Geometric and Algebraic Descriptions of a Soft Scissors Mode

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Geometric and Algebraic Descriptions of a Soft Scissors Mode

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Abstract. Some of the elementary properties of the scissors mode in deformed nuclei are recalled. The effect of a neutron skin on this excitation mode is investigated. It is shown, in the context of a geometric as well as an algebraic model, that the development of a neutron skin can give rise to an additional low-energy mode involving out-of-phase angular oscillations of the skin neutrons against the remaining nucleons of the core. This 'soft' scissors mode represents the analogue of the soft dipole mode already postulated to occur in neutron-rich nuclei.

1. The Scissors Mode

In 1984 an inelastic experiment on ^{156}Gd showed a strongly dipole-excited state around 3 MeV [1]. Subsequent experiments (for an overview, see e.g. [2]) confirmed the existence and the orbital character of such states in many other deformed nuclei and showed, moreover, that the total M1 strength is proportional to the square of the ground-state quadrupole deformation [3]. The emerging picture was that of orbitally excited scissors-type states as had been predicted in the context of a two-fluid model [4] and of the proton–neutron interacting boson model [5]. States of a similar character have also been observed in vibrational U(5) and γ-soft O(6) nuclei [6,7].

Some elementary properties of this magnetic dipole mode can be understood from a simple perspective by comparison with the corresponding electric dipole case, revealing similarities and differences. The operator for the electric dipole response is

$$\bar{T}(\mathrm{E1}) = \sum_i e(i)\bar{r}(i)$$

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$$= \underbrace{\frac{e}{2}\sum_i \bar{r}(i)}_{\bar{R}} - \underbrace{\frac{e}{2}\sum_i \tau_z(i)\bar{r}(i)}_{\bar{R}_\mathrm{p} - \bar{R}_\mathrm{n}} , \qquad (1)$$

where the summation extends over all nucleons in the nucleus. The isoscalar part corresponds to an overall linear displacement and is thus spurious. The isovector part involves a relative displacement of the proton and neutron centres of mass and excites the giant dipole resonance. In the magnetic dipole case the operator is

$$\begin{aligned} \bar{T}(\mathrm{M1}) &= \sum_i g_l(i)\bar{l}(i) + \sum_i g_s(i)\bar{s}(i) \\ &= \underbrace{0.50\sum_i [\bar{l}(i) + \bar{s}(i)]}_{\bar{J}} + \underbrace{0.38\sum_i \bar{s}(i)}_{\mathrm{small}} \\ &\quad + \underbrace{0.50\sum_i \tau_z(i)\bar{l}(i)}_{\bar{L}_\mathrm{p} - \bar{L}_\mathrm{n}} - \underbrace{4.71\sum_i \tau_z(i)\bar{s}(i)}_{\mathrm{spin-flip}} . \end{aligned} \qquad (2)$$

The isoscalar part is now predominantly a (spurious) overall rotation of the nucleus but also contains a spin term although the coefficient in front of it is fairly small. Thus the internal-excitation part of the operator is not purely isovector. Furthermore, the isovector part of the M1 operator consists of two pieces. The first corresponds to an overall *angular* displacement of the protons relative to the neutrons and is the one that excites the scissors states. It is seen, however, that a second spin-flip part is present as well. In measuring the M1 response of a nucleus, care should be taken to separate the orbital from the spin-flip contribution and this can be achieved through the comparison of electron and proton scattering [2]. Moreover, higher resolution can be attained with nuclear resonance fluorescence (γ,γ') experiments which in case of polarised photons allow to separate magnetic from electric dipole states [8].

2. A Soft Scissors Mode?

In recent years, experiments with radioactive beams from projectile fragmentation facilities have revealed [9] the presence of a neutron halo in several of the lightest nuclei on the neutron drip line. This is now understood as an effect where the last one or two neutrons are in low angular momentum orbits very near the top of the well so that their wave functions have a very extended distribution which is manifest empirically in an anomalously large matter radius. There is, however, a distinctly different phenomenon which is predicted in some Hartree–Fock calculations [10,11] to occur in heavier nuclei in which an excess of several neutrons builds up so that the neutron density actually extends out significantly further than that of the protons, resulting in a mantle of dominantly neutron matter.

The presence of this neutron 'skin' may affect collective modes of nuclear excitation which involve the out-of-phase motion of neutrons against protons, such as the giant dipole resonance (GDR) and the scissors mode, the first involving a linear displacement of neutrons relative to protons and the second an angular displacement. There is also then the possibility of a 'soft' dipole mode [12] in which the core nucleons move against the more weakly bound skin neutrons. The effect of an increasing skin thickness on the energy of these three modes was investigated [13] with a simple approach based on classical density oscillations, in which the change in the potential energy in each case was estimated from the density overlaps as a function of displacement. Not surprisingly, the effect on the first two modes was found to be minimal, while the soft dipole mode drops rapidly in energy, relative to the GDR.

In this contribution the behaviour of the scissors mode in the presence of a neutron skin is investigated in the context of a geometric as well as an algebraic model [14]. In each case the approach is illustrated for the normal scissors state and subsequently extended to the soft scissors state.

3. Geometric Description

For the geometric description of the normal and soft scissors modes we use a simple model [13] which is couched in terms of classical oscillations of the proton and neutron (core or skin) densities. The proton and neutron densities are assumed to have (axially-symmetric) Fermi distributions of the type

$$\rho_i(r,\theta;R_i,\beta_i,a_i)=\rho_{\mathrm{o}i}\left[1+\exp\frac{r-R_i(1+\beta_iY_{20}(\theta))}{a_i}\right]^{-1}\equiv\rho_{\mathrm{o}i}\tilde{\rho}(r,\theta;R_i,\beta_i,a_i), \tag{3}$$

involving the spherical coordinates r and θ, and the shape parameters R_i (radial extent), β_i (deformation) and a_i (diffuseness) for protons ($i=\mathrm{p}$) and neutrons ($i=\mathrm{n}$). For simplicity we assume in the following an equal diffuseness for protons and neutrons, $a_{\mathrm{p}}=a_{\mathrm{n}}\equiv a$. The approach of [13] consists in calculating the restoring force between two displaced density distributions. For the giant or soft dipole resonance the displacement is linear while for the normal or soft scissors resonance it is angular, that is, it involves an angle between the axes of symmetry of the proton and neutron distributions. The restoring force is assumed to arise from the proton–neutron interaction which for simplicity is taken to be a zero-range force, $\kappa\delta(\bar{r}_{\mathrm{p}}-\bar{r}_{\mathrm{n}})$, where κ is a strength parameter with dimension MeV fm^3. Combining the restoring force with the masses (dipole case) or the moments of inertia (scissors case) of the displaced components, an estimate of the energy of the corresponding mode is obtained as a function of the shape parameters of the nucleus. This approach can be viewed as a refinement of the Goldhaber–Teller model of giant resonances [15], with separate consideration of the proton and neutron shape parameters and as an extension of it towards scissors and soft excitations, the latter involving a neutron skin.

In the specific case of the scissors resonance, the potential energy of an angular oscillation involving all protons against all neutrons, depends upon a single angle $\theta_{\rm pn}$ between the axes of symmetry of the respective densities and is given by

$$V_{\rm S}(\theta_{\rm pn};R_{\rm p},\beta_{\rm p},R_{\rm n},\beta_{\rm n},a)=\kappa\int\rho_{\rm p}(r,\theta+\theta_{\rm pn};R_{\rm p},\beta_{\rm p},a)\rho_{\rm n}(r,\theta;R_{\rm n},\beta_{\rm n},a)d\bar{r}. \quad (4)$$

After an expansion in the deformation parameters β_i, the change in potential energy with respect to $\theta_{\rm pn}=0$ is found to be

$$\begin{aligned}
&\Delta V_{\rm S}(\theta_{\rm pn};R_{\rm p},\beta_{\rm p},R_{\rm n},\beta_{\rm n},a)\\
&\equiv\ V_{\rm S}(\theta_{\rm pn};R_{\rm p},\beta_{\rm p},R_{\rm n},\beta_{\rm n},a)-V_{\rm S}(\theta_{\rm pn}=0;R_{\rm p},\beta_{\rm p},R_{\rm n},\beta_{\rm n},a)\\
&\approx\ \kappa\beta_{\rm p}\beta_{\rm n}R_{\rm p}R_{\rm n}\frac{d^2}{dR_{\rm p}dR_{\rm n}}\int\rho_{\rm p}(r,\theta=0;R_{\rm p},a)\rho_{\rm n}(r,\theta=0;R_{\rm n},a)\\
&\qquad\qquad\times Y_{20}(\theta)[Y_{20}(\theta+\theta_{\rm pn})-Y_{20}(\theta)]d\bar{r}\\
&\approx\ -6\kappa\rho_{\rm op}\rho_{\rm on}\beta_{\rm p}\beta_{\rm n}R_{\rm p}R_{\rm n}aF_2(R'_{\rm p},R'_{\rm n})\theta^2_{\rm pn}, \qquad (5)
\end{aligned}$$

with $R'_i\equiv R_i/a$ and

$$\begin{aligned}
F_2(R'_{\rm p},R'_{\rm n}) &= -\alpha_{\rm p}\alpha_{\rm n}\left[\frac{(\alpha_{\rm p}+\alpha_{\rm n})(C_3(R'_{\rm p})-C_3(R'_{\rm n}))}{3(\alpha_{\rm p}-\alpha_{\rm n})^3}+\frac{C_2(R'_{\rm p})+C_2(R'_{\rm n})}{3(\alpha_{\rm p}-\alpha_{\rm n})^2}\right],\\
F_2(R',R') &= \frac{3R'^2+\pi^2-6}{54}, \qquad (6)
\end{aligned}$$

where $\alpha_i\equiv\exp(-R'_i)$ and

$$C_3(R'_i)=\frac{1}{3}(R'^3_i+\pi^2R'_i),\qquad C_2(R'_i)=R'^2_i+\frac{1}{3}\pi^2. \quad (7)$$

The density constants $\rho_{{\rm o}i}$ in (5) are obtained from the normalisation conditions

$$\begin{aligned}
Z &= \int\rho_{\rm p}(r,\theta;R_{\rm p},\beta_{\rm p},a)d\bar{r}\approx\left(\frac{4\pi}{3}R^3_{\rm p}+\frac{4\pi^3}{3}a^2R_{\rm p}+\beta^2_{\rm p}\right)\rho_{\rm op},\\
N &= \int\rho_{\rm n}(r,\theta;R_{\rm n},\beta_{\rm n},a)d\bar{r}\approx\left(\frac{4\pi}{3}R^3_{\rm n}+\frac{4\pi^3}{3}a^2R_{\rm n}+\beta^2_{\rm n}\right)\rho_{\rm on}. \qquad (8)
\end{aligned}$$

The expression (5) for the change in potential energy should now be compared with the classical result

$$\Delta V=\frac{1}{2}\mathcal{I}\omega^2\theta^2_{\rm pn}, \quad (9)$$

where $\mathcal{I}$ is the 'reduced moment of inertia' of the two subsystems (i.e., here $\mathcal{I}=\mathcal{I}_{\rm S}=\mathcal{I}_{\rm p}\mathcal{I}_{\rm n}/(\mathcal{I}_{\rm p}+\mathcal{I}_{\rm n})$) and ω is the frequency of the oscillation. Combination of (5), (8) and (9) gives an estimate of the excitation energy $\hbar\omega$ of the scissors state in terms of the strength κ, the shape parameters R_i, β_i and a, and the moments of inertia $\mathcal{I}_i$:

$$E_{\rm S} = \left[-12\kappa\hbar^2 \frac{\beta_{\rm p}\beta_{\rm n}}{\mathcal{I}_{\rm S}} ZN \frac{R'_{\rm p}R'_{\rm n}F_2(R'_{\rm p},R'_{\rm n})}{(4\pi C_3(R'_{\rm p})+\beta_{\rm p}^2R'^3_{\rm p})(4\pi C_3(R'_{\rm n})+\beta_{\rm n}^2R'^3_{\rm n})}\right]^{1/2}. \tag{10}$$

A quantitative prediction on the basis of this expression can only be obtained by taking some model approximation for the moments of inertia. A more reliable application is to consider (10) in conjunction with a similar expression for the energy of the soft scissors state, as is shown below.

The energy of the soft scissors state can be derived similarly. The two oscillating components for calculating the restoring force are in this case the protons,

$$\rho_{\rm p}(r,\theta;R_{\rm p},\beta_{\rm p},a) = \rho_{\rm op}\tilde{\rho}(r,\theta;R_{\rm p},\beta_{\rm p},a), \tag{11}$$

and the skin neutrons,

$$\rho_{\rm s}(r,\theta;R_{\rm p},\beta_{\rm p},R_{\rm n},\beta_{\rm n},a) = \rho_{\rm os}[\tilde{\rho}(r,\theta;R_{\rm n},\beta_{\rm n},a) - \tilde{\rho}(r,\theta;R_{\rm p},\beta_{\rm p},a)], \tag{12}$$

where for simplicity it is assumed that the core radius coincides with the proton radius. The normalisation conditions from where the density constants $\rho_{\rm op}$ and $\rho_{\rm os}$ can be determined are now

$$\begin{aligned} Z &= \int \rho_{\rm p}(r,\theta;R_{\rm p},\beta_{\rm p},a)d\bar{r}, \\ N - N_{\rm c} &= \int \rho_{\rm s}(r,\theta;R_{\rm p},\beta_{\rm p},R_{\rm n},\beta_{\rm n},a)d\bar{r}, \end{aligned} \tag{13}$$

where $N_{\rm c}$ is the number of neutrons in the core. The change in potential energy is found to be

$$\begin{aligned} &\Delta V_{\rm SS}(\theta_{\rm pn};R_{\rm p},\beta_{\rm p},R_{\rm n},\beta_{\rm n},a) \\ &\quad\approx -6\kappa\rho_{\rm op}\rho_{\rm os}\left[\beta_{\rm p}\beta_{\rm n}R_{\rm p}R_{\rm n}aF_2(R'_{\rm p},R'_{\rm n}) - \beta_{\rm p}^2R_{\rm p}^2aF_2(R'_{\rm p},R'_{\rm p})\right]\theta_{\rm pn}^2. \end{aligned} \tag{14}$$

If we assume equal proton and neutron deformations, $\beta_{\rm p} = \beta_{\rm n} \equiv \beta$, and take the reduced moments of inertia to be proportional to the product divided by the sum of the particle numbers in the constituent components,

$$\mathcal{I}_{\rm S} \propto \frac{ZN}{Z+N}, \qquad \mathcal{I}_{\rm SS} \propto \frac{(Z+N_{\rm c})(N-N_{\rm c})}{Z+N}, \tag{15}$$

the following ratio of soft to normal scissors energy results:

$$\begin{aligned} \frac{E_{\rm SS}}{E_{\rm S}} &= \left[\frac{Z}{Z+N_{\rm c}} \frac{4\pi C_3(R'_{\rm n})+\beta^2R'^3_{\rm n}}{4\pi C_3(R'_{\rm n}) - 4\pi C_3(R'_{\rm p}) + \beta^2(R'^3_{\rm n} - R'^3_{\rm p})}\right. \\ &\qquad \left.\times \frac{R'_{\rm n}F_2(R'_{\rm p},R'_{\rm n}) - R'_{\rm p}F_2(R'_{\rm p},R'_{\rm p})}{R'_{\rm n}F_2(R'_{\rm p},R'_{\rm n})}\right]^{1/2}. \end{aligned} \tag{16}$$

If, in first approximation, the difference between $R'_{\rm p}$ and $R'_{\rm n}$ is neglected, $R'_{\rm p} \approx R'_{\rm n} \equiv R'$, this ratio becomes

$$\frac{E_{\rm SS}}{E_{\rm S}} = \left[\frac{Z}{Z+N_{\rm c}}\frac{(6R'^2+\pi^2-6)((4\pi+3\beta^2)R'^2+4\pi^3)}{(3R'^2+\pi^2-6)(3(4\pi+3\beta^2)R'^2+4\pi^3)}\right]^{1/2} \xrightarrow{R'\to\infty} \sqrt{\frac{2Z}{3(Z+N_{\rm c})}}. \quad (17)$$

This simplified expression shows that (i) the ratio $E_{\rm SS}/E_{\rm S}$ is only weakly dependent on the deformation β and (ii) the soft scissors is expected to occur at roughly half the normal scissors energy (assuming $Z \approx N_{\rm c}$). In addition, the full expression (16) shows that $E_{\rm SS}/E_{\rm S}$ further decreases as $R'_{\rm n}$ becomes larger than $R'_{\rm p}$, especially so if the diffuseness a is small. In this respect the expected behaviour of the soft scissors state is qualitatively similar to that of the soft dipole resonance reported in [13].

4. Algebraic Description

The algebraic approach associated with the interacting boson model (IBM) [16] has previously proved particularly enlightening in characterising the scissors excitation. In this section we briefly recall the description of the scissors excitation in the context of the IBM [5], and formulate its extension to the soft scissors excitation [14]. In the following proton and neutron bosons are denoted as π and ν, to be distinguished from p and n which is used for the protons and neutrons themselves.

The incorporation of both protons and neutrons in the IBM involves the product algebra $\mathrm{U}_\pi(6) \otimes \mathrm{U}_\nu(6)$ the algebras being characterised by the number of proton and neutron bosons that correspond to a given nucleus,

$$\begin{array}{ccc} \mathrm{U}_\pi(6) & \otimes & \mathrm{U}_\nu(6) \\ \downarrow & & \downarrow \\ [N_\pi] & & [N_\nu] \end{array} . \quad (18)$$

This is the dynamical algebra of the model in the sense that a single of its (irreducible) representations determines the entire model space. The resulting model is referred to as neutron–proton interacting boson model or IBM-2 [17]. Scissors states arise in the context of the IBM-2 by virtue of the reduction of the proton and neutron U(6) algebras to a single coupled U(6) algebra,

$$\mathrm{U}_\pi(6) \otimes \mathrm{U}_\nu(6) \supset \mathrm{U}_{\pi\nu}(6), \quad (19)$$

where $\mathrm{U}_{\pi\nu}(6)$ is obtained by adding the separate generators. States in the symmetric representation $[N] \equiv [N_\pi + N_\nu]$ of $\mathrm{U}_{\pi\nu}(6)$ correspond to the usual IBM-1 states where no disctinction is made between protons and neutrons. States with mixed symmetry $[N-f, f]$ in $\mathrm{U}_{\pi\nu}(6)$ are outside IBM-1; they have an out-of-phase character in protons and neutrons and therefore are the algebraic equivalent of the scissors states [5]. The geometric interpretation of the classification (19) is that proton and neutron deformations should be the same, $\beta_\pi = \beta_\nu$. Note, however, that in contrast to the discussion in section 3 the classification is not restricted to deformed [SU(3)] nuclei but applies equally well to spherical [U(5)] and transitional [O(6)] nuclei, all cases having found experimental examples [1, 6, 7].

The starting point for the quadrupole modes of a nucleus with a neutron skin is the triple product involving an additional algebra $\mathrm{U}_{\nu_s}(6)$ with the remaining core neutrons being described by $\mathrm{U}_{\nu_c}(6)$. The dynamical algebra of the system is then

$$\begin{array}{ccccc} \mathrm{U}_\pi(6) & \otimes & \mathrm{U}_{\nu_c}(6) & \otimes & \mathrm{U}_{\nu_s}(6) \\ \downarrow & & \downarrow & & \downarrow \\ [N_\pi] & & [N_{\nu_c}] & & [N_{\nu_s}] \end{array}, \tag{20}$$

where each U(6) algebra is characterised by a number of bosons N_i coupled symmetrically to $[N_i]$.

The fact that the skin neutrons are assumed to interact weakly with the core protons and neutrons, which interact strongly with each other, is represented in the reduction of (20) by coupling the U(6) algebra of the neutron skin *after* those describing the core nucleons. The reduction thus proceeds as

$$\mathrm{U}_\pi(6) \otimes \mathrm{U}_{\nu_c}(6) \otimes \mathrm{U}_{\nu_s}(6) \supset \mathrm{U}_{\pi\nu_c}(6) \otimes \mathrm{U}_{\nu_s}(6) \supset \mathrm{U}_{\pi\nu_c\nu_s}(6). \tag{21}$$

The triple-sum algebra $\mathrm{U}_{\pi\nu_c\nu_s}(6)$ has a subalgebra structure familiar from IBM-1, and, specifically, the three usual limits [16], U(5), SU(3) and O(6), can be obtained as subchains of (21). The results derived below are not only valid in the symmetry limits but also for intermediate situations. The presence of $\mathrm{U}_{\pi\nu_c\nu_s}(6)$ in (21) implies an identical mixture of U(5), SU(3) and O(6) (i.e., equal deformations) for all three subsystems π, ν_c and ν_s. More general situations, where the deformations of the subsystems are different, can be envisaged in an algebraic treatment but are not considered here.

In the reduction (21), $\mathrm{U}_{\pi\nu_c}(6)$ is characterised by representations $[N_\pi + N_{\nu_c} - f, f]$. where $N_\pi + N_{\nu_c}$ is the number of nucleon pairs in the core. The lowest states are then contained in the representation $[N_\pi + N_{\nu_c}, 0]$, which denotes the totally symmetric coupling. The lowest states of mixed symmetry are in the next representation, $[N_\pi + N_{\nu_c} - 1, 1]$. The triple-sum algebra $\mathrm{U}_{\pi\nu_c\nu_s}(6)$ is characterised by up to three rows; the lowest couplings arising from $[N_\pi + N_{\nu_c}, 0] \times [N_{\nu_s}]$ are $[N_\mathrm{b}, 0, 0]$ and $[N_\mathrm{b} - 1, 1, 0]$, N_b denoting the total number of bosons, $N_\mathrm{b} = N_\pi + N_\nu$. Hence the first non-symmetric representation resulting from the triple-sum algebra describes *symmetric* coupling of the core nucleons and *non-symmetric* coupling of the skin neutrons. However, the non-symmetric representation $[N_\mathrm{b} - 1, 1, 0]$ of $\mathrm{U}_{\pi\nu_c\nu_s}(6)$ may also arise from the product $[N_\pi + N_{\nu_c} - 1, 1] \times [N_{\nu_s}]$. In this case, it is the core nucleons which are coupled non-symmetrically. The result is that there are now *two* scissors modes, one representing out-of-phase motion between the neutrons and protons in the core and the other denoting an oscillation between the core and the skin where, in this case, as in the soft dipole mode, the core protons are assumed to carry the core neutrons with them. Denoting these two classes of states as $|\mathrm{S}\alpha\rangle$ and $|\mathrm{SS}\alpha\rangle$, respectively, where α describes all further sublabels of U(6), we have

$$\begin{aligned}
|\mathrm{S}\alpha\rangle_a &\equiv |[N_\pi]\times[N_{\nu_c}]\times[N_{\nu_s}];[N_\pi+N_{\nu_c}-1,1]\times[N_{\nu_s}];[N_\mathrm{b}-1,1,0]\alpha\rangle, \\
|\mathrm{SS}\alpha\rangle_a &\equiv |[N_\pi]\times[N_{\nu_c}]\times[N_{\nu_s}];[N_\pi+N_{\nu_c},0]\times[N_{\nu_s}];[N_\mathrm{b}-1,1,0]\alpha\rangle, \qquad (22)
\end{aligned}$$

where the subscript a is used to distinguish (22) from other scissors states discussed below.

While the intuitive approach to the relative interaction strengths of the constituents leads to the two modes described above, it is also possible to propose a scissors mode involving an oscillation of the protons against *all* the neutrons rather than just those in the core. Similarly, the soft scissors mode always involves the skin neutrons but these can oscillate against just the protons rather than all the nucleons in the core. These additional possibilities can be represented algebraically and emerge from the remaining two possible reductions of (20),

$$\mathrm{U}_\pi(6)\otimes\mathrm{U}_{\nu_c}(6)\otimes\mathrm{U}_{\nu_s}(6)\supset\left\{\begin{array}{ll}(b) & \mathrm{U}_{\pi\nu_s}(6)\otimes\mathrm{U}_{\nu_c}(6)\\ (c) & \mathrm{U}_\pi(6)\otimes\mathrm{U}_{\nu_c\nu_s}(6)\end{array}\right\}\supset\mathrm{U}_{\pi\nu_c\nu_s}(6). \qquad (23)$$

In what follows, the classifications or limits (21) and (23) are denoted as a, b and c, respectively.

In the same way as discussed above for limit a, each of the reductions b and c gives rise to a normal and soft scissors mode, the latter being distinguished by non-symmetric coupling of the skin neutrons with one of the other constituents. The scissors states in limits b and c are

$$\begin{aligned}
|\mathrm{S}\alpha\rangle_b &\equiv |[N_\pi]\times[N_{\nu_c}]\times[N_{\nu_s}];[N_\pi+N_{\nu_s},0]\times[N_{\nu_c}];[N_\mathrm{b}-1,1,0]\alpha\rangle, \\
|\mathrm{S}\alpha\rangle_c &\equiv |[N_\pi]\times[N_{\nu_c}]\times[N_{\nu_s}];[N_\pi]\times[N_{\nu_c}+N_{\nu_s},0];[N_\mathrm{b}-1,1,0]\alpha\rangle, \qquad (24)
\end{aligned}$$

where the first corresponds to an oscillation of the core neutrons against the protons and the skin neutrons while in the second the protons oscillate against *all* neutrons. The latter assumption is usually taken for the scissors state but it is as yet unclear whether such will be the case in nuclei with a large neutron excess. The former assumption is, *a priori*, unreasonable.

The soft scissors states in limits b and c are

$$\begin{aligned}
|\mathrm{SS}\alpha\rangle_b &\equiv |[N_\pi]\times[N_{\nu_c}]\times[N_{\nu_s}];[N_\pi+N_{\nu_s}-1,1]\times[N_{\nu_c}];[N_\mathrm{b}-1,1,0]\alpha\rangle, \\
|\mathrm{SS}\alpha\rangle_c &\equiv |[N_\pi]\times[N_{\nu_c}]\times[N_{\nu_s}];[N_\pi]\times[N_{\nu_c}+N_{\nu_s}-1,1];[N_\mathrm{b}-1,1,0]\alpha\rangle, \quad (25)
\end{aligned}$$

where the first corresponds to an oscillation of the skin neutrons against the protons while in the second they oscillate against the core neutrons. Again, one of the modes, the second in this case, appears unphysical.

Thus the basis a which stemmed from the physically intuitive choice of coupling remains the most attractive. Nevertheless, the fact that the three U(6) algebras are coupled to the same final quantum numbers $[N_{\mathrm{b}}-1,1,0]$ implies that the three bases are connected by a unitary transformation and thus any one state can be expressed as a suitable mixture in one of the other bases. Moreover, any of the bases (21) or (23) can be generated with the hamiltonian

$$\hat{H} = A\,\hat{M}_{\pi\nu_c} + B\,\hat{M}_{\pi\nu_s} + C\,\hat{M}_{\nu_c\nu_s} + D\,\hat{M}_{\pi\nu_c\nu_s}, \tag{26}$$

where $\hat{M}_{ij}$ is a Majorana operator which gives zero when acting on states symmetric in $\mathrm{U}_{ij}(6)$ (i.e., characterised by $[N_i+N_j]$) and N_i+N_j when acting on states characterised by $[N_i+N_j-1,1]$. The three limits introduced above are recovered for specific choices of the parameters in the hamiltonian (26), that is, $B=C=0$ in limit a, $A=C=0$ in limit b and $A=B=0$ in limit c.

The energy contribution of (26) to the total hamiltonian can be obtained analytically for states with $\mathrm{U}_{\pi\nu_c\nu_s}(6)$ symmetry $[N_{\mathrm{b}}-1,1,0]$, to which normal and soft scissors states belong. Its matrix elements in basis a are given by

$$\begin{aligned}\left\langle \begin{matrix}\mathrm{S}\alpha\\ \mathrm{SS}\alpha\end{matrix}\middle|\hat{H}\middle|\begin{matrix}\mathrm{S}\alpha\\ \mathrm{SS}\alpha\end{matrix}\right\rangle &= A\begin{bmatrix} N_\pi+N_{\nu_c} & 0\\ 0 & 0\end{bmatrix} + D\begin{bmatrix} N_{\mathrm{b}} & 0\\ 0 & N_{\mathrm{b}}\end{bmatrix}\\ &+ \frac{B}{N_\pi+N_{\nu_c}}\begin{bmatrix} N_{\nu_c}N_{\nu_s} & -\sqrt{N_\pi N_{\nu_c}N_{\nu_s}N_{\mathrm{b}}}\\ -\sqrt{N_\pi N_{\nu_c}N_{\nu_s}N_{\mathrm{b}}} & N_\pi N_{\mathrm{b}}\end{bmatrix}\\ &+ \frac{C}{N_\pi+N_{\nu_c}}\begin{bmatrix} N_\pi N_{\nu_s} & -\sqrt{N_\pi N_{\nu_c}N_{\nu_s}N_{\mathrm{b}}}\\ -\sqrt{N_\pi N_{\nu_c}N_{\nu_s}N_{\mathrm{b}}} & N_{\nu_c}N_{\mathrm{b}}\end{bmatrix}.\end{aligned} \tag{27}$$

Note that the energy matrix is independent of α, the labels of the subalgebras of $\mathrm{U}_{\pi\nu_c\nu_s}(6)$. As a result the energy eigenvalues of $[N_{\mathrm{b}}-1,1,0]$ states are obtained by diagonalising a 2×2 matrix.

As a side result of the diagonalisation of (28), the transformation between the various normal and soft scissors states as defined in (22), (24) and (25) can be obtained. Clearly, the normal and soft scissors states in any one limit are orthogonal; e.g. ${}_a\langle \mathrm{S}\alpha|\mathrm{SS}\alpha\rangle_a = 0$. Note, however, that the modes are not orthogonal if different limits are considered, albeit involving reasonable assumptions concerning the nature of the modes. For instance, if one considers an oscillation of the protons against all neutrons for the scissors state (limit c) and an oscillation of the skin neutrons against all other nucleons for the soft scissors state (limit a), the overlap is then

$${}_c\langle \mathrm{S}\alpha|\mathrm{SS}\alpha\rangle_a = \left[\frac{N_\pi N_{\nu_s}}{(N_\pi+N_{\nu_c})N_\nu}\right]^{1/2}. \tag{28}$$

Overlaps of the kind (28) can be interpreted as recoupling (or Racah) coefficients in U(6). It is seen that they come out as square roots of ratios of the various boson numbers involved. They have a general structure which is independent of the particular unitary algebra [U(6) in this case] but depends solely on the character of the representations (symmetric $[N_i]$ and mixed-symmetric $[N_i-1,1]$) and on the order of the coupling.

The characteristic excitation of these mixed-symmetry modes is via magnetic dipole transitions. In even–even nuclei the existence of 1^+ scissors states excited in (e, e') or (γ, γ') is by now well established. The IBM-2 prediction for the M1 strength towards the 1^+ state corresponding to $|\mathrm{S}\alpha)_c$ in the above is

$$B(\mathrm{M1}; 0_{\mathrm{G}}^+ \to 1_{\mathrm{S}_c}^+) = \frac{3}{4\pi}(g_\pi - g_\nu)^2 f(N_\mathrm{b}) N_\pi N_\nu, \tag{29}$$

where g_π and g_ν are boson g factors. The function $f(N_\mathrm{b})$ is known analytically in the three limits of the IBM,

$$f(N_\mathrm{b}) = \begin{cases} 0 & \mathrm{U}(5) \\ \dfrac{8}{2N_\mathrm{b}-1} & \mathrm{SU}(3) \\ \dfrac{3}{N_\mathrm{b}+1} & \mathrm{O}(6) \end{cases} . \tag{30}$$

Equation (29) is valid for the scissors state of limit c in which all protons oscillate against all neutrons. With an appropriate choice of interpolating function $f(N_\mathrm{b})$ (e.g., the one of [18]), (29) is also valid for a mixture of the three limits where it should be identified with the summed M1 strength, which is concentrated in one state in the three symmetry limits but is fragmented over several states in general.

A similar expression can be derived for the dipole strength to the soft scissors state of limit a,

$$B(\mathrm{M1}; 0_{\mathrm{G}}^+ \to 1_{\mathrm{SS}_a}^+) = \frac{3}{4\pi}(g_\pi - g_\nu)^2 f(N_\mathrm{b}) \frac{N_\pi^2 N_{\nu_s}}{N_\pi + N_{\nu_c}}. \tag{31}$$

From (29) and (31) one finds the following simple result for the ratio of $B(\mathrm{M1})$'s to the soft and normal scissors states:

$$\frac{B(\mathrm{M1}; 0_{\mathrm{G}}^+ \to 1_{\mathrm{SS}_a}^+)}{B(\mathrm{M1}; 0_{\mathrm{G}}^+ \to 1_{\mathrm{S}_c}^+)} = \frac{N_\pi N_{\nu_s}}{(N_\pi + N_{\nu_c}) N_\nu}. \tag{32}$$

This prediction is valid in the three limits and in any intermediate situation under the assumption that the three subsystems π, ν_c and ν_s have equal deformation.

5. Concluding Remarks

The question that has *not* been addressed in this contribution is whether or not the nucleus can develop a neutron skin and whether this skin sufficiently decouples from the core to exhibit angular oscillations with respect to it. The objective of the exercise was rather to find out, *assuming* that such soft modes exist, what their expected properties are. In the case of the scissors mode the soft version is expected to occur at about half the energy of the normal one and excited by a fraction of the normal M1 strength given by (32).

As far as the experimental detection of the conjectured mode is concerned, the difficulties inherent in inverse-kinematics experiments using a neutron-rich beam are compounded still further by the absence of a suitable target since the orbital magnetic dipole states have only been observed to date in (e, e') and (γ, γ').

Finally, it should be emphasised that the algebraic analysis presented in section 4 can be applied to any three-component system of which the internal degrees of freedom of each component can be described by a unitary algebra $\mathrm{U}(n)$. Many of the features obtained for $\mathrm{U}(6)$, such as the appearance of two classes of states with mixed symmetry $[N-1,1,0]$ and the associated fragmentation of transition strength, remain valid generally. It would be of interest to study the generalisation of such three-component structures towards non-compact algebras with the purpose of describing 'borromean' systems, that is, systems that consist of pairwise unbound components but which themselves are bound.

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Notes

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