Two-particle angular correlations in e^+e^- interactions compared with QCD predictions

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EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN–EP/98–138

7 September 1998

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DELPHI Collaboration

Abstract

Two–particle angular correlations in jet cones have been measured in e^+e^- annihilation into hadrons at LEP energies ($\sqrt{s}$ = 91 and 183 GeV) and are compared with QCD predictions using the LPHD hypothesis. Two different functions have been tested. While the differentially normalized correlation function shows substantial deviations from the predictions, a globally normalized correlation function agrees well. The size of α_S^{eff} (and other QCD parameters) and its running with the relevant angular scale, the validity of LPHD, and problems due to non–perturbative effects are discussed critically.

(Accepted by Physics Letters B)

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1 Introduction

A description of multihadron production in e^+e^- reactions using QCD is difficult because of the existence of a low energy non-perturbative region. In phenomenological models the parton cascade, which can be handled with the Leading Log Approximations (LLA), is cut off at some scale $Q_0 \geq 1$ GeV and is followed by a hadronisation phase. These models with well motivated parameters have yielded good results, but there are many of them and only few are directly connected to (perturbative) QCD. It has been suggested to extend instead the parton evolution down to a lower mass scale (if possible to the pion mass scale). Using this concept, the multihadron final states can be compared directly with the multiparton final states [1]. The possibility that perturbative QCD also has some applicability in the low energy regime led in the mid-eighties to the concept of Local Parton Hadron Duality (LPHD) [2].

Our interest is the study of correlations between hadrons produced in e^+e^- annihilation using the relatively new tool of Correlation Integrals [3]. The main theoretical effort for evaluating multiparton correlations in the framework of QCD has been based on the Double Log Approximation (DLA) [1,4]. Detailed prescriptions for multiparton angular correlations in cones using the DLA have been proposed [5]. It has been pointed out [5] that the aim of such studies is not primarily a further test of perturbative QCD at a fundamental level, but rather to find out the limiting scale for its application and thereby to learn about the onset of non-perturbative confinement forces. The present comparison, however, also has to cope with substantial simplifications in the calculations of the perturbative part which are justified only at asymptotic energies, as well as with the question of how far the LPHD hypothesis is valid (see below). Therefore our goal here is simply to present the corresponding experimental data, to discover possible discrepancies, and to show whether some predictions are already fulfilled at LEP energies.

In section 2 the theoretical background is described, and the definitions and the actual QCD predictions on partonic angular correlations are given. In section 3 the experimental measurements of 2-particle angular correlations are presented and confronted to the analytical calculations using the concept of LPHD. In section 4 the measured values of α_S^{eff}, the "running" of α_S^{eff}, the range of validity of the LPHD hypothesis, and problems due to extensions to the non–perturbative region are critically discussed. Section 5 is a summary of the experimental results.

2 Theoretical background, definitions and predictions

The following short outline follows the procedures used by Ochs and Wosiek [5]. They calculated particle correlations produced in gluon cascades radiated off the initial parton. The matrix element for gluon bremsstrahlung in DLA is as follows:

$$M(k)d^3k = c_a \gamma_0^2 \frac{dk}{k} \frac{d\Theta_{pk}}{\Theta_{pk}} \frac{d\Phi_{pk}}{2\pi} \quad (1)$$

where $c_g = 1$ and $c_q = \frac{4}{9}$, p and k are the 3-momenta of the parent parton and the radiated gluon, Θ_{pk} is the angle of emission of the gluon, Φ_{pk} is the azimuthal angle of the gluon around $\vec{p}$ and

$$\gamma_0^2 = 6\alpha_S/\pi. \quad (2)$$

The inclusive n-particle densities $\rho_n(k_1, k_2, ...k_n)$ (k_i is the 3-momentum of the i-th particle) are obtained by applying the generating functional technique [6] which has been developed for QCD jets first as explained in references [7,4]. The calculations have been carried out in DLA where the integrals involved are performed only in phase space regions with dominant contributions given by the singularities of (1). Energy-momentum conservation and $q\bar{q}$ production were neglected in the calculations, and well developed cascades at very high energies were assumed; angular ordering was taken into account.

Theoretical predictions [5] concerning the emission of two partons with a relative angle ϑ_{12} - within a cone with half opening angle Θ around the jet axis - have been evaluated using two correlation functions defined as follows [3]:

$$r(\vartheta_{12}) = \frac{\rho_2(\vartheta_{12})}{\rho_1 \otimes \rho_1(\vartheta_{12})} \tag{3}$$

$$\tilde{r}(\vartheta_{12}) = \frac{\rho_2(\vartheta_{12})}{\bar{n}^2(\Theta)} \tag{4}$$

with the correlation integrals [3] $\rho_2(\vartheta_{12}) = \int_\Theta d^3k_1 d^3k_2 \rho_2(k_1, k_2)\delta(\vartheta_{12} - \vartheta(k_1, k_2))$ and $\rho_1 \otimes \rho_1(\vartheta_{12}) = \int_\Theta d^3k_1 d^3k_2 \rho_1(k_1)\rho_1(k_2)\delta(\vartheta_{12} - \vartheta(k_1, k_2))$ where $\rho_1(k)$ is the single particle distribution and $\bar{n}(\Theta)$ is the mean multiplicity of partons emitted into the Θ-cone. The quantities in eqns. (3) and (4) exhibit very different structures. $\rho_2(\vartheta_{12})$ consists of 2 terms $\rho_2(\vartheta_{12}) = C_2(\vartheta_{12}) + \rho_1 \otimes \rho_1(\vartheta_{12})$ where only $C_2(\vartheta_{12})$ describes the genuine correlations, $\rho_1 \otimes \rho_1(\vartheta_{12})$, on the other side, is obtained from the single particle spectra. Consequently $r(\vartheta_{12})$ is given essentially by the normalized C_2-term whereas it turns out that the dominant term of $\tilde{r}(\vartheta_{12})$ is given by $\rho_1 \otimes \rho_1(\vartheta_{12})$.

Distinct predictions for $r(\vartheta_{12})$ and $\tilde{r}(\vartheta_{12})$ have been evaluated which depend essentially only on the QCD parameters Λ and n_f, where the latter is the effective number of flavours involved in the parton cascade [5]:

a) At high energy and for sufficiently large angles $\vartheta_{12} \leq \Theta$ the following power law is expected:

$$r(\vartheta_{12}) = \left(\frac{\Theta}{\vartheta_{12}}\right)^{0.5\gamma_0} \tag{5}$$

and the scale determining γ_0 is given by $Q \approx P\Theta$, where $P = \frac{\sqrt{s}}{2}$ is the momentum of the primary parton.

b) For asymptotically high energies the quantity

$$\frac{\ln(r(\vartheta_{12}))}{\sqrt{\ln\frac{P\Theta}{\Lambda}}} \approx 2\beta(\omega(\epsilon, 2) - 2\sqrt{1-\epsilon}) \tag{6}$$

with

$$\epsilon = \frac{\ln\frac{\Theta}{\vartheta_{12}}}{\ln\frac{P\Theta}{\Lambda}} \quad , \tag{7}$$

$$\beta^2 = 12(11 - \frac{2}{3}n_f)^{-1} \quad = 1.56 \;\; for \;\; n_f = 5$$

and

$$\omega(\epsilon, n) = n\sqrt{1-\epsilon}(1 - \frac{1}{2n^2}\ln(1-\epsilon)) \tag{8}$$

is expected to be independent of the cone opening angle Θ and primary momentum P, meaning that it is a scaling function.

c) Transforming $\tilde{r}(\vartheta_{12})$ to the $\tilde{r}(\epsilon)$ and dividing by factors depending on $\sqrt{\ln(\frac{P\Theta}{\Lambda})}$ a new function $Y(\epsilon)$ is obtained which is expected to be independent of Θ and the primary momentum P, meaning that $Y(\epsilon)$ is a scaling function:

$$\tilde{r}(\epsilon) = \vartheta_{12}\tilde{r}(\vartheta_{12}) \ln \frac{P\Theta}{\Lambda} \tag{9}$$

$$Y(\epsilon) = -\frac{\ln(\tilde{r}(\epsilon)/b)}{2\sqrt{\ln(\frac{P\Theta}{\Lambda})}} = 2\beta(1 - 0.5\omega(\epsilon, 2)) \quad , \quad b = 2\beta\sqrt{\ln(\frac{P\Theta}{\Lambda})} \tag{10}$$

The scale $Q \approx P\Theta$ in these formulae is given by the cone half opening angle Θ which is the upper limit for the angle of emission of the first hard gluon in a cascade (eqn.(1)). The degrading of Q along the cascade is taken into account by the specific dependence of (6) and (10) on ϵ and, via eqn.(7), on ϑ_{12}, the second angle under consideration. It should be noted that the QCD parameter Λ enters into eqns. (6) and (10) within the factor $(\ln \frac{P\Theta}{\Lambda})^{0.5}$ when transforming the directly measurable quantities of (3) and (4).

The predictions of [5] and the present study use the lowest order QCD relations between the coupling α_s and the QCD scale Λ.

$$\alpha_s = \frac{\pi\beta^2}{6} \frac{1}{\ln(\frac{Q}{\Lambda})} \tag{11}$$

When transforming the data to compare with predictions, the values $\Lambda = 0.15$ or 0.3 GeV (in section 3) are used. On the other hand, when trying to obtain α_S or Λ directly from the data, the notation α_S^{eff} and Λ_{eff} (in section 4) is used. The reason for this is discussed in Section 4. All experimental measurements concern hadronic states. When comparing to the partonic states considered in the analytical calculations, the hypothesis of LPHD has to be used.

3 Comparison with the data

3.1 The data sample

The analysis uses about 600,000 selected e^+e^- events collected by DELPHI at $\sqrt{s} =$ 91 GeV in 1994. These statistics are adequate for our study. A sample of about 1200 high energy events at $\sqrt{s} = 183\ GeV$ collected in 1997 is used to investigate the energy dependence. The calculated hadron energy was required to be greater than 162 GeV (corresponding to a mean energy of 175 GeV). The standard cuts for hadronic events and track quality were applied as used in earlier studies of correlations [8]. The special procedures for selecting the high energy events are described in [9]. Detailed Monte Carlo studies were done using the JETSET 7.4 PS model [10]. Some results of ref. [5] using HERWIG [11] will be also mentioned for comparison.

Corrections were applied using events generated from a JETSET Monte Carlo simulation which had been tuned to reproduce general event characteristics [12]. These events were examined at

* **Generator level**
where all charged final–state particles (except electrons and muons) with a lifetime larger than 10^{-9} seconds have been taken,

* **Detector level**
  which includes distortions due to particle interactions with the detector material, other detector imperfections such as limited resolution, multi–track separation and detector acceptance, and the event selection procedures.

Using these events, the various angular correlation functions of order n, A_n, studied below were corrected ("bin by bin") by

$$A_n^{cor} = C_n A_n^{raw}, \quad C_n = \frac{A_n^{gen}}{A_n^{det}} \tag{12}$$

where the superscript "raw" indicates the correlation function calculated directly from the data, and "gen" and "det" denote those obtained from the JETSET Monte Carlo events at generator and detector level respectively. The simulated data at detector level were found to agree excellently with the corrected experimental data, e.g. fig.1a (for the definition of variables used in this figure see section 2). The measurement error on the relative angle ϑ_{12} between two outgoing particles was determined to be of order 0.5° (in the case of good Vertex Detector hits, even down to 0.1°).

In addition, all phenomena which were not included in the analytical calculations had to be corrected for, (i) initial state radiation, (ii) Dalitz decays of the π^0, (iii) residual K_s^0 and Λ decays near the vertex, and (iv) the effect of Bose–Einstein correlations. These corrections were estimated, "bin by bin" like those in eqn. (2), by switching the effects on and off. They were all small ($< 2\%$). The total correction factor C_n^{tot} is the product of all individual correction factors.

Fig. 1a shows an example of the corrections. The correction factor C_n^{tot} shown in fig. 1a for $\Theta = 45°$ is also valid for all $r(\vartheta_{12})$ with $15° \leq \Theta \leq 60°$. Systematic errors were obtained from the C_n^{tot} according to $\Delta A_n^{corr} = \pm|A_n^{raw}(C_n^{tot} - 1)/2|$. To maintain clarity, the following figures present the corrected data with statistical errors only. The systematic errors are presented in the tables.

3.2 Comparison of the measured correlations with the predictions

The unnormalized correlation function $\rho_2(\vartheta_{12})$ (in eqn. 3) was measured by counting pairs of particles in the relevant angular regions (defined by bins: $\vartheta_{12} \pm \frac{1}{2}$binwidth) [3]. The axis of the Θ-cone was experimentally determined by the sphericity axis. The denominator of (3), $\rho_1 \otimes \rho_1(\vartheta_{12})$, was evaluated using the method of event mixing [13], where particles are selected randomly from different events. It is then calculated in the same way as for real events (see also [14]). This method reveals all correlations, including those from hard gluon radiations. It is the normalization demanded by the analytical calculations in [5]. The values of ϑ_{12} vary from $\vartheta_{12} = \Theta$ down to $\vartheta_{12} \approx 1°$.

In the following, the predictions introduced in Section 2 are compared with the corrected experimental data:

a) $r(\vartheta_{12})$ is expected to rise with $\ln \frac{\Theta}{\vartheta_{12}}$ or ϵ and then level off for small ϑ_{12}. In fig. 1b, this dependence is investigated for several cone opening angles Θ (60, 45, 30 and 15 degrees). As expected, the slopes become smaller with bigger cone openings.

b) The expected scaling properties of the quantity $\frac{\ln(r(\vartheta_{12}))}{\sqrt{\ln P\Theta/\Lambda}}$ (eqn. (6)) are tested in figs. 2a and b.
   The dependence on the cone opening angle Θ is shown in fig. 2a. It can be seen that for $15° \leq \Theta \leq 60°$ the dependence on Θ is very weak already at $\sqrt{s} = 91\ GeV$,

in agreement with the predictions of eqn. (6). This scaling with respect to the variable ϵ is especially good for broader cones; for smaller values of Θ, uncertainties in the determination of the jet axis are expected to cause deviations. The shape predicted by eqn. (6) is shown as the dashed line in fig. 2a differing appreciably from the measurement. There is a "hook" in the data at small ϵ, the shape of the data is only similar to that predicted in the sense that it is rising and levelling off; the data are much smaller and flatter. Thus at LEP energies the analytic QCD calculations do not describe quantitatively the 2-particle angular correlations r(ϑ_{12}). The dependence on energy is shown in fig. 2b for a cone opening angle $\Theta = 45°$. The distribution of $\frac{\ln(r(\vartheta_{12}))}{\sqrt{\ln P\Theta/\Lambda}}$ at $\sqrt{s} = 183\ GeV$ (full circles) is much steeper than that at $\sqrt{s} = 90\ GeV$ (open circles). The agreement with the JETSET Monte Carlo simulation[1] is good (full resp. open triangles). The data show that at LEP energies a limiting function of ϵ is possibly reached only for $\epsilon \lesssim 0.2$ (for large relative angles ϑ_{12}). At $\sqrt{s} = 183\ GeV$, for larger values of ϵ, the measurement seems to become steeper than the prediction (dashed line in fig. 2b).

c) The expected scaling properties of $Y(\epsilon)$ (eqn. (10)) are tested in figs. 3a and b. The energy dependence of $Y(\epsilon)$ is shown in fig. 3a - for the cone opening of $\Theta = 45°$ - at $\sqrt{s} = 91\ GeV$ (open circles) and at $\sqrt{s} = 183\ GeV$ (full circles). Both distributions agree very well with each other in the whole ϵ region, therefore exhibiting scaling in energy. There is also good agreement with the coresponding JETSET simulations on both the partonic and hadronic level, which supports parton hadron duality[2].
In fig. 3b it is shown that Y(ϵ) is independent of Θ, it is therefore also scaling in the cone opening angle as predicted. Apart from the region of small ϵ (large ϑ_{12}) the overall agreement of data and prediction (dashed line) is good, especially when choosing $\Lambda = 0.3\ GeV$ in fig. 3b. Note that no arbitrary normalization has been applied in fig. 3.

Tables 1 and 2 give the numerical values of $r(\vartheta_{12})$ and $\tilde{r}(\vartheta_{12})$, restricting to each second bin.

4 Discussion

So far only qualitative statements have been made when comparing experimental data with the analytical calculations. Considering that the theoretical predictions contain important parameters of the strong sector of the standard model (α_S, Λ, n_f) one may try to discuss several items by estimating numerical values for these parameters from the data. Here some aspects of the 2-particle angular correlation measurements are discussed in a framework for future phenomenological advances.

* **Can α_S be measured using QCD predictions on angular correlations?**
In principle this should be possible, since the theoretical descriptions of angular correlations [5] do not contain arbitrary parameters (as in fragmentation models). However, it has to be assumed that LPHD is valid. It has to be remembered that the theoretical descriptions are derived from the double logarithmic approximation and can be regarded only as a first and simplified approach. Therefore α_S^{eff} and

[1]The high energy data at $\sqrt{s} = 183\ GeV$ exhibit a broad distribution of effective energies with a mean value $\sqrt{s_{eff}} = 175\ GeV$ (see sect. 3.1). Consequently also the Monte Carlo calculations have been performormed at this slightly lower energy.

[2]A more detailed comparison of partonic and hadronic levels in the context of the HERWIG Monte Carlo supporting local parton hadron duality and scaling in energy up to $\sqrt{s} = 1800\ GeV$ is given in [5].

$\Theta = 15^o$				$\Theta = 60^o$			
ϑ_{12}/Θ	$r(\vartheta_{12})$	$\pm\Delta$stat.	$\pm\Delta$syst.	ϑ_{12}/Θ	$r(\vartheta_{12})$	$\pm\Delta$stat.	$\pm\Delta$syst.
0.9643	0.923	0.003	0.015	0.9535	1.025	0.003	0.005
0.8313	0.927	0.003	0.016	0.7843	0.949	0.002	0.003
0.7167	0.955	0.003	0.019	0.6451	0.944	0.002	0.001
0.6179	0.992	0.003	0.022	0.5306	0.946	0.002	0.003
0.5327	1.029	0.003	0.025	0.4364	0.957	0.002	0.007
0.4592	1.066	0.004	0.027	0.3590	0.975	0.002	0.012
0.3959	1.104	0.004	0.029	0.2953	0.996	0.002	0.018
0.3413	1.141	0.005	0.029	0.2429	1.026	0.002	0.023
0.2943	1.172	0.005	0.028	0.1998	1.061	0.003	0.028
0.2537	1.195	0.006	0.025	0.1643	1.098	0.003	0.032
0.2187	1.220	0.006	0.020	0.1352	1.138	0.003	0.036
0.1885	1.243	0.007	0.014	0.1112	1.181	0.004	0.038
0.1626	1.255	0.008	0.005	0.0915	1.221	0.004	0.039
0.1401	1.269	0.010	0.004	0.0752	1.260	0.005	0.037
0.1208	1.269	0.011	0.014	0.0619	1.290	0.005	0.033
0.1042	1.265	0.013	0.025	0.0509	1.313	0.006	0.027
0.0898	1.260	0.015	0.035	0.0419	1.327	0.008	0.019
0.0774	1.262	0.017	0.044	0.0344	1.337	0.009	0.010

Table 1: Numerical values of $r(\vartheta_{12})$ for various $\frac{\vartheta_{12}}{\Theta}$ (with stat. and syst. errors) for cone openings $\Theta = 15^o$ and 60^o (fig. 1b).

$\Theta = 45^o$							
ϑ_{12}/Θ	$r(\vartheta_{12})$	$\pm\Delta$stat.	$\pm\Delta$syst.	binw.	$\tilde{r}(\vartheta_{12})$	$\pm\Delta$stat.	$\pm\Delta$syst.
0.9557	0.968	0.003	0.002	0.06961	0.04596	0.00017	0.00071
0.7938	0.907	0.002	0.003	0.05782	0.05077	0.00018	0.00195
0.6593	0.908	0.002	0.005	0.04802	0.04990	0.00017	0.00290
0.5476	0.927	0.002	0.009	0.03989	0.04717	0.00015	0.00366
0.4549	0.949	0.002	0.013	0.03313	0.04385	0.00014	0.00382
0.3778	0.975	0.002	0.017	0.02752	0.04014	0.00013	0.00361
0.3138	1.010	0.002	0.022	0.02286	0.03578	0.00012	0.00337
0.2607	1.044	0.003	0.027	0.01898	0.03124	0.00011	0.00291
0.2165	1.085	0.003	0.031	0.01577	0.02665	0.00009	0.00244
0.1798	1.125	0.003	0.035	0.01310	0.02222	0.00008	0.00191
0.1494	1.166	0.004	0.037	0.01088	0.01806	0.00007	0.00145
0.1241	1.204	0.004	0.038	0.00904	0.01429	0.00006	0.00106
0.1030	1.243	0.005	0.036	0.00751	0.01110	0.00005	0.00075
0.0856	1.272	0.005	0.033	0.00623	0.00833	0.00005	0.00053
0.0711	1.299	0.006	0.028	0.00518	0.00625	0.00004	0.00031
0.0590	1.309	0.007	0.021	0.00430	0.00459	0.00003	0.00017
0.0490	1.324	0.009	0.013	0.00357	0.00330	0.00003	0.00010
0.0407	1.310	0.010	0.003	0.00297	0.00234	0.00002	0.00003

Table 2: Numerical values of $r(\vartheta_{12})$ (fig. 1b) and $\tilde{r}(\vartheta_{12})$ (fig. 3a) for various $\frac{\vartheta_{12}}{\Theta}$ (with stat. and syst. errors) for cone opening $\Theta = 45^o$. The bin width is also given since it is needed for transforming $\tilde{r}(\vartheta_{12})$ to $Y(\epsilon)$.

Λ_{eff} obtained from fitting the data are to be considered only as effective parameters of the observables.

* **Values of α_S^{eff} obtained**
  Although no agreement of data and predictions were obtained in fig. 2a concerning the overall shape (it was verified that even varying the QCD parameters $0.04 \leq \Lambda \leq 0.8$ and $2 \leq n_f \leq 5$ cannot lead to an overall agreement), one could try to get some information from the slopes alone. Fig. 4a collects the values of α_S^{eff} obtained by fitting the anomalous dimension $\gamma_0 = \sqrt{\frac{6\alpha_S}{\pi}}$ in equation (5) for $r(\vartheta_{12})$. The fit range $5.7° \lesssim \vartheta_{12} \lesssim 13°$ has been chosen from fig. 1b by selecting the reasonably linear piece of $r(\vartheta_{12})$ with the steepest increase in the log–log plot. The hook in the data at larger angles, which is thought in [5] to be due to missing energy-momentum conservation in the calculations, prevents any meaningful fits being performed in this region.
* **The dependence on the choice of the jet axis**
  A possibly substantial systematic error on α_S^{eff} might arise from the poor determination of the jet–axis. The sphericity axis deviates, of course, from the "true" $q\bar{q}$–axis which has been adopted in the calculations. This problem has been investigated in a JETSET Monte Carlo study [16], showing that choosing the sphericity axis instead of the "true" $q\bar{q}$–axis decreased α_S^{eff} by about 30% (because of a smearing effect). Such Monte Carlo corrections for α_S^{eff} have been estimated for different Θ's and the corrected values are given in fig. 4a.
* **The dependence of α_S^{eff} on the opening angle Θ of the jet cone**
  In the theoretical predictions, the value of Q which sets the scale for (hard) gluon emission into the Θ-cone is set to $Q \approx P\Theta$, with $P = 45.05$ GeV/c. This causes e.g. a Θ–dependence of r(ϑ_{12}). In particular, it is expected that the slopes of $r(\vartheta_{12})$ in fig. 1b increase with decreasing Θ. This is indeed the case, as can be seen from the rise of α_S^{eff} for smaller cone openings Θ (fig. 4a). One cannot conclude that the observed "running" of α_S^{eff} is due to QCD alone. In a Monte Carlo study with JETSET on parton and hadron levels, similar Θ dependences even for fixed α_s are observed. Further investigations with different Monte Carlo models and with data at higher energies will be necessary to clarify this question.
  Fig. 4a also shows that a better agreement with the data can be obtained for small values of Λ and when the value of n_f is decreased from 5 to 3 or even lower. It should be noted that in the ideal case the theoretical calculations should decrease n_f according to the decreasing number of open flavors in the parton evolution ($n_f = 5 \rightarrow n_f = 3 \rightarrow n_f = 2$). It is argued [17] that the main contribution comes from $n_f = 3$. Our measurement favours a value significantly lower than 5.
* **The function $\tilde{r}(\vartheta_{12})$**
  This special scaling function found by Ochs and Wosiek [5] seems to be less sensitive to shortcomings of the DLA. The data show scaling in both in energy (fig. 3a) and in Θ (fig 3b). The "transformation" of the experimental measurement to the expression in eqn. (10) requires a specific value of Λ. Fig. 4b shows that the measured function $Y(\epsilon)$ is remarkably sensitive to the value of Λ chosen. The data "prefer" a value of $\Lambda_{eff} \approx 0.3$ GeV in order to be in good agreement with the prediction [5] (dashed line).
* **MLLA (Modified LLA)**
  In the ideal case the values of Λ_{eff} (or α_s^{eff}) should be the same when obtained from $r(\vartheta_{12})$ (fig. 4a) or from $\tilde{r}(\vartheta_{12})$ (fig. 4b). But whereas low values for α_s^{eff} (or

Λ_{eff}) are obtained in fig. 4a, it is demonstrated in fig. 4b that here the data prefer a $\Lambda_{eff} \approx 0.3$ GeV. It has to be noted that DLA takes only the leading singularities in both cases which could lead to different redefinitions of Λ_{eff}. In an improved calculation (e.g. MLLA) the difference should be diminished. In this context it is interesting to point to the fact that γ_0 which has been used in eqn. (5) to determine α_s^{eff} is the DLA anomalous dimension describing the multiplicity growth. Similar to the observation with $r(\vartheta_{12})$, in this latter case the DLA also predicts larger values than measured [16] but the agreement is substantially improved in MLLA [17]. Up to now no MLLA calculations exist for both $r(\vartheta_{12})$ and $\tilde{r}(\vartheta_{12})$.

* **LPHD**

The predictions of the correlations are only at the parton level. To make a meaningful comparison with the measured hadronic correlations, the influence of hadronisation has to be considered.

This was studied in the context of the HERWIG model by Ochs and Wosiek [5] for $r(\vartheta_{12})$, and LPHD was shown to hold in the region of not too small angles ϑ_{12} ($\gtrsim 3°$). This means that no pronounced difference exists between the partonic level when the parton cascade is continued down to $Q_0 \approx$ a few 100 MeV and the hadronic level where, starting from $Q_0 \geq 1$ GeV and adding hadronisation, the last step involves decays via known resonances. This similarity suggests that in the HERWIG model, the cascade decay of resonances might be similar to the last steps of the implemented "QCD" cascade, extended to the non-perturbative regime and that resonance decay does not destroy the correlation pattern at moderate angles.

Buschbeck et al [16] have shown that in the JETSET model with $\Lambda = 0.15$ GeV, on the other hand, the angular correlations on the partonic and hadronic levels deviate if Q_0 is as low as $2m_\pi$. In reference [16] a Monte Carlo study demonstrates that $r(\vartheta_{12})$ on the partonic level resembles that on hadronic level for $\vartheta_{12} \geq 5°(\epsilon \leq 0.42)$ if the cutoff Q_0 was chosen to be 0.6 GeV. For small angles $\lesssim 5°$, LPHD was not valid and resonance decays strongly influenced the correlation functions. The difference between HERWIG and JETSET concerning the validity of LPHD in the case of $r(\vartheta_{12})$ is presumably due to the different assumptions about the evolution of the parton cascades and the different hadronisation schemes[3]. In the angular region which has been selected in the present study to determine α_S^{eff} via linear fits to $r(\vartheta_{12})$, both models show only small disturbances due to resonance decay.

LPHD is reasonably fulfilled for $Y(\epsilon)$ in both JETSET (shown in this study) and HERWIG (shown in ref. [5]).

5 Summary and outlook

Two-particle angular correlations have been measured using data collected by the DELPHI detector (e^+e^- annihilations into hadrons at $\sqrt{s} = 91$ GeV and some low statistics data at 183 GeV) and have been compared with analytical predictions [5] for the corresponding parton correlations using the DLA of QCD and the concept of LPHD. Some of the predictions are fulfilled while others fail, namely:

The ϵ–scaling property implies that the correlation functions $r(\vartheta_{12})$ and $\tilde{r}(\vartheta_{12})$ are (after some transformation) functions of the scaling variable ϵ only, but not of the cone opening angle Θ nor of the jet momentum P. The correlation in the relative angle $r(\vartheta_{12})$ fulfills this scaling for the opening angle Θ rather well, but using data from $\sqrt{s} = 91$ to 183 GeV the scaling with jet momentum P is satisfied only in the small ϵ region. There

[3] Note also the different definitions of Q_0 in the Monte Carlos programs.

are substantial deviations of the measured $r(\vartheta_{12})$ from the predicted shape. On the other hand, the measured correlation function $\tilde{r}(\vartheta_{12})$, using a different normalization, is already rather close to the asymptotic predictions at LEP energies and exhibits scaling in Θ and P.

While the theoretical calculations use only first order relations and many approximations, the values of α_S and Λ obtained by fitting the data can be considered only as effective parameters, α_S^{eff} and Λ_{eff}. There is better agreement when the effective number n_f of flavors participating in the parton cascade is decreased well below 5. Generally, the largest deviations between data and theory occur at large angles ϑ_{12}. This indicates that including energy–momentum conservation (in particular the recoil effects) in the analytical calculations is necessary.

The question of validity of LPHD remains open in case of $r(\vartheta_{12})$, since previous studies using different models (JETSET, HERWIG) gave different answers. It could be shown in this study, however, that in case of $\tilde{r}(\vartheta_{12})$ LPHD is reasonably satisfied using JETSET, which is in agreement with published findings using HERWIG [5].

In summary, this first experimental study of angular correlations has provided support for some of the theoretical predictions. Not only are the data far away from the asymptotic energy, but also various approximations have been used. Therefore the sizeable deviations found in the comparison are not surprising. More checks on more refined predictions are desirable. It is to be hoped that these observations may provide valuable information which can be used to improve further the QCD calculations.

Acknowledgements

We thank W. Kittel, P. Lipa, J.-L. Meunier, W. Ochs, R. Peschanski and J. Wosiek for valuable discussions and stimulation.
We are greatly indebted to our technical collaborators, to the members of the CERN-SL Division for the excellent performance of the LEP collider, and to the funding agencies for their support in building and operating the DELPHI detector.
We acknowledge in particular the support of
Austrian Federal Ministry of Science and Traffics, GZ 616.364/2-III/2a/98,
FNRS–FWO, Belgium,
FINEP, CNPq, CAPES, FUJB and FAPERJ, Brazil,
Czech Ministry of Industry and Trade, GA CR 202/96/0450 and GA AVCR A1010521,
Danish Natural Research Council,
Commission of the European Communities (DG XII),
Direction des Sciences de la Matière, CEA, France,
Bundesministerium für Bildung, Wissenschaft, Forschung und Technologie, Germany,
General Secretariat for Research and Technology, Greece,
National Science Foundation (NWO) and Foundation for Research on Matter (FOM), The Netherlands,
Norwegian Research Council,
State Committee for Scientific Research, Poland, 2P03B06015, 2P03B03311 and SPUB/P03/178/98,
JNICT–Junta Nacional de Investigação Científica e Tecnológica, Portugal,
Vedecka grantova agentura MS SR, Slovakia, Nr. 95/5195/134,
Ministry of Science and Technology of the Republic of Slovenia,
CICYT, Spain, AEN96–1661 and AEN96-1681,
The Swedish Natural Science Research Council,
Particle Physics and Astronomy Research Council, UK,
Department of Energy, USA, DE–FG02–94ER40817.

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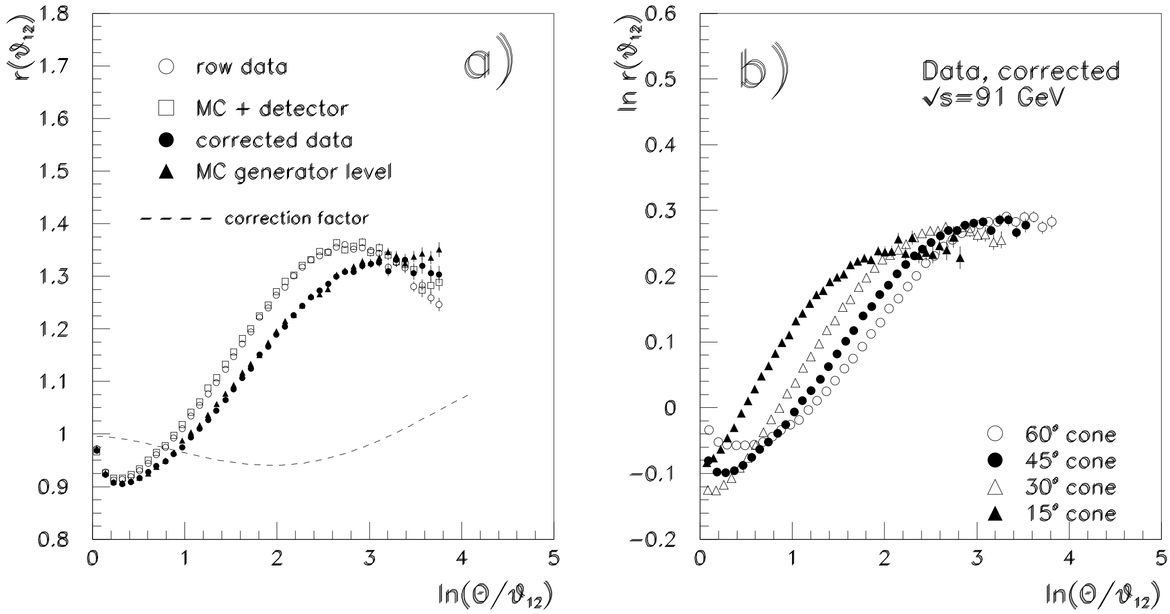

Figure 1: The normalized two–particle correlation function $r(\theta_{12})$, eqn. (3). **a)** The correction factor for $\Theta = 45°$; the dashed line is a best fit to the "bin by bin" correction factors C_n^{tot} including corrections for detector effects, Bose–Einstein correlations, Dalitz Pairs and residual K_S^0 and Λ decays. **b)** The corrected functions in forward cones of different half opening angles Θ; the values of ϑ_{12} range from $\vartheta_{12} = \Theta$ (left) down to $\vartheta_{12} \approx 1°$ (rightmost points). The errors are statistical only.

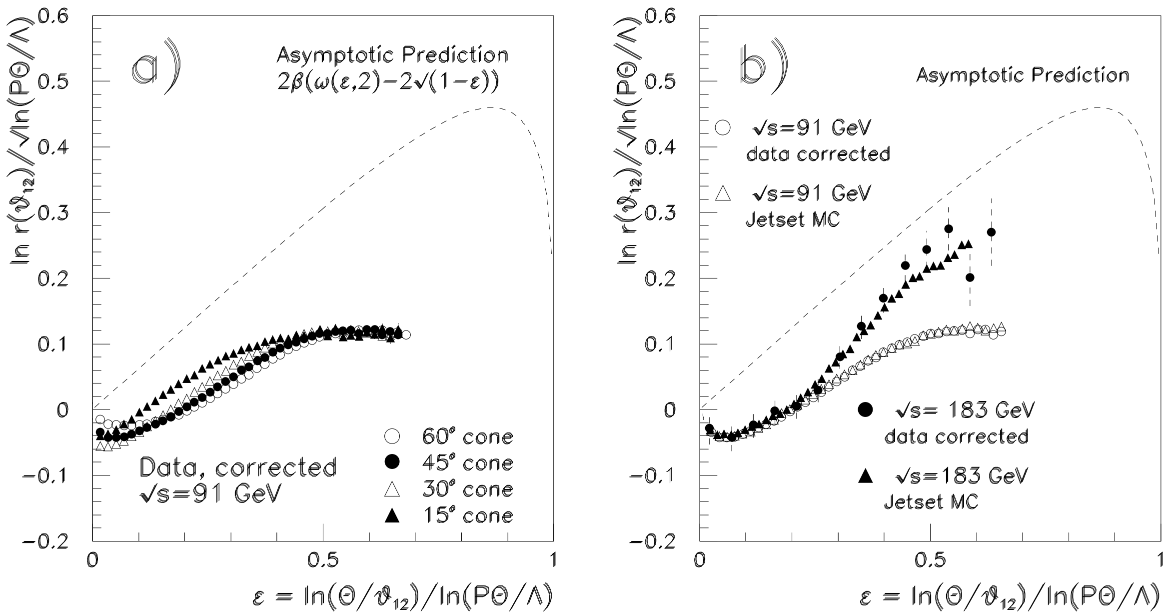

Figure 2: **a)** The data of fig. 1b are rescaled and plotted against the scaling variable ϵ in order to test eqn. (6), dashed line. **b)** The corrected data for $\Theta = 45°$, at 91 GeV (open circles) and at 183 GeV (full circles), using $\Lambda = 0.15\ GeV$, are shown together with JETSET Monte Carlo calculations (open resp. full triangles).

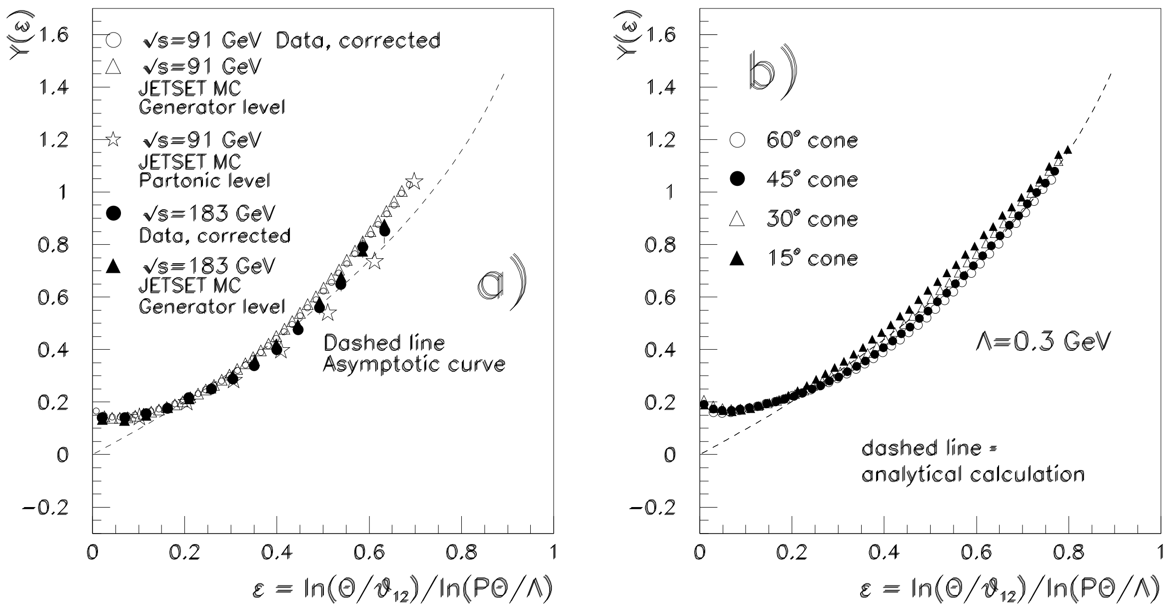

Figure 3: An energy independent scaling function $Y(\epsilon)$ (eqn. (10)) is extracted from the 2-body angular correlation function defined by eqn. (4). The dashed lines represent the asymptotic prediction eqn. (10). Statistical and systematic errors are smaller than 0.01 for the 91 GeV data. **a)** The corrected data for $\Theta = 45°$, at 91 GeV (open circles) and at 183 GeV (full circles), using $\Lambda = 0.15\ GeV$, are shown together with Monte Carlo calculations (open resp. full triangles). The range of ϑ_{12} corresponds to that in fig. 1. **b)** Test of the Θ-scaling behaviour of the data as predicted by eqn. (10), using $\Lambda = 0.3\ GeV$.

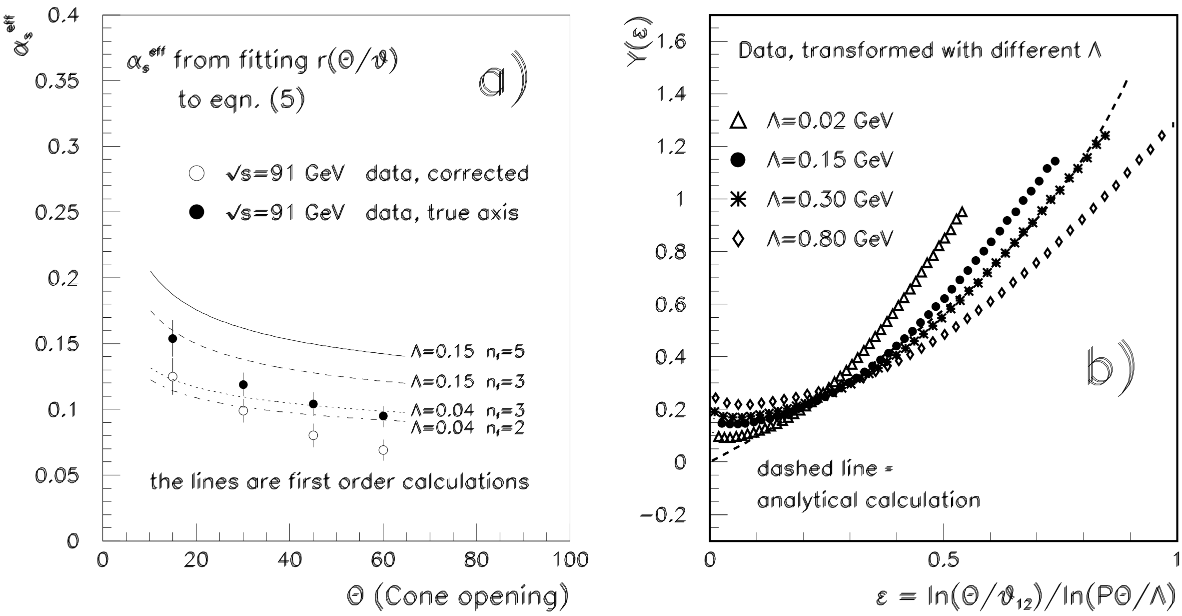

Figure 4: **a)** The measured values α_S^{eff} (∘) from eqn. (5) for different values of Θ are compared with lowest order QCD predictions eqn. (11), with $Q \approx P\Theta$, for different values of Λ and n_f . Applying Monte Carlo corrections for choosing the true axis (of the initial parton) increases the value for α_S^{eff} (•). The errors shown are systematic ones only, since the statistical errors are much smaller. **b)** Variation of the measured $Y(\epsilon)$ (see also fig. 3) by choosing different values of Λ.