic neutrons. An interesting concept of laser source was developed in several places [32], it takes advantage of the stepwise resonant laser ionisation process to produce highly selected ions.

An other exciting concept to get multi-charged ions, is to have a very simple one-charge source near the target and to transform, later on, the mono-charged beam onto a multi-charged one by means of an EBIS [33] or an ECR. Promising results were recently obtained at Grenoble where an efficiency of 3% was obtained with the on-line ionisation  $Rb^{1+}$ ->  $Rb^{9+}$  through an ECR [34].

## 4.3 Fragment separator

Tens or more different species are simultaneously produced in the target. some of them with a much greater yield than the expected one. A first selection is operated by the source. The selectivity can be high (Resonant Laser Ion Source, Neg. Surface Ion Source) or poor (other sources). The worse situation is with the multicharged ion sources where the q/m combinations result in a large number of ions having nearly the same magnetic rigidity. It is of the utmost importance to do a first separation at the source output in order to avoid transporting away possibly highly active beams (without forgetting the space charge problems). For a one charge ion source, a simple magnet can provide sufficient resolution (200-400) to eliminate all but the isobaric parasitic beams. The separation of the isobars require a much larger resolving power (20000-40000). High mass and energy resolution analysers were proposed [35] and built, or are under construction in many places. These two-stage separators benefit from an intermediate acceleration that allows the elimination of all the ions scattered by the residual gas or by the edges of the analysing slits.

An other separator, LAMS, is proposed at SPIRAL [36] which, using a bunched beam, transforms the mass variations into time of flight differences and then, using a debuncher, in energy differences allowing separation in a small magnet. The calculated resolving power of LAMS is 2000 but the separation cannot be complete due to tails between the bunches.

# 4.4 Beam identification

The species must be identified at some point during the acceleration process. The usual electromagnetic separation methods are usually not sufficient to do that. At SPIRAL, we have chosen to use nuclear detection methods [37].

The beam extracted from the source at a few tens of kV will be first analysed by a dipole with a moderate resolution of about 200. The analysed beam will then be periodically deviated towards the identification station. The method varies with the disintegration channel of the ion. The simplest case is for short lived nuclei emitting a well identified γ ray (ex: <sup>18</sup>Ne). In this case, a simple Ge junction, followed by a counting chain is sufficient to identify the nuclei and measure the intensity. If the identification of the  $\gamma$  ray is ambiguous (ex:  $^{29}\text{Al},\,\text{E}\gamma\text{=-}1$ 273 keV,  $T_{1/2}$ =6.6 mn but also <sup>29</sup>P, E $\gamma$ =1 273 keV,  $T_{1/2}$ =4.1 s), one will have to use a chopper that will allow the lifetime to be measured. If the period is longer (ex:  $^{13}N$ ,,  $T_{1/2}$ =9.96 mn), it will be necessary to collect the ions on a tape and transport the activity up to the Ge detector which will be placed in a shielded area. Some nuclei emit only  $\beta$ - particles (ex <sup>6</sup>He) which will be seen by a plastic detector and a photo-multiplier, others are proton emitters which will need a silicon detector.

The identification being specific, the detection must be flexible enough to be adapted to the desired nuclei.

## 4.5 Acceleration and diagnostics

When the beam is very weak, the "classical" diagnostics are inefficient to proceed with the usual settings of the beam lines' magnets and the post-accelerator.

The method which is currently used at Louvain-la-Neuve [38] is to produce a stable but abundant beam of very near g/m species (ex:  $^{18}$ O and  $^{18}$ F,  $\Delta$ m/m=9.9  $^{10^{-5}}$ ) which is then used to tune the beam line and the cyclotron. Once the cyclotron is well tuned with the stable beam, a shift in the magnetic field of  $\Delta B/B=$  $-\Delta m/m$  allows one to obtain the settings for the radioactive beam. An additional reduction of the accelerating voltage allows one to attain a complete isobaric separation by increasing the number of turns (For a cyclotron,  $R \sim 2\pi HN$ , H being the harmonic number and N the number of turns). The cost to pay is a reduction of the extraction efficiency which falls by a factor of 10 or more. A similar process is proposed to tune SPIRAL, nevertheless, stable ions with very near q/m values do not always exist. As soon as the step becomes noticeable, the method becomes questionable as the field variation is not simply equal to the current variation. The RF frequency may then be changed, the isochronism is not strictly conserved in the cyclotron and the buncher phases must be retuned.

As soon as the energy of the beam becomes sufficient, nuclear detection methods can be once more succesfully applied as diagnostic tools. An experiment was recently made at GANIL with a plastic scintillator handled by the main radial probe of the CSS2. With the help of an external P.M. and the associated electronics, it was possible to tune very precisely the field of the CSS with beams of a few ions/s. The detection threshold was less than 40 MeV in total and can certainly be improved. The lifetime of the scintillator is limited, but it is not a problem to frequently change it.

## 4.6 Activation and contamination

The production target is obviously a very contaminating object, especially if it contains uranium and fission products. The most important part of the contamination will be trapped in the target and in the source which will become very active after some days of use. The gaseous part of the contamination will be concentrated in the pumping system of the target/source assembly. The oil of the primary pumps will be activated and treated as nuclear wastes. The exhaust gas emitted by the pumps must be collected in balloons and stored long enough to permit a sufficient reduction of the activity.

It is nearly impossible to prevent the contamination due to neutrals from propagating slowly along the beam line and the accelerator tube. After a long running time, this can result of highly contaminated parts of the accelerator complex.

already mentioned, another way of activation/contamination is the propagation of avtivity by the beam itself and particularly by the parasitic beams. If the first analysis is not sufficient to stop all of them, as it is usually the case for existing installations, the contamination will propagate up to other parts of the accelerator such as, in the case of a cyclotron, the injection and the extraction elements.

### 5 CONCLUSION

This rapid survey of RIB's production and acceleration methods shows that these beams open new fields of very active research. The demand for medium to high energy beams opens for the accelerator physicist new and challenging ideas in the design of the accelerators and of the beam handling systems. The nuclear detection allows the control of weak beams of very rare elements.

The design of the second generation facilities shows a rapid evolution of the domain, promising that very exotic beams will be soon available.

Nearly all the primary beams can be used, but they are not equivalent. There are domains of the nuclei chart where one approach is better than the others. Consequently, one can expect a specialisation of the existing and future facilities.

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