Group Theory in a Nutshell for Physicists

P. van Isacker

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A. Zee

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Many books have been written about group theory's applications to physics. Some have an arid, mathematically rigorous style that often obscures physical insight. Other, less formal presentations usually cannot deliver the necessary know-how for practical applications. In *Group Theory in a Nutshell for Physicists*, Anthony Zee, a physicist at the University of California, Santa Barbara, combines clarity of presentation with mathematical detail at a level of rigor acceptable to physicists. The result is a tour de force that guides readers through the universe of group theory and leads them to recent applications in particle physics, cosmology, and condensed matter.

The book is unique in its laid-back presentation. It is peppered with colorful stories about famous mathematicians and physicists and includes frequent interjections from fictitious characters. Particularly helpful are the mutterings of Dr. Feeling, who supplies intuitive understandings of formal definitions or theorems, and the observations by Confusio, who (not surprisingly) points out issues of possible confusion. The book is ideally suited to accompany a graduate course on symmetries in physics because of its pedagogical approach, the detail of its illustrative examples, and its many exercises. Readers need to be familiar with the basics of quantum mechanics, but little other advance knowledge is required since the

book starts with a brief review of linear algebra and a reminder of the properties of matrices.

After its mathematical refresher, the book turns to a detailed presentation of the representation theory of finite groups and the introduction of Lie groups. Early on, Zee introduces Lie algebras by way of three-dimensional rotations; the classification of those algebras by roots, weights, and Dynkin diagrams comes later. The book's mathematically detailed material is interspersed with group theoretical applications to physical systems. Given the author's distinguished career in particle physics, it is not surprising that most of the examples come from that field, but Zee occasionally ventures out to other areas with examples relevant to condensed-matter and atomic physics.

The book makes only a single mention of group theory applied to atomic nuclei (my field of expertise), and that appears as a footnote when Zee discusses the Elliott model. James Philip Elliott's application of $SU(3)$ is admittedly of less fundamental importance than the use of group theory in particle physics, but the mathematics behind Elliott's application is more sophisticated. After reading about how the finest minds in the particle-physics community struggled to get the eightfold way right, I can only admire Elliott's achievement even more, as he developed the $SU(3)$ model of nuclei essentially by himself.

The book does not comprehensively discuss the representation theory of the symmetric group of permutations, and the author even advises readers to stay clear of the diagrammatic machinery of Young tableaux. That may be sensible advice when one is dealing with low-dimensional representations. However, as the dimension

of the representation increases, as is the case, for example, in quantum many-body physics, Zee's treatment in terms of either totally symmetric or totally antisymmetric tensors rapidly becomes cumbersome, and Young tableaux are called for.

Eugene Wigner, who introduced group theory into quantum mechanics and is therefore one of the heroes of the book, famously wrote about the "unreasonable effectiveness" of mathematics. In the final chapters of his text, Zee forcefully makes the case for the unreasonable effectiveness of group theory and buttresses his case with many compelling examples. Group theory can generate everything from the Dirac equation for the electron to the equations that describe the expanding universe. Indeed, all known particles can be unified within the framework of the Lie group $SU(5)$.

With *Group Theory in a Nutshell for Physicists*, Zee convincingly demonstrates that group theory governs the physical universe, and he gives aspiring physicists the tools to understand its applications to their work.

Piet Van Isacker
Large Heavy Ion Accelerator (GANIL)
Caen, France

Piet Van Isacker is a researcher with the French Alternative Energies and Atomic Energy Commission (CEA), working at France's Large Heavy Ion Accelerator (GANIL). In his research he applies group theoretical methods to study quantum mechanical many-body systems, in particular atomic nuclei.