Suppression, persistence and reentrance of superfluidity in overflowing nuclear systems

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Based on a microscopic description of superfluidity in overflowing nuclear systems, it is shown that continuum coupling plays an important role in the suppression, the persistence and the reentrance of pairing. In such systems, the structure of the drip-line nucleus determines the suppression and the persistence of superfluidity. The reentrance of pairing with increasing temperature leads to additional critical temperatures between the normal and superfluid phases.

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Overflowing many-body fermionic systems exist in various situations going from the crust of neutron stars [1, 2] to ultra-cold atoms [3, 4]. Interestingly, these systems offer the possibility to study the coupling between two fluids with very different pairing properties [5, 6]. In such systems, one fluid is localized inside an initial container, such as for instance a nuclear potential, and a second fluid is overflowing towards a larger container. Being in different environments, these two fluids can acquire different pairing gaps. In this paper, we address the question of the coupling between the two superfluids and their finite temperature properties.

At the transition between its outer and inner crust, neutron stars provide an example of such microscopic overflowing systems, commonly called neutron dripping [1]. In the outer crust, nuclei form a Coulomb lattice which gets more and more neutron rich as the density increases. When the maximum number of neutrons that a nucleus can sustain is reached, the overproduced neutrons drip out of nuclei. These neutrons populate the continuum states and shall be described within the band theory (see Refs. [7, 8] and references therein). In the present work, we shall consider the spherical Wigner-Seitz approximation to the band theory which limitations are discussed for instance in Ref. [8]. It should be noted that nuclei surrounded by an infinite neutron gas could exist as a stable configuration in neutron stars where they are bounded by gravitation, while isolated nuclei that exist for instance on earth are limited to the drip lines.

The first prediction of the suppression of pairing in overflowing $Z = 50$ nuclear systems was performed in Ref. [5]. It was proposed to attribute this suppression to the large coherence length of the weakly-superfluid neutrons gas: the neutron gas can penetrate the dense nuclear system and could impose its weak pairing field. It was however also noted in the same work that neutrons are dripping out of the double-magic nucleus ^{176}Sn. In a recent work, the pairing gap of the last occupied state was also predicted to be quenched in overflowing nuclear systems, metallic grains and cold-atoms [6]. It was concluded that the suppression of superfluidity is a generic fact of a fermionic superfluid overflowing from a narrow container into a much wider one. It shall be noted that in this work the nuclear model was mainly limited to ℓ=0 single particle states (s-states). The description of nuclear systems requires however the inclusion of $\ell > 0$ single particle states for the bound and the mean-field resonance states. It is indeed well known that, close to the drip lines, resonance states play an important role for pairing properties and it is referred generally as continuum coupling [9, 10]. It has recently regained some attention in nuclei close to the drip-line (see for instance Refs. [11, 12] and references therein). The general question of continuum coupling in overflowing superfluid systems and various phenomenons such as persistence, suppression and reentrance of pairing remains to be studied. Pairing correlations in the ground state of weakly-bound nuclei are commonly described by the Hartree-Fock-Bogoliubov (HFB) theory [13, 14]. In most of the HFB calculations the continuum is discretized by solving the HFB equations with box boundary conditions [15]. In this paper a systematic analysis based on several overflowing isotopic chains is performed and it is shown that pairing quenching and continuum coupling are strongly related. A pairing reentrance phenomenon with increasing temperature is predicted for $Z = 50$ overflowing systems.

In the present work, an HFB approach in coordinate representation is employed. This model has already been applied to describe nuclei and Wigner-Seitz cells in a fully self-consistent framework (see Ref. [16] and references therein). The Skyrme SLy4 interaction [17] is used in the mean-field channel, and is completed by the ISS pairing force which is adjusted to the BCS pairing gap predicted by bare nucleon-nucleon potentials [16]. All the bound states are considered, the angular momentum goes up to $J_{max} = 27/2$, and we take a large box radius R_{box}=27.4 fm ensuring an adequate description of the continuum. An almost constant neutron density at the edge of the Wigner-Seitz cell is obtained using mixed Dirichlet-Von Neumann boundary conditions. The expected nuclei in neutron stars located at the transition between the outer crust and the inner crust have a proton number Z around 30 to 50, depending on the models [18]. We therefore selected 8 isotopes given in Table I and located around Z=28, 40 and 50. It is well adapted to perform a spherical HFB calculation since these nuclei are predicted spherical near the neutron drip-line from a de-

Isotope	Z	N_{drip}	group	N_{res}
Ni	28	60	$\mathcal{A}_1$	3.0
Kr	36	82	$\mathcal{A}_2$	0.0
Sr	38	82	$\mathcal{A}_2$	0.0
Zr	40	84	$\mathcal{A}_1$	2.2
Mo	42	90	$\mathcal{A}_1$	8.0
Ru	44	92	$\mathcal{A}_1$	3.0
Sn	50	126	$\mathcal{A}_2$	0.0
Te	52	126	$\mathcal{A}_2$	0.0

TABLE I: Isotope acronym, number of protons Z, number of neutrons of the last nucleus before the drip line N_{drip}, for the selected set. Isotopes for which the drip-line nucleus is non-magic (magic) belongs to the group $\mathcal{A}_1$ ($\mathcal{A}_2$, respectively). The total occupation number of resonance states for the drip-line nuclei N_{res} is shown in the last column (see the text for more details).

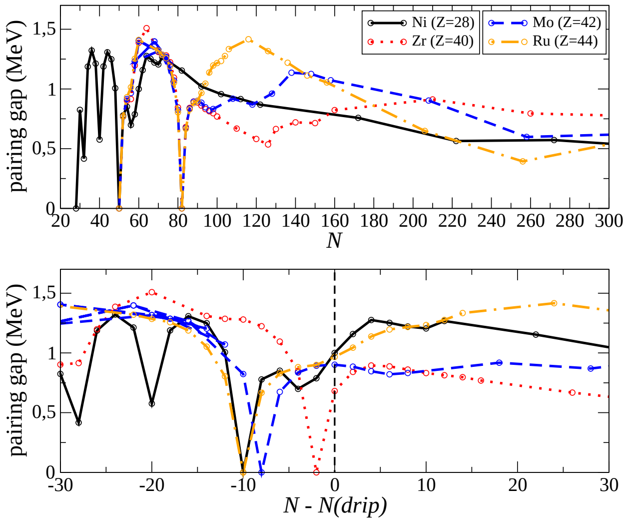

FIG. 1: (color online) Neutron pairing gaps versus neutron density N (top panel) and versus $N - N(drip)$ (bottom panel) for the isotopes in the group $\mathcal{A}_1$ (see the text for more details).

formed HFB model based on the Gogny interaction [19].

For these selected isotopes, the neutron-drip number N_{drip}, defined as the neutron number of the last nucleus before the two-neutrons separation energy $S_{2n}(N) = E(N) - E(N-1)$ changes its sign [14], is given in table I. The total neutron occupation number of resonance states N_{res} for the drip-line nuclei is also given. The calculation of N_{res} requires the identification of resonance states which, in our case, are defined as positive energy states with rms radii lower than half the size of the box. It is interesting to remark from Table I that the drip-line nuclei with a number of neutrons that coincides with the well-known magic numbers (82 and 126) have no states in the continuum. The shell occupation in this case is 0 or 1, as in magic nuclei. We therefore group Kr, Sr, Sn and Te isotopes in the same group $\mathcal{A}_2$. The persistence of magicity with the same magic numbers as in stable nuclei might be due to the sphericity of these nuclei. This is a well known stabilization effect of magicity in nuclear structure [14]. In the case of Ni, Zr, Mo and Ru, the drip-line occurs in more complicated shell structure partially involving resonance states and these isotopes are grouped in $\mathcal{A}_1$.

The drip-line isotopes identified in Table I can be considered as the seed nuclei from which the overflow occurs. The dripping of neutrons shall therefore be influenced by the microscopic structure of these seed nuclei. Nuclei belonging to the group $\mathcal{A}_1$ have a resonance occupation number N_{res} which goes from 2.2 to 8.0 particles. This number reveals the large continuum coupling. The number of particles in the resonance states is zero for nuclei belonging to the group $\mathcal{A}_2$, since in the magic nuclei located at the drip-line, pairing correlations are quenched and continuum coupling therefore largely suppressed. From a microscopic analysis, the structure of drip-line nuclei is qualitatively different between the group $\mathcal{A}_1$ and $\mathcal{A}_2$. In the following, we show how these differences impact the quenching of pairing correlations at overflowing.

Figs. 1 and 2 display the average neutron pairing gaps versus the neutron number (top panel) and the difference between the neutron number and the neutron drip $N - N_{drip}$ (bottom panel). The average neutron pairing gap is obtained from the neutron pairing field given by the HFB solution and is defined as,

$$\Delta \equiv \frac{\int d^3r \Delta(r) \kappa(r)}{\int d^3r \kappa(r)}, \qquad (1)$$

where $\Delta(r)$ and $\kappa(r)$ are the pairing field and pairing tensor obtained from the solution of the finite temperature HFB model [2, 28]. Fig. 1 represents the neutron pairing gaps for the $\mathcal{A}_1$ isotopes while in Fig. 2 are shown the neutron pairing gaps for the $\mathcal{A}_2$ isotopes. In the case of nuclei from the $\mathcal{A}_1$ group, the neutron pairing gap persists beyond the drip line (see bottom panel of Fig. 1). The continuum coupling preserves the pairing diffusivity around the Fermi energy: the occupancy of the scattering states beyond the drip-line does not suppress superfuidity. The presence of an overflowing neutron gas has therefore a limited effect on the pairing gap for the $\mathcal{A}_1$ isotopes, showing the persistence of superfluidity. In the case of the $\mathcal{A}_2$ isotopes, a suppression of the average pairing gap just beyond the drip line is observed (Fig. 2). The presence of a magic nucleus at the drip line have therefore a strong influence on the pairing gap at and beyond the drip line: the large spacing between the last occupied state and the first excited state (resonance state) suppresses superfluidity. Figs. 1 and 2 provide a clear illustration that the shell structure of the drip-line nucleus determines the suppression or the persistence of

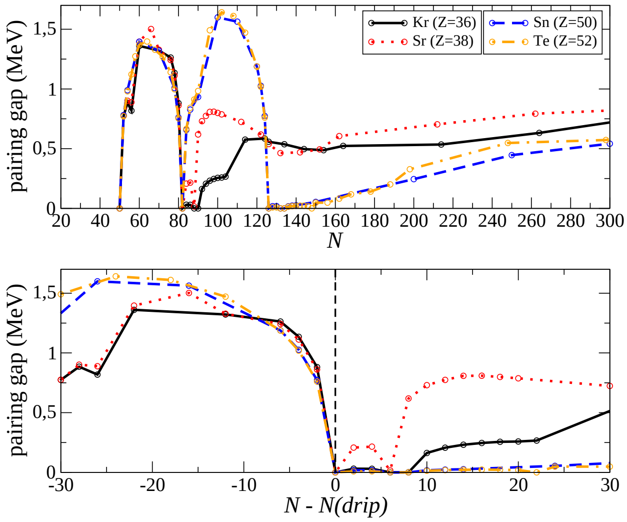

FIG. 2: (color online) Same as Fig. 1 for isotopes in the $\mathcal{A}_2$ group.

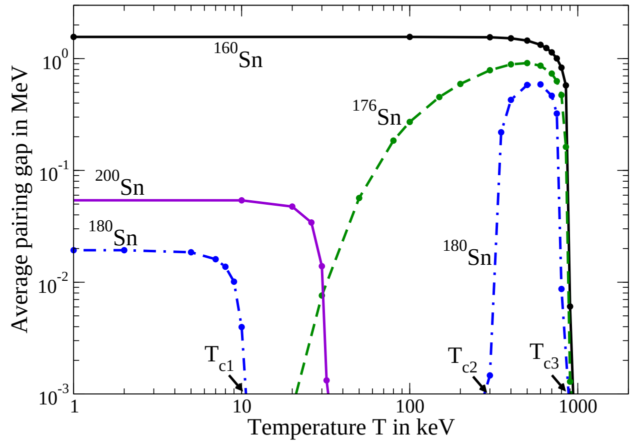

FIG. 3: (Color online) Temperature averaged neutron pairing gap versus temperature for ^{160,176,180,200}Sn. A superfluid reentrant effect is observed for ^{176}Sn and ^{180}Sn.

superfluidity, even in overflowing nuclear systems.

In Ref. [6], the quenching of pairing in overflowing systems has been predicted upon overflow of trapped fermions. It should be underlined that in the present work, the mean-field in nuclear systems have a centrifugal term which gives rise to the mean-field resonance states and continuum coupling, whereas the conclusions obtained in Ref. [6] are applicable only to overflowing systems involving $\ell = 0$ single particle states. In more general systems, like in the crust of neutron stars, a more sophisticated theoretical model such as the present HFB one predicts a suppression of superfluidity only if the continuum coupling is quenched, by the shell structure for instance. It shall however be mentioned that for the description of pairing in nuclei close to the transition point the HFB model shall be complemented by particle-projection methods.

The persistence of superfluidity upon overflow of bound neutrons might not be the only consequence of continuum coupling: in the case where pairing is suppressed, the increase of temperature may generate the reentrance of superfluidity. The finite-temperature HFB model [2, 28], is employed to study the reentrance of pairing in the thermal state of overflowing even-even nuclear systems. The temperature-averaged pairing gap for ^{160,176,180,200}Sn is shown in Fig. 3. In ^{160}Sn and ^{200}Sn it behaves as expected from HFB theory: the pairing gap vanishes at the critical temperature $T_c = 0.57\Delta(T \approx 0)$ (see Ref. [29] and references therein). In the case of ^{176}Sn and ^{180}Sn, the reentrance of superfluidity is observed with increasing temperature. This reentrance is induced by the presence of resonances states in the spectrum of these nuclear systems: Being slightly too high in energy, these states are not occupied at zero temperature (see Table I), while at finite temperature, they can be partially occupied from the Fermi-Dirac distribution. At low temperature, the pairing correlations can therefore be switched on allowing the reapparition of the superfluid state. The reentrance critical temperature depends on the step in energy between the last occupied bound state and the first resonance one, which changes from one system to another, as observed in Fig. 3 for ^{176}Sn and ^{180}Sn. In the case of ^{200}Sn, this energy step is too large to give rise to the reentrance of superfluidity before the highest critical temperature is reached. The ^{180}Sn overflowing system has an interesting phase diagram including three critical temperatures: with increasing temperature, two of them correspond to the vanishing of superfluidity ($T_{c1} \sim 11$ keV and $T_{c3} \sim 1$ MeV) and one to its reappearance ($T_{c2} \sim 300$ keV). The lowest critical temperature T_{c1} is associated to the transition from the superfluid to the normal state in the overflowing neutron gas. The highest critical temperature T_{c3} is similar for ^{160,176,180}Sn, indicating that superfluidity has been restored in the seed nucleus of ^{176,180}Sn between T_{c2} and T_{c3}. This superfluidity is mainly built on resonances populated at finite temperature. More generally, pairing reentrance in hot systems is observed for isotopes belonging to the group $\mathcal{A}_2$ where resonances are too high in energy to participate to pairing at zero temperature but close enough to the last occupied state to be reached at finite temperature.

Reentrance of superfluidity at finite temperature have been predicted in nuclear systems such as in odd-nuclei [20], rotational motion of nuclei [21], and the deuteron pairing channel in asymmetric infinite matter [22]. It was aslo predicted in polarised ^{3,4}He [23–25] and in spin asymmetric cold atom gas [26, 27]. In all these systems, pairing at zero temperature is generated by an attraction among Fermions of different spin or isospin. Superfluidity is therefore maximum in spin or isospin symmetric systems for which there is a matching of the Fermi levels of the constituent Cooper-pairs.

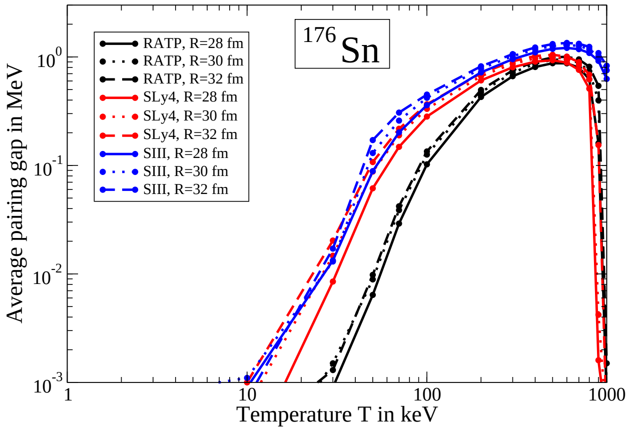

FIG. 4: (Color online) Temperature averaged neutron pairing gap versus temperature for ^{176}Sn and for several box radii and for three Skyrme interaction (RATP, SLy4 and SIII). See the text for more details.

Breaking the spin or isospin symmetry disfavor pairing while temperature in asymmetric systems acts in favor of restoring the broken symmetry and can eventually induce a reentrance of pairing. In the present case, the condition for pairing to be analyzed is related to the equal amount of occupied and unoccupied states. At shell closure, this symmetry is maximally broken. The reentrance phenomenon analyzed in this paper is based on the restoration of this condition for pairing at finite temperature where resonance states play a major role.

Now we analyze the stability of the reentrance prediction and we focus on the prediction for ^{176}Sn. The dependence on the Skyrme interaction is first analyzed. Among various Skyrme interactions (SLy4, RATP, SKI5, SKM*, SGII, SIII), we selected the ones that predict ^{176}Sn at the drip-line (group $\mathcal{A}_2$), that are: SLy4, RATP, SKI5 and SIII. We also found that SLy4 and RATP a f7/2 resonance state at low-energy (between 1 and 2 MeV), SIII a p3/2 state around 1 MeV while and the lower-energy resonance state with SKI5 is found above 4 MeV. We found that SLy4, RATP and SIII Skyrme interactions predict a reentrance phenomenon in ^{176}Sn, as illustrated on Fig. 4, while SKI5 does not. We also show on Fig. 4 that all the interactions having a low-energy resonance in the window 0 to 2 MeV above the Fermi energy (RATP, SIII and SLy4) behave similarly, confirming our previous conclusions that the reentrance phenomenon is related to the vicinity of resonance states. The dependence on the choice of the Skyrme interaction (RATP, SLy4 and SIII) is also shown. The size of the box have also been varied from 28 to 30 fm, and the effect is shown to be small on Fig. 4.

In summary, we have investigated the pairing properties of nuclear systems upon overflowing superfluid neutrons. Suppression, persistence and reentrance of superfluidity can occur in these finite systems. From a systematic HFB calculations on 8 isotopic chains, the pairing properties is shown to be strongly correlated to the continuum coupling, both at zero and finite temperature. At zero temperature, the coupling between the seed nucleus and the gas is weak, and a formal separation of their properties into a nucleus plus a gas provides a qualitative understanding of the suppression and the persistence of superfluidity. With increasing temperature in the normal state, the Fermi-Dirac distribution can populate the resonance states giving rise to the reentrance of superfluidity. The pairing correlations in the nuclear system are switched on again and consecutive critical temperatures are predicted.

The understanding of the suppression, persistence and reentrance of superfluidity in nuclear systems, deeply related to the continuum coupling, opens wide perspectives for discoveries in weakly bound nuclei, as well as it sheds new light on the transition between the inner and outer crusts in neutron stars. The role of resonances around the neutron drip not only changes the microscopic understanding of the neutron drip-out mechanism, but it also modifies the thermal properties of the crust through the strength of the pairing interaction [30]. The temperatures at work during cooling are typically of the order of 10 to 500 keV [1] and coincide with the critical temperatures of the reentrance phenomenon. As a consequence, the novel pairing reentrance phenomenon analyzed in this paper is expected to modify the thermodynamical and cooling properties in the crust of neutron stars. The links with other superfluid Fermi-systems shall be investigated in the near future. For instance in cold Fermionic atoms overflowing from an inner trap to a larger one [3, 4], it is known that stable path close to the centrifugal energy are the classical analog of the quantal resonances. It will therefore be interesting to investigate the role of these stable paths on the superfluid properties of overflowing cold Fermionic atoms.

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