

SIMULTANEOUS RECONSTRUCTION AND SEPARATION IN A SPECTRAL CT FRAMEWORK

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► **To cite this version:**

S. Tairi, S. Anthoine, C. Morel, Y Boursier. SIMULTANEOUS RECONSTRUCTION AND SEPARATION IN A SPECTRAL CT FRAMEWORK. ITWIST 2016, Aug 2016, Aalborg, Denmark. <<http://www.itwist16.es.aau.dk/>>. <in2p3-01427020>

HAL Id: in2p3-01427020

<http://hal.in2p3.fr/in2p3-01427020>

Submitted on 5 Jan 2017

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INTRODUCTION

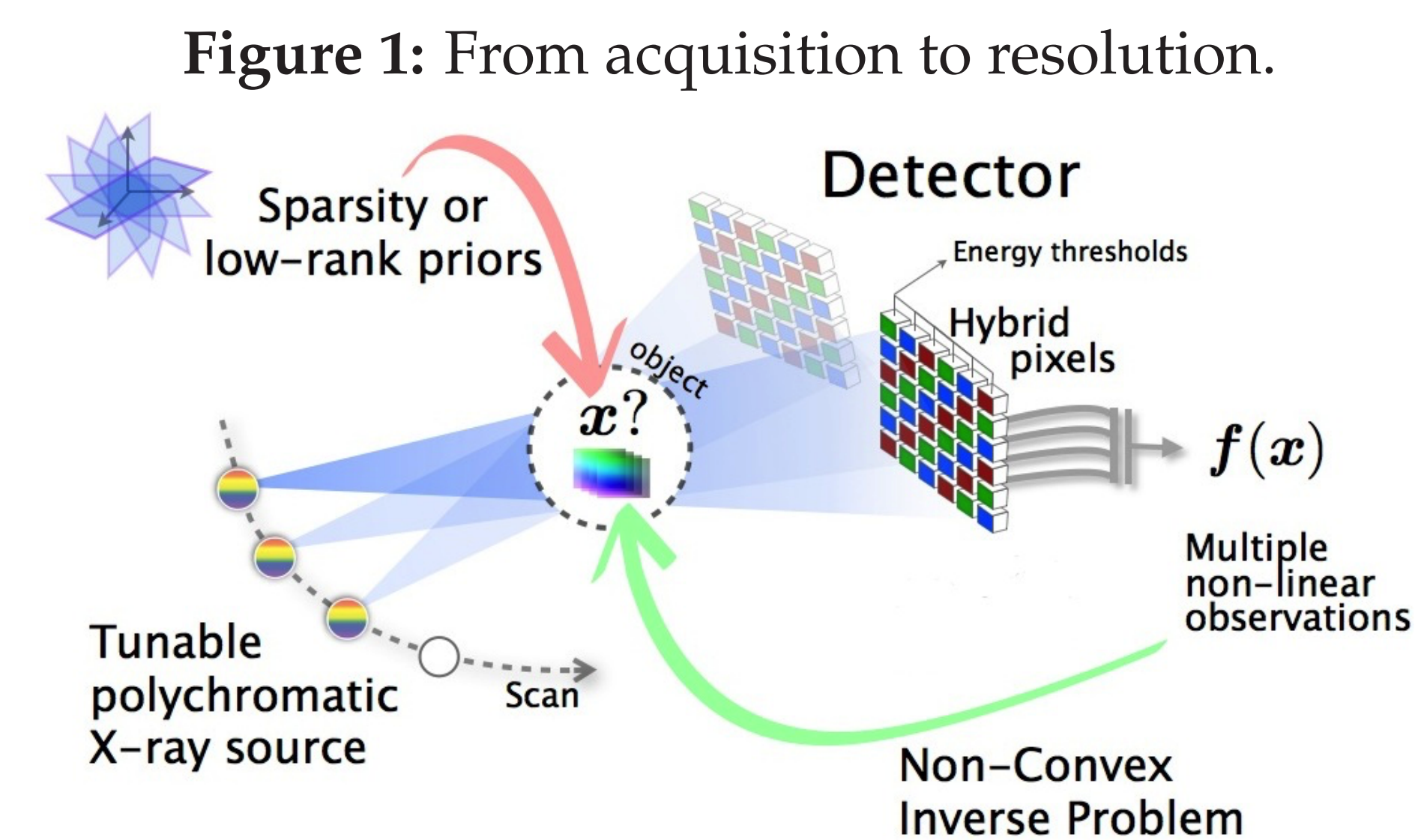
The advent of new X-ray detection technology by hybrid pixel cameras working in a photon-counting mode paves the way to the development of **spectral CT**.

This allows to simultaneously separate and reconstruct the physical components of an object and finds applications in many areas of imaging.

Our framework allows to iteratively reconstruct an image from a spectral CT data by setting a **polychromatic model** that encompasses constraints (positivity, sparsity) and solving an **ill-posed inverse and non-convex problem**.

Preliminary results are obtained on simulated images containing four elements (water, Iodine, Yttrium and Silver).

trium and Silver).



FORWARD MODEL

Acquisition

A Computerized-Tomography (CT) scan is obtained by shining a X-Ray light modulated by metallic filters through a rotating object. **In the spectral setting, one explicitly exploits the spectral polychromaticity of the source attenuated through different metallic filters.**

The Beer-Lambert law governs the measurements:

$$y^m = \int_{\mathbb{R}^+} f^m(E) e^{-\int_{\mathcal{L}^p} \mu(l, E) dl} dE \quad (1)$$

where $f^m(E) = I_0(E)F_i^q(E)D^r(E)$ denotes the total spectral inputs:

- $\mu(l, E)$: absorption coefficients of the object for l on the line of sight \mathcal{L}^p (to be found),
- I_0 : X-ray source's intensity energy spectrum,
- F_i : filter's attenuation energy spectrum,
- D : detector's efficiency energy spectrum.

Absorption maps model

The absorption maps are naturally the sums of the contributions of each of their components; moreover the **spectral signature is physically independent of the spatial location** of a component:

$$\mu(l, E) = \sum_{k=1}^K a^k(l) \sigma^k(E), \quad (2)$$

with $a^k(l)$ the concentration of component k at point l , and $\sigma^k(E)$ its interaction cross section. Eq. (1) now reads:

$$y^m = \int_{\mathbb{R}^+} f^m(E) e^{-\sum_{k=1}^K \sigma^k(E) \int_{\mathcal{L}^m} a^k(l) dl} dE \quad (3)$$

Discretized forward model

The energy E is discretized in N bins, and the 3D-volume where the object lives in D voxels. The forward discretized model reads:

$$Y = (F \odot e^{-SA\Sigma}) \mathbb{1}_N, \quad (4)$$

- $Y \in \mathbb{R}^M$: discretized measured data;
- $F \in \mathbb{R}^{M \times N}$: dictionary of energy modulating filters;
- $S \in \mathbb{R}^{M \times D}$: X-ray Transform operator;
- $A \in \mathbb{R}^{D \times K}$: concentration matrix: $A[d, k] = a_k(v_d)$;
- $\Sigma = (\sigma_1(\cdot), \dots, \sigma_K(\cdot))^T \in \mathbb{R}^{K \times N}$: dictionary of the interaction cross sections of the K components: $\Sigma[k, n] = \sigma_k(E_n)$.

($\mathbb{1}_N = (1, \dots, 1)^T$ and \odot is the Hadamard product.)

METHOD

The measurements Y are noisy realizations of the perfect measurement described by Eq. (4). The inverse problem of recovering A , the matrix containing the concentration coefficient maps of the K components of the object, is solved by minimizing:

$$J(A) = D(Y, (F \odot e^{-SA\Sigma}) \mathbb{1}_N) + R(A) \quad (5)$$

with

- $D(Y, Z)$ a discrepancy measure (negative log-likelihood for Gaussian or Poisson noise),
- $R(A)$ a regularization term that models our a priori on A .

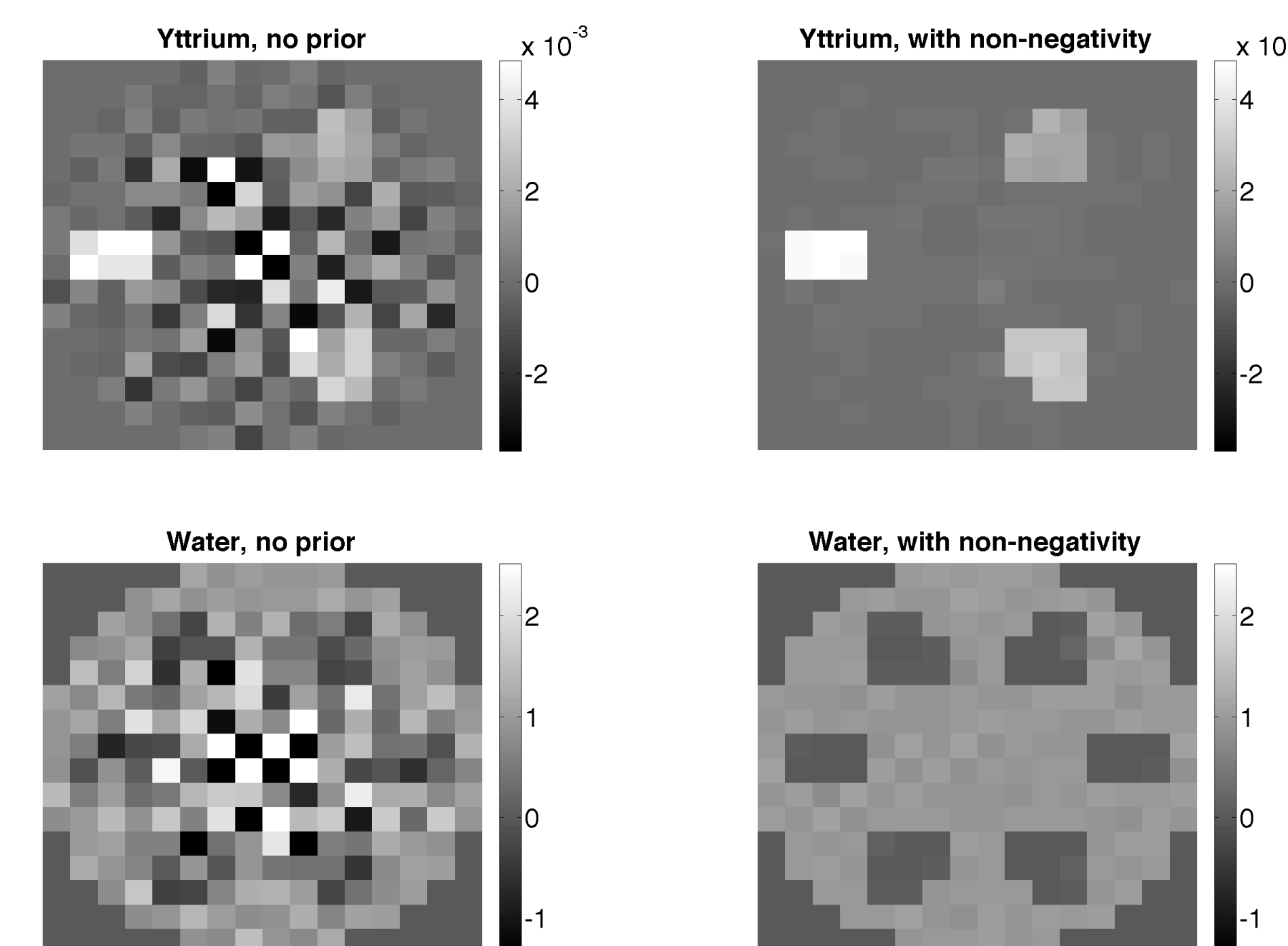
With $R(A)$ the non-negativity constraint, Eq. (5) is minimized with a classical trust-region algorithm

Trust-Region Algorithm

Require: A_0, Δ_0
 for $k = 1, 2, 3..$ do
 $m_k(p) = J(A_k) + g_k^T p + \frac{1}{2} p^T B_k p$ ▷ Quadratic model.
 $p_k \leftarrow \text{ArgMin}_{\|p\| < \Delta_k} m_k(p)$ ▷ Step calculation.
 $R_k \leftarrow \frac{f(X_k) - f(X_k + p_k)}{m_k(0) - m_k(p_k)}$ ▷ Ratio actual/predicted reduction.
 if R_k acceptable then
 $A_{k+1} \leftarrow A_k + p_k$
 Update Δ_{k+1}
 else
 Reduce Δ_{k+1}
 end if
 end for
 return A_k

RESULTS

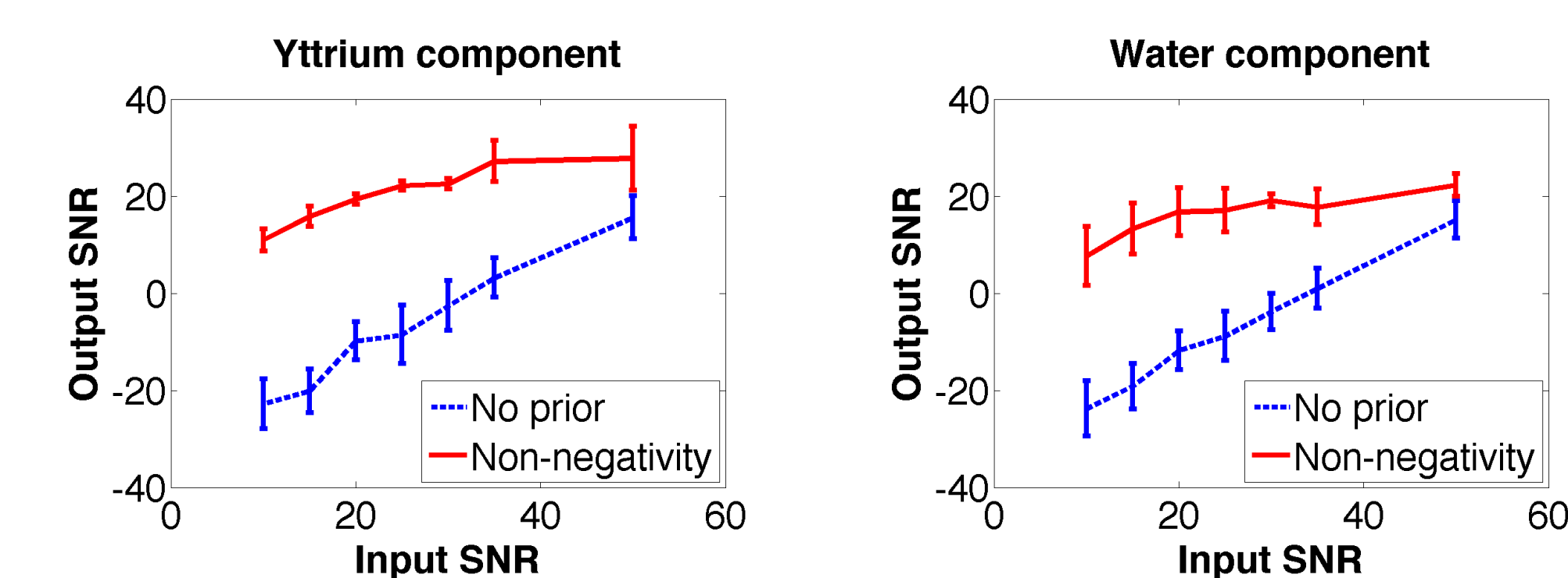
We have generated a **contrast phantom** made of one large cylinder filled with water and six smaller tubes filled with contrast agents. Three tubes contain Yttrium at different concentrations, two contain Silver and one contains Iodine ($K = 4$).



We have simulated a set of tomographic scans

smeared by Gaussian noise with **5 different metallic filters** F_i , which are ideal pass-band filters around the discontinuities specifying the contrast agents. The discretized sizes are $M = 1440$, $D = 256$, and $N = 43$.

We have then minimized Eq. (5) with the non-negativity constraint using a trust-region algorithm and report the results obtained for 10 realizations of noise. The figures show that the low-rank model allows to reconstruct simple maps from non-linear polychromatic measurements. The output SNR evolves linearly with the input SNR.



CONCLUSION

We have established a new flexible model for spectral CT reconstruction. Results obtained with a classical Trust-Region approach on simulated data

pave the way to future developments including sparsity constraints.