Using TensorFlow for amplitude fits
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To cite this version:

HAL Id: in2p3-01917564
http://hal.in2p3.fr/in2p3-01917564
Submitted on 9 Nov 2018

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PyHEP Workshop, Sofia, 7–8 July, 2018
Overview

1. Amplitude analysis
2. TensorFlow
3. TensorFlowAnalysis
4. Performance
Amplitude analysis
Amplitude analysis

Amplitude analysis: introduction

Amplitude fits:

- Extract information about intermediate states in multi-body decays
- PDFs can be **computationally expensive** to evaluate
  - Complex models (in both meanings of ‘complex’)
  - Many free parameters
  - Multi-dimensional phase space
  - Often numerically integrated
- Writing fitters can be **labour-intensive** without the right framework

Used in:

- Hadron spectroscopy
  - Discovery of pentaquarks
- Measurement of CKM parameters
  - $CP$ violation
  - $\gamma$ angle
Amplitude analysis: tools

Amplitude fitting:
- Laura++: [https://laura.hepforge.org/](https://laura.hepforge.org/)
  - C++ with ROOT as its only dependency
  - Powerful tool for Dalitz plot fits
  - Can do time-dependent fits
  - Single-threaded, but many clever optimisations
- MINT: [https://twiki.cern.ch/twiki/bin/view/Main/MintTutorial](https://twiki.cern.ch/twiki/bin/view/Main/MintTutorial)
  - C++ interface
  - Can do 3- and 4-body final states
  - Can be used as a generator in the LHCb simulation package Gauss

Generic GPU-based fitting:
- GooFit: [https://github.com/GooFit](https://github.com/GooFit)
  - C++ with python bindings
  - Has a third-party library for amplitude fits
- Ipanema-β: [https://gitlab.cern.ch/bsm-fleet/Ipanema](https://gitlab.cern.ch/bsm-fleet/Ipanema)
  - Based on pyCUDA
  - HEP-specific functions
  - Lacks amplitude analysis functions

Tool for covariant tensors:
- qft++: [https://github.com/jdalseno/qft](https://github.com/jdalseno/qft)
Existing frameworks lack functionality and/or flexibility to cover all cases that might be encountered in amplitude analysis. Users may spend a lot of time altering the framework itself to suit their needs, e.g.:

- Non-scalars in the initial/final states
- Complicated relationships between parameters
- Fitting projections of the full phase space
- Fitting partially-reconstructed decays

For \( n \)-body final states with complicated models, we need:

- Speed (of computation)
- Speed (of development)
- Flexibility
Amplitude analysis: similarities with machine learning

Maximum-likelihood fitting (particularly amplitude analysis) is very similar to machine-learning:

- Large amounts of data — many evaluations of the same function
- Complicated models
- Optimisable parameters
- Minimisation (cost function/NLL)
- Both abbreviate to ‘ML’

Many of the challenges faced in amplitude analysis have been overcome for machine learning.
TensorFlow
Open source library developed by Google: https://www.tensorflow.org/

- Primarily a machine learning library, but the core functionality is suitable for other tasks
  - Symbolic mathematics
- High-performance numerical computation using dataflow graphs
  - Calling functions builds a directed graph, which can then be optimised and compiled
- TF can find analytic derivatives of a graph
- Python, C++ and Java interfaces
- Runs on many architectures out-of-the-box, including GPUs
Functions: symbolic *dataflow graphs*

- Each node is an operation
- Edges represent the flow of data

Data: *tensors* (*n*-dimensional arrays)

- Input and output of mathematical operations
- Operations are *vectorised*

Input:

- Placeholders: used to represent data when building dataflow graphs.
- Variables: can change value during a session, e.g. fit parameters.

Output:

- Numpy arrays

Evaluation:

- Construct a ‘session’
- Run the session by passing a graph and a dict relating placeholders to data samples

\[
a \times \text{tf.sin}(w \times x + p)
\]
import tensorflow as tf

# Define input data (x) and model parameters (w,p,a)
x = tf.placeholder(tf.float32, shape = (None))
w = tf.Variable(1.)
p = tf.Variable(0.)
a = tf.Variable(1.)

# Build graph
f = a*tf.sin(w*x+p)

# Create TF session and initialise variables
init = tf.global_variables_initializer()
sess = tf.Session()
sess.run(init)

# Run calculation of y by feeding data to tensor x
f_data = sess.run(f, feed_dict = {x: [1., 2., 3., 4.]})
print f_data # [ 0.84147096, 0.90929741, 0.14112, -0.7568025 ]
TensorFlow: features for amplitude analysis

Vectorisation:
• Most functions will calculate element-wise over a tensor
• Ideal for maximum-likelihood fits, where the same function must be evaluated repeatedly for a large number of points

Analytic gradient:
• TF can derive analytic gradients from graphs
• Greatly speed up convergence when passed to a minimiser

Partial execution:
• TF can cache parts of a graph unaffected by changes in parameters
• In practice, this does not work as expected, but one can manually inject the value of a tensor when running a session

Minimisation:
• TF has minimisers for training machine-learning algorithms...
• ... which not particularly suitable for fitting
  • No uncertainties on parameters
  • Cannot do likelihood scans
TensorFlowAnalysis
TensorFlowAnalysis: introduction

TensorFlow alone is almost a suitable framework for amplitude fits. TensorFlowAnalysis (https://gitlab.cern.ch/poluekt/TensorFlowAnalysis/) adds some crucial features:

- Read/write ROOT ntuples
- Fit parameter class (extends tf.Variable)
- Interface to Minuit
- Toy generation
- Fit fractions
- Functions commonly for calculating amplitudes
  - Kinematics: lorentz vectors, boosts, rotations, two-body momenta, helicity angles...
  - Dynamics: lineshapes, form factors...
  - Helicity amplitudes, LS couplings, Zemach tensors...
  - Elements of covariant formalism (polarisation vectors, $\gamma$ matrices...)
- Phase space classes
  - Check if a datapoint is within the phase space
  - Generate uniform distributions
  - Return/calculate specific variables from a datapoint
Simple 2D Dalitz plot model:

```python
# Phase space object
phsp = DalitzPhaseSpace(ma, mb, mc, md)
# Fit parameters
mass = Const(0.770)
width = FitParameter("width", 0.150, 0.1, 0.2, 0.001)
a = Complex( FitParameter("Re(A)", ...), FitParameter("Im(A)", ...)
# Fit model as a function of 2D tensor of data
def model(x):
    m2ab = phsp.M2ab(x) # Phase space class provides access to
    m2bc = phsp.M2bc(x) # individual kinematic variables
    ampl = a*BreitWigner(mass, width, ...)*Zemach(...) + ...
    return Density(ampl)
```
TensorFlowAnalysis: fitting

- Using MC integration for the normalisation:

```python
# Call the model on placeholders to build the dataflow graphs
model_data = model(phsp.data_placeholder)
model_norm = model(norm.data_placeholder)
# Assemble into a negative log-likelihood graph to be minimise
nll = UnbinnedNLL(model_data, Integral(model_norm))
```

- Input data samples are numpy arrays:

```python
# Both samples of the form data[event][variable]
data_sample = ReadNTuple(tree, [branches...])
norm_sample = sess.run(phsp.RectangularGridSample(400, 400))
```

- Minimise the NLL with Minuit:

```python
result = RunMinuit(sess, nll, {phsp.data_placeholder: data_sample,
                                   phsp.norm_placeholder: norm_sample})
WriteFitResults(result, "result.txt")
```
TensorFlowAnalysis: fitting

Straightforward to modify the NLL to add functionality, e.g.:

- **Weighted fit:**

```python
# Assume the weight is the last element in the list
def event_weight(datapoint, norm = 1.):
    return tf.transpose(datapoint)[-1] * norm
integral = WeightedIntegral(model_norm, event_weight(norm_ph))
weight_correction = sum([dp[-1] for dp in data_sample])
    /sum([dp[-1]**2 for dp in data_sample])
nll = UnbinnedWeightedNLL(model_data, integral,
                          event_weight(data_ph, norm = weight_correction))
```

- **Simultaneous fit:**

```python
norm = Integral(model1_norm) + Integral(model2_norm)
nll = UnbinnedNLL(model1_data, norm) + UnbinnedNLL(model2_data, norm)
```
• Complex combinations of parameters:

```python
def HelicityCouplingsFromLS(ja, jb, jc, lb, lc, bls):
    a = 0.
    for ls, b in bls.iteritems():
        # Where b is a Complex(FitParameter(...), FitParameter(...))
        l = ls[0]
        s = ls[1]
        coeff = math.sqrt((l+1)/(ja+1))*Clebsch(jb, lb, jc, -lc, s, lb-lc)
                      *Clebsch(l, 0, s, lb-lc, ja, lb-lc)
        a += Const(coeff)*b
    return a
```
Some recent features allow the user to quickly build an amplitude model of an $n$-body decay.

The Particle class:
- Holds intrinsic properties and mother/daughter relationships
- Useful to quickly define different decay chains within an amplitude model
- Handles rotations and boosts

HelicityMatrixDecayChain:
- Takes the head Particle of the decay chain and a dict of helicity amplitude parameters
- Builds a dict of matrix elements in the helicity formalism for a specific decay chain

PHSPGenerator and NBody:
- Construct a phase space object given the mother mass and a list of final-state daughter masses
Some issues with using TensorFlow for amplitude fits:

- Python 2 only (for now)
- TF not readily available on LXplus
  - Binary distributions available from debian-based distros and Mac
  - Available from pip without machine-specific optimisations
  - Can install from source: tricky (especially with CUDA) but possible.
- Memory usage can be several GB:
  - Especially with analytic gradient/large datasets/complicated models
  - Limiting for consumer-grade GPUs
- Double precision essential
  - Limiting for consumer-grade GPUs
- Slow RAM–VRAM transfer
  - Has been mitigated since earlier versions of TFA
- Errors at graph execution time are hard to debug
  - Dedicated debugger: https://www.tensorflow.org/programmers_guide/debugger
TensorFlowAnalysis: plans

- Port to python 3
- Expand the library
  - K-matrix formalism
  - Analytical coupled-channel approaches
- Save/load compiled graphs
  - Graph-building can sometimes take longer than minimisation
- Optimisations of CPU and memory usage; better caching
- More symbolic maths
  - Sympy, in particular, works well with TF
- Self-documentation
  - Generate LaTeX description of formulae entering the fit
- Automatic code generation: share standalone models with theorists
Performance
Benchmark runs (fit time only), compare 2 machines:

- CPU1: Intel Core i5-3570 (4 cores), 3.4GHz, 16 Gb RAM
  GPU1: NVidia GeForce 750Ti (640 CUDA cores), 2 Gb VRAM
- CPU2: Intel Xeon E5-2620 (32 cores), 2.1GHz, 64 Gb RAM
  GPU2: NVidia Quadro p5000 (2560 CUDA cores), 16 Gb VRAM

Two isobar models:

- $D^0 \rightarrow K^0_S \pi^+ \pi^-$: 18 resonances, 36 free parameters
- $\Lambda_b \rightarrow D^0 p \pi^-$: 3 resonances, 4 non-resonant amplitudes, 28 free parameters
## Performance

<table>
<thead>
<tr>
<th></th>
<th>Iterations</th>
<th>Time, sec</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>CPU1</td>
</tr>
<tr>
<td>$D^0 \rightarrow K^+_S\pi^+\pi^-$, 100k events, 500 × 500 norm.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Numerical grad.</td>
<td>2731</td>
<td>488</td>
</tr>
<tr>
<td>Analytic grad.</td>
<td>297</td>
<td>68</td>
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<tr>
<td>$D^0 \rightarrow K^+_S\pi^+\pi^-$, 10M events, 1000 × 1000 norm.</td>
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<td></td>
</tr>
<tr>
<td>Numerical grad.</td>
<td>2571</td>
<td>3393</td>
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<tr>
<td>Analytic grad.</td>
<td>1149</td>
<td>1587</td>
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<tr>
<td>$\Lambda_b^0 \rightarrow D^0 p\pi^-$, 10k events, 400 × 400 norm.</td>
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<td></td>
</tr>
<tr>
<td>Numerical grad.</td>
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<tr>
<td>Analytic grad.</td>
<td>425</td>
<td>33</td>
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<tr>
<td>$\Lambda_b^0 \rightarrow D^0 p\pi^-$, 100k events, 800 × 800 norm.</td>
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<td></td>
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<tr>
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<td>910</td>
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<tr>
<td>Analytic grad.</td>
<td>390</td>
<td>133</td>
</tr>
</tbody>
</table>
• TensorFlow is a good basis for an amplitude fitting framework
• High-performance architectures can be exploited without expert knowledge
• Models written in TFA are portable and can, with small effort, work standalone from TF: easy to share with theorists
• Flexibility of TFA allows for rapid and simple development of complicated fits
• TensorFlowAnalysis package: library to perform amplitude analysis fits. In active development, used for a few ongoing baryonic decay analyses at LHCb.