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Superdeformed bands of odd nuclei in A = 190 region in the quasiparticle picture

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We study the properties of the superdeformed (SD) bands of ¹⁹⁵Pb and ¹⁹³Hg by the cranked Hartree-Fock-Bogoliubov method. Our calculations reproduce the flat behavior of the dynamical moment of inertia of two of the SD bands of ¹⁹⁵Pb measured recently. We discuss possible configuration assignments for the observed bands 3 and 4 of ¹⁹⁵Pb. We also calculate the two interacting SD bands of ¹⁹³Hg. Our analysis confirms the merit of density-dependent pairing forces as compared to seniority pairing interactions. [S0556-2813(97)01703-2]

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I. INTRODUCTION

The dynamical moment of inertia $\mathcal J$ of most superdeformed (SD) bands observed in nuclei of the $A \approx 190$ region are increasing functions of the angular velocity ω [1–14]. Recently, Farris *et al.*, [12] found that \mathcal{J} is almost constant versus ω for the two lowest SD bands in ¹⁹⁵Pb. This new feature is particularly interesting. In the same nucleus two other bands have been observed which display the usual increasing trend. It appears natural to attempt an explanation of these various behaviors of the SD bands of ¹⁹⁵Pb in terms of their quasiparticle (qp) structure. According to most theoretical investigations in the A = 190 region, the neutron qp's relevant for neutron numbers above the N = 112 gap are built on the [752]5/2, [512]5/2, and [624]9/2 orbitals [15-18]. In this work, we analyze the properties of the SD bands of the two odd-N neighboring nuclei ¹⁹⁵Pb and ¹⁹³Hg [5] which today provide the richest information set on the neutron structure in the A = 190 superdeformed well. Our work is based on the cranked Hartree-Fock-Bogoliubov (HFB) approach which has been shown to reproduce with a good accuracy the SD band properties of even nuclei [19,20]. As in Ref. [18], the mean-field method has been corrected by means of the Lipkin-Nogami prescription [21–23] to take into account the finite number of nucleons. The nucleonnucleon effective interaction in the particle-hole channel is the Skyrme force within the Skm* parametrization [24]. In the pairing channel we use a zero-range force with a surfacepeaked density dependence as described in Ref. [20]. In a previous study of the yrast SD band of ¹⁹⁴Pb, this type of force was shown to improve significantly the alignment properties which determine the saturation of \mathcal{J} for large values of ω .

Our method of solution of the HFB equation combines the imaginary-time evolution method to determine the basis which diagonalizes the mean-field Hamiltonian and a diagonalization of the HFB Hamiltonian matrix to construct the canonical basis. Details can be found in Ref. [18]. The different bands of the odd nuclei we are concerned with, are described by the self-consistent creation of the appropriate qp [25] on the even vacuum. This requires some care in the numerical treatment, since, except at zero angular velocity, parity, and signature are the only quantum numbers available for sorting qp's. Therefore, we rely mostly on continuity properties versus ω to follow a given SD band.

The first part of this paper is devoted to a study of \mathcal{J} and qp Routhian (qpR) properties of ¹⁹⁵Pb. In the second part we analyze how the structure of the pairing force affects SD properties by comparing the results for ¹⁹³Hg with the density-dependent interaction with those of Ref. [25] in which a seniority interaction was used.

II. SUPERDEFORMATION IN ¹⁹⁵Pb

A. Dynamical moment of inertia

In Fig. 1 we compare the dynamical moments of inertia \mathcal{J} of the four observed bands [12] with those calculated for the seven SD bands built on the [752]5/2, $\alpha = \pm 1/2$, [512]5/2, $\alpha = \pm 1/2$, [624]9/2, $\alpha = \pm 1/2$, and [642]3/2, $\alpha = -1/2$ orbitals. The SD bands built on the intruder ν [752]5/2 bands display a small variation of \mathcal{J} for $\hbar \omega \ge 0.24$ MeV. It seems therefore reasonable to assign them to the first and second experimental bands. The observed significant signature splitting is also reproduced and suggests that the band with lowest moment of inertia has a positive signature. For bands 3 and 4 the authors of Ref. [12] have argued that they may be built on ν [624]9/2 orbitals. On the other hand, the theoretical part of their analysis indicated that transition energies associated with ν [624]9/2 and ν [512]5/2 bands would be almost identical. This is supported by our results.

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FIG. 1. (a) Experimental dynamical moments of inertia of the four SD bands [12] of ¹⁹⁵Pb. (b) HFB dynamical moments of inertia of the lowest SD bands of ¹⁹⁵Pb. (The negative signature bands are denoted with black symbols.)

Indeed we find that the moment of inertia of the ν [624]9/2 bands is very similar to that of the ν [512]5/2 bands for $\hbar\omega$ \geq 0.24 MeV. Both sets of \mathcal{J} agree qualitatively with those observed for bands 3 and 4. Based on the sole information given by the moment of inertia, it is therefore not possible to decide whether bands 3 and 4 should be labeled ν [624]9/2 or ν [512]5/2. We only note a small signature splitting for the ν [512]5/2 when $\hbar \omega \ge 0.3$ MeV. A complementary piece of information is provided by the excitation energy $E_{\rm rel}$ of the bands with respect to each other. It is generally believed that this quantity which is directly available in the calculation (see Table I) is correlated with the observed relative population of the bands. In Table I, the reference energy corresponds to the state of ν [752]5/2, $\alpha = -1/2$ band of the angular momentum I = 32.5; it has been calculated by averaging two energies of I = 31.5 and 33.5. The relative excitation energies $E_{\rm rel}$ of the other negative-signature bands were calculated in the same way. According to Table I, the ν [752]5/2 bands are the lowest, the ν [512]5/2 bands are second lowest and the ν [624]9/2 ν [642]3/2 bands are the most excited. Therefore, for the ν [752]5/2 bands, the agreement of

TABLE I. Calculated relative excitation energies $E_{\rm rel}$ of the six SD bands in ¹⁹⁵Pb at I = 32.5. The reference band is ν [752]5/2, $\alpha = -1/2$ band. $E_{\rm rel}$ of the negative-signature bands were calculated from averages of energies of I = 31.5 and 33.5.

Band	$E_{\rm rel} [{ m MeV}]$
ν [752]5/2, $\alpha = +1/2$	0.125
ν [752]5/2, $\alpha = -1/2$	0.000
ν [512]5/2, $\alpha = +1/2$	0.168
ν [512]5/2, $\alpha = -1/2$	0.172
ν [624]9/2, $\alpha = +1/2$	0.297
ν [624]9/2, $\alpha = -1/2$	0.297
ν [642]3/2, $\alpha = -1/2$	0.368



FIG. 2. Calculated charge quadrupole moments Q_c of the ν [512]5/2, ν [624]9/2 bands and the ν [642]3/2, $\alpha = -1/2$ band of ¹⁹⁵Pb. Correspondence between the symbols and the SD bands is the same as in Fig. 1(b).

our calculation with experiment concerns both the magnitude and behavior of moments of inertia and the excitation energy. On the other hand, we are led led to assign a ν [512]5/2 structure to bands 3 and 4. We note, however, that the energy differences in Table I are of the order of 0.1 MeV. Such a precision is below the limit of physical credibility of a calculation such as ours when it comes to the relative position of orbitals. We shall return to this point when discussing the crossing phenomenon in ¹⁹³Hg.

B. Deformation

The charge quadrupole moments Q_c of the ν [512]5/2 and ν [624]9/2 bands are shown in Fig. 2. They differ by approximately 0.3 b which is probably too small to be measured. As expected for bands with large *m* values no signature splitting is found. The quadrupole moments of the intruder bands are slightly more different: 19.96 eb at I = 32.5 ($\hbar \omega = 0.314$ MeV) for $\alpha = +1/2$ and 19.47 eb at I = 31.5 ($\hbar \omega = 0.290$ MeV) for its signature partner. Differences between the magnetic moments of the ν [512]5/2 and ν [624]9/2 bands can lead to different crosstalks between the bands. However, their values are rather similar ($\approx 12.4 \ \mu_N$, μ_N being the nuclear magneton, at $I \approx 31$) for the four bands.¹ Both the quadrupole and the magnetic moments do not provide a relevant signature to establish the nature of the third and fourth SD bands.

C. Quasiparticle energies

Let us now consider the evolution of the qpR's versus ω . As the mean-fields are self-consistently modified by the

¹We have taken the cranking axis as approximate quantization axis in evaluating the magnetic moment at $\hbar \omega \approx 0.3$ MeV. Semmes *et al.* [26] have calculated the magnetic moments in the framework of the particle-rotor model with the strong-coupling scheme. They obtained a value of -0.48 for the *g* factor of the ν [512]5/2⁻ band of ¹⁹³Hg.



FIG. 3. Neutron qpR of ν [752]5/2, $\alpha = -1/2$ SD band. The thick dotted curve indicates the occupied qpR. Full (dashed) curves correspond to positive parity and positive (negative) signature, and dot-dashed (dotted) curves to negative parity and positive (negative) signature.

creation of a qp, the behavior of the qpR's are not the same for the six SD bands and differ also from those calculated for ¹⁹⁴Pb. In Figs. 3 and 4 we show the neutron qpR's for the ν [752]5/2, $\alpha = -1/2$ (after a crossing with [512]5/2, $\alpha =$ -1/2) and ν [624]9/2, $\alpha = +1/2$ bands respectively. In each figure, the thick curve indicates which qp is occupied.

In self-consistent calculations, such as those discussed in the present work, the interpretation of qp diagrams requires some care. First of all let us recall that qp energies and Routhians are not observables: they are not eigenvalues of a Hermitian operator which stay the same for all nuclei as for instance the charge quadrupole operator or the Hamiltonian. On Figs. 3 and 4 we have drawn the qp energies (eigenvalues of the HFB Hamiltonian which depends both on the nucleus and on the configuration) corresponding to the self consistent vacuum associated with two different 1-qp excitations. These diagrams differ from those of the two neighboring even isotopes. Two factors play a role. First, the Fermi level is displaced to ensure that the average number of particles is equal



FIG. 4. Neutron qpR of ν [624]9/2, α = +1/2 SD band. The thick full curve corresponds to the occupied qpR. The other drawing conventions are the same as those used in Fig. 3.

to 113. Second, the excitation of a 1-qp creates specific time odd contributions to the mean field which do exist even in the absence of rotation. These terms are responsible for a splitting between signature partner Routhians at $\hbar \omega = 0$. Of course, the creation of a 1-qp excitation in the signature partner orbital (and a change $\omega \leftrightarrow -\omega$) would simply lead to a similar qp diagram with an exchange of signature for all the curves. A question still not understood is the sign and the magnitude of these splittings. In particular, we notice that they are especially large for the pair out of which the qp is created. On the other hand, in a study of excited states in an even nucleus, one can consider a 2-qp state built on a signature partner pair. In such case, time reversal invariance is restored at $\hbar \omega = 0$ and no splitting occurs in the absence of cranking. However, due to self-consistency effects, the qp spectrum will differ from that of the 0-qp configuration. One of the main consequences of the modification of the Routhian spectrum is the displacement of level-crossings frequencies compared to the values expected from the consideration of the qp spectrum of the neighboring even nuclei. Finally, let us mention that in the definition of the "experimental" Routhians constructed from the experimental total energies, the breaking of the time reversal symmetry is not taken into account.

Figures 3 and 4 show that the flat \mathcal{J} behavior for the ν [752]5/2 bands is strongly correlated with the curvature of the associated Routhians which is markedly different from those of other qpR's. Moreover, the fact that the average curvature of the ν [752]5/2, $\alpha = +1/2$ Routhian is the smallest is consistent with the low moment of inertia of the corresponding band.

The accident in the theoretical \mathcal{J} for the [752]5/2 and [512]5/2 bands for $0.1 \le \hbar \omega \le 0.2$ MeV is generated by a band crossing. Such a feature can always happen when two crossing bands have the same quantum numbers. In case of a band crossing, our convention is to denote bands with the qp configuration which characterizes them for large values of $\hbar\omega$. For values of $\hbar\omega$ near 0.15 MeV, both for the ν [752]5/2 and ν [512]5/2 bands, we have not been able to obtain solutions satisfying the angular-momentum constraint accurately. We have met numerical instabilities caused by the near degeneracy both in energy and angular momentum. A correct physical solution of this problem requires a selfconsistent configuration mixing calculation which is beyond the scope of this study. As a band crossing has not been observed in ¹⁹⁵Pb, we infer that the calculation shown in Fig. overestimates the energy difference between the 3 ν [752]5/2, $\alpha = -1/2$ and ν [512]5/2, $\alpha = -1/2$ energies at $\omega = 0$ by at least 0.1 MeV. A possible way to remedy this deficiency, is to correct the mean field in such a way that the energy of the ν [512]5/2, $\alpha = -1/2$ is pushed up, leading to a crossing below the lowest observed $\hbar\omega$. According to the above discussion, the associated excitation energy of the ν [512]5/2 SD bands will increase and possibly modify our assignation for band 3 and 4, leading to a better agreement with the conclusions of Ref. [12].

Let us mention the relative position of ν [624]9/2, $\alpha = +1/2$ in Fig. 4. The qpR is not the lowest in Routhians having the positive parity and positive signature. It is anticipated, however, that a particle-type qpR (ν [624]9/2) becomes lower than a hole-type one (ν [642]3/2) in ¹⁹⁵Pb when



FIG. 5. Experimental dynamical moment of inertia \mathcal{J} of ¹⁹³Hg for bands 1 (solid triangle) and 4 (solid circle). Our results are indicated by open triangles for the ν [512]5/2, $\alpha = -1/2$ configuration and by open circles for the ν [752]5/2, $\alpha = -1/2$ one.

their energies are comparable for the yrast SD band of ¹⁹⁴Pb. Given the present uncertainty of mean-field calculation concerning the detailed relative location of the qpR's, we cannot disregard ν [624]9/2 as a candidate for the configuration of bands 3 and 4.

On Fig. 2 and in Table I we have also reported result for the negative signature band built on the ν [642]3/2, α =-1/2 quasiparticle. Although the Routhian of this state is lower than that of the ν [624]9/2 in the quasiparticle spectrum of ¹⁹⁴Pb, Table I shows that the [642]3/2 SD band is more excited. This is a direct consequence of self-consistent effects.

III. SUPERDEFORMATION IN ¹⁹³Hg

Let us now see how these considerations can be extended to the analysis of the nucleus ¹⁹³Hg. So far six SD bands including two identical bands have been observed [5]. Within the HFB method we have already performed a study using the same Skyrme force parametrization for the mean field [25]. However, in this earlier work, pairing correlations were described with a seniority interaction. Figure 5 shows together with the experimental data the results of the present analysis limited to the two interacting bands ν [752]5/2, $\alpha =$ -1/2 band (band 4) and ν [512]5/2, $\alpha = -1/2$ band (band 1) with a zero-range density dependent pairing force. For each band a separate HFB calculation has been performed. Our calculation shows that this dual self-consistent HFB analysis is able to reproduce the observed band interaction. This is a significant improvement over the calculation of Ref. [25] in which no interaction was found. It provides an additional indication of the superiority of a surface-type zerorange pairing force over a seniority interaction. On the other hand, the angular velocity at which bands interact is found at $\hbar \omega \approx 0.15$ MeV instead of the observed value $\hbar \omega \approx 0.25$ MeV. This difference may reflect an inaccuracy of the relative location of the relevant qpR's in ¹⁹³Hg which would then be consistent with our discussion on the position of orbitals [752]5/2 and [512]5/2 in ¹⁹⁵Pb.

IV. CONCLUSION

In summary, we have analyzed the properties of the SD bands of two odd-*N* nuclei ¹⁹⁵Pb and ¹⁹³Hg by making use of the cranked HFB method. Our self-consistent calculation has confirmed the general belief that the flat behavior of \mathcal{J} in bands 1 and 2 of ¹⁹⁵Pb is related to the curvature of the neutron intruder qpR. For bands 3 and 4 we have found that configurations based on the [624]9/2 and [512]5/2 are in competition. Both the moment of inertia, the quadrupole moments and the magnetic moments of these four SD bands are very similar. In particular, within the HFB method these quantities do not provide a relevant mean to decide the nature of the observed bands 3 and 4. We have also calculated the two interacting SD bands of ¹⁹³Hg with qualitative success. The results of this analysis provide additional support for an effective pairing force acting predominantly at the nuclear surface. The quantitative inaccuracy on the position of the crossing frequencies ($\Delta \hbar \omega \approx 0.1$ MeV) could be an indication that qpR associated with the [512]5/2 is too low by 0.1 MeV relative to the rest of the spectrum. It is an interesting and open question whether it is possible to determine an effective interaction with the same global qualities of the SkM* force which could also achieve a better precision as regards the single-particle energies.

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