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# Measurement of the $\mathrm{B}_{d}^{0}$ oscillation frequency using kaons, leptons and jet charge 

DELPHI Collaboration


#### Abstract

A measurement of the mass difference, $\Delta m_{d}$, between the two physical $\mathrm{B}_{d}^{0}$ states has been obtained from the analysis of the impact parameter distribution of a lepton emitted at large transverse momentum $\left(p_{t}\right)$ relative to the jet axis and from the analysis of the flight distance distribution of secondary vertices tagged by either a high $p_{t}$ lepton or an identified kaon. In the opposite hemisphere of the event, the charge of the initial quark has been evaluated using a high $p_{t}$ lepton, a charged kaon or the mean jet charge. With 1.7 million hadronic $Z^{0}$ decays recorded by DELPHI between 1991 and 1993, $\Delta m_{d}$ is found to be:


$$
\Delta m_{d}=0.531_{-0.046}^{+0.050}(\text { stat. }) \pm 0.078 \text { (syst.) } \mathrm{ps}^{-1}
$$

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[^0]
## 1 Introduction

In the Standard Model, the $\mathrm{B}_{q}^{0}-\overline{\mathrm{B}_{q}^{0}}(q=d, s)$ mixing is a direct consequence of second order weak interactions. Starting with a $\mathrm{B}_{q}^{0}$ meson produced at time $t=0$, the probability, $\mathcal{P}$, to observe a $\mathrm{B}_{q}^{0}$ decaying at the time $t$ can be written, neglecting effects from CP violation:

$$
\mathcal{P}\left(\mathrm{B}_{q}^{0} \rightarrow \mathrm{~B}_{q}^{0}\right)=\frac{\Gamma_{q}}{2} e^{-\Gamma_{q} t}\left(\cosh \left(\frac{\Delta \Gamma_{q}}{2} t\right)+\cos \left(\Delta m_{q} t\right)\right)
$$

where $\Gamma_{q}=\frac{\Gamma_{q}^{H}+\Gamma_{q}^{L}}{2}, \Delta \Gamma_{q}=\Gamma_{q}^{L}-\Gamma_{q}^{H}$ and $\Delta m_{q}=m_{q}^{L}-m_{q}^{H} . L$ and $H$ denote respectively the light and heavy physical states. The oscillation period gives a direct measurement of the mass difference between the two physical states.

For $\mathrm{B}_{d}^{0}$ mesons, the Standard Model predicts that $\Delta \Gamma_{d} \ll \Delta m_{d}$. Therefore the previous expression simplifies to:

$$
\mathcal{P}\left(\mathrm{B}_{d}^{0} \rightarrow \mathrm{~B}_{d}^{0}\right)=\Gamma_{d} e^{-\Gamma_{d} t} \cos ^{2}\left(\frac{\Delta m_{d} t}{2}\right)
$$

and similarly:

$$
\mathcal{P}\left(\mathrm{B}_{d}^{0} \rightarrow \overline{\mathrm{~B}_{d}^{0}}\right)=\Gamma_{d} e^{-\Gamma_{d} t} \sin ^{2}\left(\frac{\Delta m_{d} t}{2}\right) .
$$

Measurements of time integrated probabilities for mixing $\chi_{q}=\frac{x_{q}^{2}}{2\left(1+x_{q}^{2}\right)}$ with $x_{q}=\frac{\Delta m_{q}}{\Gamma_{q}}$ have been already derived from $\Upsilon(4 \mathrm{~S})$ and LEP results [1]. Time dependent oscillations of $\mathrm{B}_{d}^{0}$ mesons have also been measured at LEP [2].

The analysis presented here gives a measurement of $\Delta m_{d}$ based on data taken by DELPHI at LEP1 from 1991 to 1993. The principle of the method is the following: after having divided the charged and neutral particles from $Z^{0}$ decay into two hemispheres separated by the plane transverse to the sphericity axis, a "production sign" is defined on one side, which is correlated to the sign of the initial quark at the production point; in the other hemisphere the flight distance of the B hadron is evaluated and a "decay sign" is defined, correlated to the $\mathrm{B}^{0} / \overline{\mathrm{B}^{0}}$ nature of the decaying hadron. Three tagging procedures have been used, which give a measurement of the charge of the $b$ or $\bar{b}$ quark when it decays inside the $B$ hadron:

- Direct semileptonic decays of $b$ quarks produce a negative lepton, with a branching fraction close to $10 \%$ per each lepton flavour. These leptons have usually a larger transverse momentum, relative to the axis of the jet they belong to, than those produced in lighter flavour decays.
- The dominant decay chain $b \rightarrow c \rightarrow s$ generates an excess of $\mathrm{K}^{-}$relative to $\mathrm{K}^{+}$in B hadron decays (apart for $\overline{\mathrm{B}_{s}^{0}}$ mesons for which similar numbers of kaons of the two signs are expected). Kaons from B hadron decays can be isolated by requiring that their measured trajectory has a significant offset relative to the position of the $Z^{0}$ decay.
- Finally the value of the mean charge of particles produced in a jet is sensitive to the charge of the $b$ quark producing the jet.
In section 2 the components of the DELPHI detector which are important for this analysis are described. Section 3 presents the event selection, particle identification and Monte-Carlo simulation. In section 4 a measurement of $\Delta m_{d}$ is obtained from the dilepton event sample alone. In section 5 the analysis is extended to other tagging procedures and conclusions are given in section 6 . As the extended analysis includes most of the dilepton sample, only this last measurement has been quoted as the final result.


## 2 The DELPHI detector

The events used in this analysis were collected at LEP running near the $Z^{0}$ peak with the DELPHI detector [3]. The performance of the detector is detailed in [4]. The relevant parts for lepton identification are the muon chambers and the electromagnetic calorimeters. The Vertex Detector is used in combination with the central tracking devices to measure precisely the charged particle trajectories close to the beam interaction point.

The DELPHI reference frame is defined with $z$ along the $e^{-}$beam, $x$ towards the centre of LEP and $y$ upwards. Angular coordinates are $\theta$, measured from $z$, the azimuth, $\phi$, from the $x$-axis, while $R$ is the distance from the $z$-axis.

The muon chambers are drift chambers located at the periphery of DELPHI. The barrel part $(-0.63<\cos (\theta)<0.63)$ is composed of three sets of modules, each of two active layers, and gives $z$ and $R \phi$ coordinates. In the forward part, two layers of two planes give the $x$ and $y$ coordinates in the transverse plane. The precision of these detectors has to be taken into account for muon identification: it has been measured to be $\pm 1 \mathrm{~cm}$ in $z$ and $\pm 0.2 \mathrm{~cm}$ in $R \phi$ for the barrel part, and $\pm 0.4 \mathrm{~cm}$ for each of the two coordinates given by the forward part. The number of absorption lengths determines the hadron contamination and is approximatively 8 at $90^{\circ}$.

Electrons are absorbed in electromagnetic calorimeters; the High density Projection Chamber (HPC) covers the barrel part and provides three dimensional information on electromagnetic showers with 18 radiation lengths thickness. Calorimeters in the endcap regions are not used in this analysis because their acceptance is not matched with the solid angle covered by the vertex detector.

During the relevant period of data taking (1991 to 1993), the Vertex Detector (VD) [5] consisted of three cylinders of silicon strip detectors, at average radii of $6.3,9$ and 11 cm . This detector measured the coordinates of charged particle tracks in the transverse plane with respect to the beam direction with a precision of $\pm 8 \mu \mathrm{~m}$. The association of this detector to the central tracking system of DELPHI, consisting of the Time Projection Chamber (TPC) and the Inner and Outer Detectors, gave a $\sqrt{24^{2}+(69 / p)^{2}} \mu \mathrm{~m}(p$ in $\mathrm{GeV} / c$ units) precision on the impact parameter of charged particles with respect to the primary vertex. The 192 sense wires of the TPC also measure the specific energy loss, or $\mathrm{dE} / \mathrm{dx}$, of charged particles, as the $80 \%$ truncated mean of the amplitudes of the wire signals, with a minimum requirement of 30 wires. This $\mathrm{dE} / \mathrm{dx}$ measurement is available for $75 \%$ of charged particles in hadronic jets, with a precision which has been measured to be $\pm 7.5 \%$. It has been used for electron identification.

To identify kaons with momenta between 3 and $15 \mathrm{GeV} / c$ (this range corresponds to a typical kaon from a B decay), the gas radiator of the barrel Ring Imaging CHerenkov detector (RICH) is used: below $8.5 \mathrm{GeV} / c$, it works in the "veto" mode (kaons and protons give no Cherenkov photons and are thus distinguished from pions and leptons, but not from each other); above this threshold, kaons are distinguished from all other charged particles by measuring the radius of the ring of detected Cherenkov photons. A complete description of this detector is given in [6].

## 3 Event Selection

Hadronic decays of the $Z^{0}$ were selected by requiring the total energy of the charged particles in each hemisphere to exceed 3 GeV (assuming all charged particles to be pions), the total energy of the charged particles to exceed 15 GeV and at least 5 charged particles
with momenta above $0.2 \mathrm{GeV} / c$. For the dilepton analysis it has been required that detectors for lepton identification were operational. These requirements gave 204000 events in 1991, 589000 in 1992 and 572000 in 1993 with an efficiency ranging between $81 \%$ and $85 \%$. For the other analysis, the event thrust axis is required to be well within the acceptance of the Vertex Detector and the RICH through the condition $45^{\circ}<\theta_{\text {thrust }}<$ $135^{\circ}$. This selected 458000 events in 1992, 457000 in 1993 (the RICH information was available in $2 / 3$ of this sample), and 930000 events have been kept in the simulated sample.

Simulated events have been generated using the JETSET parton shower model[7] and including the full detector simulation[8]. Parameters in this simulation have been adjusted from previous studies [9] and those relevant for this analysis have been summarized in Table 1 [10].

| Parameter | mean value |
| :---: | :---: |
| $\mathrm{B}_{d}$ lifetime | 1.57 ps |
| $\mathrm{B}_{s}$ lifetime | 1.58 ps |
| $\mathrm{B}^{+}$lifetime | 1.63 ps |
| $b$-baryons lifetime | 1.18 ps |
| $\operatorname{Br}(b \rightarrow \ell)$ | 0.11 |
| $\operatorname{Br}(b \rightarrow c \rightarrow \bar{\ell})$ | 0.081 |
| $\operatorname{Br}(b \rightarrow \bar{c} \rightarrow l)$ | 0.008 |
| $\chi_{s}$ | 0.5 |

Table 1: Relevant parameters used in the simulation.

B hadron semileptonic decays have been simulated using the model of ISGW [11] with a fraction of $30 \%$ for $D^{* *}$ production.

Each selected event is divided into two hemispheres separated by the plane transverse to the sphericity axis. A clustering analysis based on the JETSET algorithm LUCLUS with default parameters is used to define jets using both charged and neutral particles [7]. These jets are used to compute the $p_{t}$ of each particle of the event, as the transverse momentum of this particle with respect to the axis of the jet it belongs to, after having removed this particle from its jet.

### 3.1 Lepton identification

A minimum momentum of $3 \mathrm{GeV} / c$ is required for muons and electrons.
Muon chamber information is associated to the information coming from the central tracking devices of DELPHI to identify muons in the regions $53^{\circ}<\theta<127^{\circ}$ (barrel part) and $20^{\circ}<\theta<42^{\circ}, 138^{\circ}<\theta<160^{\circ}$ (forward part) [4].

The identification of electrons is performed using information coming from the HPC in the region $45^{\circ}<\theta<135^{\circ}$, and the ionization measured in the Time Projection Chamber. Electrons from photon conversion are rejected. In the acceptance region of electromagnetic calorimeters and muon chambers, the global identification efficiencies for
muons and electrons in the selected momentum range and the corresponding probabilities for a hadron to be misidentified as a lepton are given in Table 2. These values have been obtained using the detailed simulation code of the DELPHI detector, DELSIM [4], and have been checked on real data using selected events samples such as $\mathrm{K}_{s}^{0} \rightarrow \pi^{+} \pi^{-}$, $\mathrm{Z}^{0} \rightarrow \mu^{+} \mu^{-}$, converted photons before the HPC, $\gamma \gamma \rightarrow \ell^{+} \ell^{-}$and hadronic $\tau$ decays [12].

|  | $\mathcal{P}(\ell \rightarrow \ell) \%$ | $\mathcal{P}(h \rightarrow \ell) \%$ |
| :---: | :---: | :---: |
| muon | 86. | 0.7 |
| electron | 60. | 0.4 |

Table 2: Mean values of lepton identification probabilities for real leptons and for hadrons, with momenta larger than $3 \mathrm{GeV} / \mathrm{c}$ and in the angular regions defined in the text.

### 3.2 Kaon identification

Charged kaons were identified in the RICH using the standard DELPHI algorithm [4]. The efficiencies were evaluated from the simulation and verified using pure subsamples from $\mathrm{K}^{0}, \Lambda, \phi$ decays: the probability for a $\mathrm{K}^{ \pm}$within the barrel RICH angular acceptance to be identified is $65 \%$ for $3<p<8.5 \mathrm{GeV} / c$ and $80 \%$ for $8.5<p<15 \mathrm{GeV} / c$. The probability for a pion to be seen as a kaon is $8 \%$, that for a proton to be seen as a kaon is $68 \%$ for $3<p<8.5 \mathrm{GeV} / c$ and $26 \%$ for $8.5<p<15 \mathrm{GeV} / c$.

## 4 Measurement of $\Delta m_{d}$ from the dilepton sample

The dilepton sample is considered first because this is the simplest channel to select and results can be compared with similar analyses performed in other experiments [2].

Both muons and electrons are selected with a $p_{t}$ larger than $1 \mathrm{GeV} / c$. If several leptons are found in a given hemisphere, only the one with the highest $p_{t}$ is kept. The $p_{t}$ cut dependence of the final result will be considered as a possible systematic uncertainty. Each lepton is considered in turn to extract time information and its charge is compared with the charge of the lepton in the opposite hemisphere, to tag the oscillation. The lepton containing time information is required to be associated to at least two hits in different layers of the Vertex Detector (VD), while there is no requirement of VD information for the other lepton. As a result, each $Z^{0}$ decay containing two leptons in opposite hemispheres may lead to one or two time measurement(s).

In real data, the selected dilepton sample yields 1073 time measurements associated with a same sign correlation, and 2151 associated with an opposite sign correlation. 2055 muons and 1169 electrons are selected on the measurement side, and the tagging side sample contains 2057 muons plus 1167 electrons. The efficiency of the VD selection has been found to be $88 \pm 1 \%$ for muons and $82 \pm 1 \%$ for electrons.

The composition of the total sample expected from the simulation (all sign combinations mixed together) is given in Table 3. The main component is from direct $b$ semileptonic decays. The fraction of cascade $(b \rightarrow c \rightarrow \bar{\ell})$ decays is of the order of $10 \%$ and the fraction of charm events remains small. The cut at $1 \mathrm{GeV} / \mathrm{c}$ on the lepton transverse momentum has been optimized knowing that the signal from oscillations depends on the difference between the fractions of direct and cascade semileptonic decays, and on the
number of selected events. The remaining events contain the following categories, with small associated fractions: semileptonic decays in a $b$ decay chain, fake leptons in a $b$ decay chain, charged particles coming from the primary vertex in $b \bar{b}$ events and light quark events.

| Lepton origin | Relative fraction |
| :---: | :---: |
| $b \rightarrow \ell$ | $79.4 \%$ |
| $b \rightarrow c \rightarrow \bar{\ell}$ | $9.6 \%$ |
| other leptons in a $b$ decay chain | $3.0 \%$ |
| fake leptons in a $b$ decay chain | $2.7 \%$ |
| primary vertex particles in a $b$ hemisphere | $1.5 \%$ |
| $c \rightarrow \bar{\ell}$ lepton | $3.3 \%$ |
| $u, d, s \rightarrow$ lepton | $0.5 \%$ |

Table 3: Composition of the simulated sample on the impact parameter measurement side.

### 4.1 Measurement of the $B$ decay time using the lepton impact parameter

The measurement of $\Delta m_{d}$ is extracted from a study of the impact parameter distribution obtained from same sign and opposite sign dilepton events: a primary vertex is reconstructed in the transverse ( $R \phi$ ) plane for each event, through an iterative vertex fit including the beam profile information [13,4], where at each step the track contributing most to the $\chi^{2}$ is removed until none contributes more than $5 \chi^{2}$ units. The impact parameter $\delta$ of the lepton is measured with respect to the primary vertex position and has a positive or negative lifetime sign depending on the relative position of the primary vertex and the intersection of the lepton with the jet along the jet direction. $t_{\delta}=\delta / \mathrm{c}$ is the time sensitive variable which is used in the analysis.

The precision on the impact parameter is dominated by the accuracy on the primary vertex determination (approximatively $\pm 50 \mu \mathrm{~m}$ in the horizontal and $\pm 20 \mu \mathrm{~m}$ in the vertical direction).

The correlation between $t_{\delta}$ and the true proper time of the B meson is smeared by the decay kinematics and by the energy distribution of B mesons. The effective proper time resolution $\left(t_{\delta}-t_{B}^{\text {true }}\right) / t_{B}^{\text {true }}$ has a width of $70 \%$ and a $40 \%$ shift towards lower values due to the reconstruction procedure which is performed in the transverse plane only; such a resolution is in fact sufficient to study the slow $\mathrm{B}_{d}^{0}$ oscillation.

The data and simulated samples are composed of events with one $t_{\delta}$ measurement in a single hemisphere, and of events with $t_{\delta}$ measurements in both hemispheres. The data sample contains 249 same sign and 455 opposite sign events with one measurement, 412 same sign and 848 opposite sign events with two measurements. When $\Delta m_{d}$ is set to 0.45 $\mathrm{ps}^{-1}$ in the simulation, the simulated sample contains 752 same sign and 1418 opposite sign events with one measurement, and 1407 same sign and 3032 opposite sign events with two measurements.

### 4.2 Fitting method and results

The $t_{\delta}$ distribution obtained from same sign events has been divided by the $t_{\delta}$ distribution obtained using opposite sign events, and the resulting distribution will be referred to as the $R_{++/+-}\left(t_{\delta}\right)$ distribution (Figure 1). Each $t_{\delta}$ measurement enters once in this distribution. In the absence of $\mathrm{B}_{d}^{0}$ oscillations, the distribution is expected to be flat at large $t_{\delta}$; the observed time dependence comes from $\mathrm{B}_{d}^{0}$ oscillations.

The $R_{++/+-}$distributions obtained from the data and from the simulation have been compared using a binned $\chi^{2}$ fit with $\Delta m_{d}$ as the only free parameter. For a given $\Delta m_{d}$ in the fit the simulated data have been adjusted to conform to the probabilities given in the introduction. Events with one and two time measurements have been treated separately in the fit. For events with two measurements, the two-dimensional distributions $R_{++/+-}\left(t_{\delta 1}, t_{\delta 2}\right)$ have been compared. The binning used in this fit and in the following has been defined such that each bin or each box contains at least 10 events. To satisfy this condition, 3 bins were used for events with a single $t_{\delta}$ measurement (grouping the last 3 bins of Figure 1 together), and a $3 \times 3$ grid was used for events with two measurements.

The $\Delta m_{d}$ measurement obtained in this way is

$$
\Delta m_{d}=0.47 \pm 0.08 \mathrm{ps}^{-1}
$$

and the corresponding $\chi^{2}$ value is found to be 7.5 for 11 degrees of freedom.
Figure 2 shows the fraction of same sign correlations, $F_{++}$, corresponding to the final $\Delta m_{d}$ measurement. This representation shows the different components of the same sign sample, listed in Table 4.

| Event type | fraction |
| :---: | :---: |
| $\overline{\mathrm{B}_{d}^{0}} \rightarrow \mathrm{~B}_{d}^{0} \rightarrow \bar{l}$ | 0.121 |
| $\overline{\mathrm{~B}_{s}^{0}} \rightarrow \mathrm{~B}_{s}^{0} \rightarrow \bar{l}$ | 0.105 |
| $b \rightarrow c \rightarrow \bar{l}$ | 0.209 |
| unmixed $\overline{\mathrm{B}^{0}}, \mathrm{~B}^{ \pm}, \Lambda_{b} \rightarrow l$ | 0.455 |
| $b \rightarrow$ fake $l$ | 0.032 |
| others | 0.078 |

Table 4: Composition of the same sign sample on the $t_{\delta}$ measurement side.

It can be seen from Figure 2 that the time evolution is due to mixed $\mathrm{B}_{d}^{0}$ events, the behaviour of the other components being flat. The amount of mixed $\mathrm{B}_{d}^{0}$ and mixed $\mathrm{B}_{s}^{0}$ in the same sign sample are similar, as expected since $f_{d} \chi_{d}$ is known to be roughly equal to $f_{s} \chi_{s}$. In these expressions $f_{d}$ and $f_{s}$ are the respective fractions of $\mathrm{B}_{d}^{0}$ and $\mathrm{B}_{s}^{0}$ mesons in a $b$ jet.

Since cascade decays fake mixed $\mathrm{B}^{0}$ decays, there is a large contribution from $b \rightarrow c \rightarrow \bar{l}$ events. The direct $b \rightarrow l$ contribution is close to $50 \%$ because the oscillation may have occurred in the opposite (tagging) hemisphere. The contributions from other categories of events are small: $b \rightarrow$ fake lepton, $b \rightarrow X \rightarrow l$ decay chains, primary vertex particles in $b \bar{b}$ events, and light quark ( $u d s c$ ) events (roughly one third of the $7.8 \%$ quoted in Table 4 for each of these last three categories).

### 4.3 Study of systematic uncertainties

The systematic uncertainty on the above measured value is computed by varying, in turn, the relevant parameters of the simulation and then re-making the $\Delta m_{d}$ fit.

Four dominant effects have been studied (see Table 5):

- B hadron lifetimes: the largest contribution comes from the uncertainty on the $\mathrm{B}_{d}^{0}$ lifetime.
- B hadron production rate: the fractions $f_{d}$ and $f_{s}$ of $\mathrm{B}_{d}^{0}$ and $\mathrm{B}_{s}^{0}$ mesons produced in the fragmentation of a $b$ quark have been obtained from the measurements of the integrated oscillation rates $\bar{\chi}$ and $\chi_{d}$ obtained at LEP, by other experiments and at the $\Upsilon(4 \mathrm{~S})[14]$. The fraction of $b$-baryon production, $f_{b-\text { baryon }}=(11.6 \pm 3.2 \%)$ has been taken from measurements of $\Lambda_{c}$ production in $c$ jets [15], assuming that it is similar for $\Lambda_{b}$ in $b$ jets and using a rate of $(2 \pm 2 \%)$ for $\Xi_{b}$ states. From the expressions:

$$
1=f_{d}+f_{u}+f_{s}+f_{b-b a r y o n} \text { and } \bar{\chi}=f_{d} \chi_{d}+f_{s} \chi_{s}
$$

and assuming $f_{d}=f_{u}$ because, for B mesons, as opposed to D mesons, no asymmetry between the $\mathrm{B}_{d}^{0}$ and $\mathrm{B}^{-}$production rates is expected from the decay of excited states one obtains:

$$
f_{d}=\frac{\left(1-f_{b-b a r y o n}\right) \chi_{s}-\bar{\chi}}{2 \chi_{s}-\chi_{d}} \text { and } f_{s}=\frac{2 \bar{\chi}-\left(1-f_{b-b a r y o n}\right) \chi_{d}}{2 \chi_{s}-\chi_{d}} .
$$

Using $\chi_{s}=0.5, \bar{\chi}=0.116 \pm 0.006$ and $\chi_{d}=0.168 \pm 0.016$ one obtains $f_{d}=$ $(39.2 \pm 2.2 \%)$ and $f_{s}=(10.0 \pm 2.2 \%)$. The uncertainties on $f_{d}$ and on $f_{s}$ appear to be weakly correlated. The error on $f_{d}$ is dominated by the uncertainty on $f_{b-\text { baryon }}$ and the error on $f_{s}$ receives similar contributions from the errors on $\bar{\chi}$ and on $\chi_{d}$. As a consequence, in the following, it has been assumed that $f_{d}$ and $f_{s}$ vary independently and that their corresponding variation is compensated by a change in $f_{b-\text { baryon }}$.

- Lepton origin: uncertainties on direct and cascade branching fractions remain the second source of uncertainty after the effect from $f_{d} / f_{s}$. Uncertainties on direct and cascade semileptonic branching fractions have been treated independently and the quoted values include the additional uncertainties coming from the modelling of the decays.
- Finally, $\chi_{s}$ is varied separately, and the corresponding variation on $\Delta m_{d}$ is 0.015 $\mathrm{ps}^{-1}$.

It has been checked that the contribution from the uncertainty on the $b$ quark fragmentation distribution is negligible by fitting the simulated $b$ fragmentation function with a Peterson function, and changing the $\epsilon$ parameter so that the mean fraction of the beam energy, taken by the B meson, varies between 0.68 and 0.72 .

Performing the fit on the sample of events which give only one $t_{\delta}$ measurement leads to:

$$
\Delta m_{d}=0.46_{-0.12}^{+0.15} \mathrm{ps}^{-1}
$$

and using only events with two $t_{\delta}$ measurements gives:

$$
\Delta m_{d}=0.47_{-0.10}^{+0.11} \mathrm{ps}^{-1}
$$

It has also been checked that the results found using muons or electrons only, as well as 1992 or 1993 data only, were compatible within the statistical errors. Because of the limited statistics, these last checks were done using a global one-dimensional fit to the $R_{++/+-}\left(t_{\delta}\right)$ distribution with each $t_{\delta}$ measurement entering once.

| Parameter | Central value | Uncertainty | Effect on $\Delta m_{d}\left(\mathrm{ps}^{-1}\right)$ |
| :---: | :---: | :---: | :---: |
| $B_{d}$ lifetime | 1.57 ps | $\pm 0.05 \mathrm{ps}$ | $\mp 0.012$ |
| global $b$ lifetime | 1.567 ps | $\pm 0.020 \mathrm{ps}$ | $\mp 0.003$ |
| $f_{d}$ | 0.392 | $\pm 0.022$ | $\mp 0.010$ |
| $f_{s}$ | 0.100 | $\pm 0.022$ | $\mp 0.045$ |
| $\operatorname{Br}(b \rightarrow l)$ | 0.110 | $\pm 0.010$ | $\pm 0.050$ |
| $\operatorname{Br}(b \rightarrow c \rightarrow \bar{l})$ | 0.081 | $\pm 0.008$ | $\mp 0.046$ |
| $c \rightarrow \bar{l}$ fraction | 0.030 | $\pm 0.003$ | $\pm 0.003$ |
| Background fraction | 0.034 | $\pm 0.007$ | $\mp 0.015$ |
| $\chi_{s}$ | 0.50 | -0.03 | +0.015 |

Table 5: Contributions to the systematic error on $\Delta m_{d}$.

Finally, the $p_{t}$ cut was varied (keeping the same momentum cuts), and no systematic effect was observed (Figure 3).

Adding in quadrature the contributions to the systematic error given in Table 5 yields the result:

$$
\Delta m_{d}=0.47 \pm 0.08(\text { stat. })_{-0.08}^{+0.09}(\text { syst. }) \mathrm{ps}^{-1} .
$$

## 5 Measurement of $\Delta m_{d}$ using leptons, $\mathrm{K}^{ \pm}$and jet charge

In the following, the analysis of section 4 has been extended to include additional final states using identified charged kaons and the mean jet charge. The B decay distance will also be evaluated in space to enhance the sensitivity of the measurement to oscillations.

Tagging procedures used to define the samples of mixed and unmixed events candidates are described along with the algorithm used to measure the B hadron decay distance. The fitting procedure used to measure $\Delta m_{d}$ is then given and the sources of systematic uncertainties are analyzed. The tagging probabilities and decay distance parametrizations have been evaluated for seven categories of hemispheres: $u / d, s, c, \mathrm{~B}^{ \pm}, \mathrm{B}_{d}^{0}, \mathrm{~B}_{s}^{0}, b$-baryons.

For each category and each tagging indicator the probabilities $\mathcal{P}_{\text {tag }}^{\text {right }}$ and $\mathcal{P}_{\text {tag }}^{\text {urong }}$, to have a right sign or a wrong sign, have been obtained from simulation. They have been defined as "right" ("wrong") if the "production" sign is the same as (opposite to) the sign of the quark or of the antiquark emitted in the hemisphere, assuming that its flavour is always beauty. For $\mathrm{B}_{d}^{0}$ mesons these probabilities include the effect of the time-integrated oscillation: they have been computed for different values of $\Delta m_{d}$ and the corresponding values are compatible with a linear interpolation. The $\mathrm{B}_{s}^{0}$ oscillation frequency is assumed to be high enough to give constant values, independent of the decay distance. Table 6 gives the results.

### 5.1 Tagging procedures

The measurements of the signs of three variables have been used, in a combined way, to establish the presence of a $b$ or a $\bar{b}$ quark in a given hemisphere ("production" sign) and to identify a B or a $\overline{\mathrm{B}}$ meson when it decays ("decay" sign). These variables are:

- The charge of leptons emitted at $p_{t}$ larger than $1 \mathrm{GeV} / c$. Lepton selection cuts are similar to those applied in section 4. If two particles of opposite sign fulfill these conditions, in the same hemisphere, the event is not kept. This "production" sign is defined in $9 \%$ of the $b$ hemispheres and the ratio right sign/wrong sign to correctly identify a $b$ or a $\bar{b}$ quark is about 5 .
- The charge of kaons produced at a secondary vertex. This tag is based on the dominant decay chain $b \rightarrow c \rightarrow s$ which implies that the sign of a secondary kaon is strongly correlated to the sign of the decaying quark. This has been studied in detail and measured in [16], and the fractions of decays with a right/wrong sign kaon (excluding ambiguous cases with a $\mathrm{K}^{+}$and a $\mathrm{K}^{-}$in the same hemisphere ) are roughly $50 \%$ and $10 \%$ respectively. Kaons have been identified by the RICH detector in the range $3<p<15 \mathrm{GeV} / c$. An additional cut is applied to favour a secondary $K^{ \pm}$from a $B$ by requiring that its impact parameter with respect to the main vertex be larger than 1.5 times the measurement error. This production sign is defined in $23 \%$ of cases, i.e. more frequently than the lepton sign, but the ratio right sign/wrong sign, of about 2, is lower. More background is also expected from $u, d, s, c$ flavors.
- The mean jet charge, which is a weighted sum of the charges of particles belonging to the most energetic jet in the hemisphere. It depends on the charge of the quark producing this jet and is defined as:

$$
Q_{j e t}=\left(\sum q_{i} p_{i}^{\kappa}\right) /\left(\sum p_{i}^{\kappa}\right)
$$

where, in the denominator, the sum is extended also to neutral particles. $p_{i}$ and $q_{i}$ are the individual particle momenta and charges. The weighting exponent $\kappa$ is chosen to be 0.6 to optimize the discrimination [17] (however its precise value is not crucial). The mean value of $Q_{j e t}$ is slightly biased by nuclear interactions in the detector: in practice the sign of $\left(Q_{j e t}-0.015\right)$ gives the right $b / \bar{b}$ nature in $62 \%$ of the cases.
The $b$ purity, which is the fraction of $b \bar{b}$ events in the selected sample, is improved by applying a $b$-tagging condition obtained from the values of the impact parameters, $\delta$, of the $n_{\text {tr }}$. charged particles in the jet, relative to the event main vertex:

$$
\frac{1}{n_{t r .}-1} \sum_{i=1}^{n_{t r .}-1} \frac{\delta_{i}^{2}}{\sigma_{\delta_{i}}^{2}}>2
$$

The track with the largest contribution has been excluded from the sum to reduce effects of secondary interactions, decays or poor measurements. Uncertainties on track impact parameters, $\sigma_{\delta}$, include track measurement errors and uncertainties on the $Z^{0}$ decay point.

To optimize the discrimination and to reduce double counting, the "production" sign has been obtained using the following indicators for a hemisphere:

- Jet sign and lepton sign $(j+l)$ in agreement: this discriminates better than the lepton sign alone. This indicator is defined in $8 \%$ of the $b$ hemispheres.
- Jet sign and kaon sign in agreement $(j+\mathrm{K})$ : this indicator is defined in $16 \%$ of the $b$ hemispheres.
- Jet sign alone with $b$-tagging condition $(j+b$ tag $)$ : this is used when the other criteria are not defined, provided $\left|Q_{j e t}-0.015\right|>0.1$. This condition gives on average $67 \%$ of right sign assignments and is defined for $39 \%$ of the $b$ hemispheres.

Events with a kaon and a lepton having a sign in agreement with the jet charge enter twice.

The probabilities, per event hemisphere, to measure a production sign have been obtained from the simulation and are given in Table 6.

To define the decay signs (see Table 7 discussed below), only leptons and kaons have been used because the mean jet charge is not sensitive enough to the transformation of a $\mathrm{B}^{0}$ into a $\overline{\mathrm{B}^{0}}$ inside a jet. Leptons have been selected in the same way as before but, for kaons, the condition on the impact parameter has been removed, in order not to bias the B meson flight distance distribution.

| Category | $\begin{array}{\|c\|} \hline \hline \mathcal{P}_{\text {tag }}^{\text {right }} \\ \mathcal{P}_{\text {tag }}^{\text {urg }} \\ (l+j) \\ \hline \end{array}$ | $\begin{aligned} & \hline \hline \mathcal{P}_{\text {tag }}^{\text {right }}(\mathrm{K}+j) \\ & \mathcal{P}_{\text {tag }}^{\text {trong }}(\mathrm{K}+j) \\ & \hline \end{aligned}$ | $\begin{array}{\|l\|} \hline \hline \mathcal{P}_{\text {tag }}^{\text {right }} \\ \mathcal{P}_{\text {tag }}^{\text {wiong }} \\ (j+b \operatorname{tag}) \end{array}$ | $\begin{array}{ll} \hline \mathcal{P}_{\text {tag }}^{\text {right }} & \text { (total) } \\ \mathcal{P}_{\text {tag }}^{\text {urong }} & \text { (total) } \\ \hline \end{array}$ |
| :---: | :---: | :---: | :---: | :---: |
| $u / d$ | $0.0072(01)$ | 0.0247(02) | 0.0500(03) | $0.0819(03)$ |
|  | $0.0044(01)$ | $0.0152(02)$ | $0.0305(02)$ | $0.0501(02)$ |
| $s$ | $0.0075(02)$ | 0.0387(04) | 0.0495(04) | 0.0957(06) |
|  | $0.0025(01)$ | $0.0166(02)$ | 0.0279(03) | 0.0470(04) |
| c | $0.0041(01)$ | 0.0577(05) | 0.0740 (06) | $0.1358(08)$ |
|  | $0.0170(03)$ | $0.0364(04)$ | $0.1025(07)$ | 0.1559(09) |
| $\mathrm{B}^{ \pm}$ | 0.0716 (08) | 0.1434(11) | 0.2586(15) | $0.4736(20)$ |
|  | 0.0073(03) | $0.0327(05)$ | $0.1262(10)$ | $0.1662(12)$ |
| $\mathrm{B}_{d}^{0}$ | $0.0565(07)$ | $0.1023(09)$ | $0.2303(14)$ | $0.3891(18)$ |
|  | - $0.020 \delta m$ | - $0.036 \delta \mathrm{~m}$ | - $0.025 \delta \mathrm{~m}$ | - $0.081 \delta m$ |
|  | $0.0214(04)$ | 0.0520(06) | 0.1620(12) | 0.2354(14) |
|  | + $0.020 \delta m$ | + $0.033 \delta m$ | + 0.026 淐 | + $0.079 \delta m$ |
| $\mathrm{B}_{s}^{0}$ | 0.0380(10) | $0.0898(16)$ | $0.2036(24)$ | $0.3314(31)$ |
|  | $0.0356(10)$ | $0.0659(13)$ | $0.1675(22)$ | 0.2690(27) |
| b-baryons | 0.0604(23) | 0.0971(29) | $0.1818(40)$ | 0.3393(54) |
|  | $0.0070(08)$ | $0.0664(24)$ | 0.1711(39) | $0.2445(46)$ |

Table 6: Probabilities of right/wrong sign in the production hemisphere. For $B_{d}^{0}$, the quoted values correspond to $\Delta m_{d}=0.50 \mathrm{ps}^{-1}$ and have a linear dependence on $\delta m=\Delta m_{d}-0.50$. The numbers in parentheses are the 1 -standard deviation statistical uncertainties only on the last digits.

### 5.2 Measurement of the $B$ decay distance

As the $\mathrm{B}_{d}^{0}-\overline{\mathrm{B}_{d}^{0}}$ oscillation period is expected to be large it is not crucial to have a very accurate evaluation of the B decay time. As the $b$ quark fragmentation distribution is peaked at large values, the $B$ decay distance distribution is still sensitive enough and has been used throughout this analysis.

The interaction point has been determined following the approach explained in section 4.1. To evaluate the $B$ decay point, advantage was taken of the gathering of $B$ decay products around the jet axis by selecting well-measured charged particles with at least 2 points associated in the Vertex Detector and situated within $25^{\circ}$ of the thrust axis of the most energetic jet in the hemisphere. A "pseudo-secondary vertex" has been fitted using this set of particles which is a mixture of primary and secondary particles, often including further decay products of a D coming from the B itself with probabilities that
are independent of the B flight. In order to keep long flight distances no cut has been applied on the $\chi^{2}$ probability of this vertex. As a result, the expectation value, $\lambda$, of the distance from the primary vertex to this pseudo-secondary vertex depends linearly on the actual flight as shown in Figure 4. The tails in the distributions have been removed partially by a cut on the longitudinal error of the vertex fit. This procedure does not bias the decay distance distribution because it does not rely on the actual value of the $\chi^{2}$ of the vertex fit. Figure 5 compares the normalized $\lambda$ distribution for data and simulated events. An additional smearing of the simulated distribution has been performed by adding a random term distributed according to a Breit-Wigner function, with a halfwidth of $140 \mu \mathrm{~m}$, so that the two distributions agree for negative values of $\lambda$.

### 5.3 Parametrization of the decay distance distributions

The basic assumption is that the probabilities of correctly defining the production sign, the decay sign and the distance estimate are independent for a given flavour. As a consequence the two hemispheres can be parametrized separately but the different B hadrons have to be distinguished.

Let $\lambda$ be the flight distance estimator and $\mathcal{P}_{\text {cat }}(\lambda)$ the probability density distribution to observe a given value of $\lambda$ in a hemisphere for a given category. For $b$ categories the distribution $\mathcal{P}_{\text {cat }}(\lambda)$ is the convolution of the exponential $b$-decay time distribution with the $b$ quark fragmentation function and with the resolution function on $\lambda$. As a result, it is difficult to write a reliable analytic expression for $\mathcal{P}_{\text {cat }}$. The simplest solution is to take this distribution from the simulation. Small values of $\lambda$ are largely contaminated by the $u, d, s, c$ background and contain very little information on the oscillation frequency, because they correspond to times where the $\sin ^{2}\left(\Delta m_{d} t / 2\right)$ factor is small. For $\lambda$ larger than 2 mm the $\mathcal{P}_{\text {cat }}(\lambda)$ distributions are close to exponentials. Figure 6 shows how the slopes corresponding to different $B$ hadrons, simulated with the same lifetime, exhibit slight differences due to the topological selection because the fractions of primary and secondary particles at the pseudo-secondary vertex are not exactly the same. For $u / d, s$ categories, the shape of $\mathcal{P}_{\text {cat }}(\lambda)$ is dominated by the resolution (see Figure 7), while, for the $c$ category, charm decays give also a significant contribution. The $\mathcal{P}_{\text {cat }}(\lambda)$ distributions have been parametrized using the exponential of a fourth order polynomial in the range $\lambda>2 \mathrm{~mm}$.

These distributions have been normalized so that $\int_{\lambda_{\text {min }}}^{\infty} \mathcal{P}_{\text {cat }}(\lambda)=1$.
As for the quark tagging procedures, at decay time the B meson can be correctly signed or not by the lepton or kaon charges and the corresponding probabilities ( $\mathcal{P}_{\text {dec,cat }}^{\text {right/urong }}$ ) have been evaluated for all values of $\lambda$ larger than $\lambda_{\text {min }}$.

For charged B mesons or $b$-baryons the final probability density distribution to measure a decay distance $\lambda$ is then:

$$
\mathcal{P}_{\text {dec,cat }}^{\text {right/wrong }}(\lambda)=\mathcal{P}_{\text {dec,cat }}^{\text {right/wrong }} \mathcal{P}_{\text {cat }}(\lambda)
$$

For $\mathrm{B}_{d}^{0}$ mesons these probabilities are different if they have oscillated ( $\mathcal{P}_{\text {dec,osc }}^{\text {right } / \text { wrong }}$ ) or not $\left(\mathcal{P}_{\text {dec, noosc }}^{\text {right/wrong }}\right)$ and have been evaluated separately. Table 7 shows that the right/wrong probabilities are not far from being simply exchanged when the $\mathrm{B}^{0}$ has oscillated.

Then the distance-dependent probability may be written:

$$
\mathcal{P}_{\text {dec }}^{\text {right } / \text { wrong }}(\lambda)=\left[\mathcal{P}_{\text {dec, noosc }}^{\text {right } / \text { wrong }}\left(1-\mathcal{P}_{\text {osc }}(\lambda)\right)+\mathcal{P}_{\text {dec,osc }}^{\text {right } / \text { wrong }} \mathcal{P}_{\text {osc }}(\lambda)\right] \mathcal{P}_{B_{d}^{0}}(\lambda)
$$

where $\mathcal{P}_{\text {osc }}(\lambda)$ is the probability, for a $\mathrm{B}_{d}^{0}$ decaying at $\lambda$, to have oscillated. It has been parametrized by a polynomial of degree 4 or 5 in $\lambda$ for 16 different values of $\Delta m_{d}$ between
0.2 and $0.95 \mathrm{ps}^{-1}$ (see Figure 8); a smoothed interpolation then gives its value at any $\Delta m_{d}$.

Finally one can define, for a given pair of hemispheres in a hadronic event, the probability to find a like/unlike sign correlation between them as a function of $\lambda$ :

$$
\begin{gathered}
\mathcal{P}_{\text {tag,dec }}^{\text {like sign }}(\lambda)=\sum_{\text {flav }} R_{\text {flav }}\left[\mathcal{P}_{\text {tag,flav }}^{\text {right }} \mathcal{P}_{\text {dec,flav }}^{\text {urong }}(\lambda)+\mathcal{P}_{\text {tag,flav }}^{\text {wrong }} \mathcal{P}_{\text {dec,flav }}^{\text {right }}(\lambda)\right] \\
\mathcal{P}_{\text {tag,dec }}^{\text {unlike sign }}(\lambda)=\sum_{\text {flav }} R_{\text {flav }}\left[\mathcal{P}_{\text {tag,flav }}^{\text {right }} \mathcal{P}_{\text {dec,flav }}^{\text {right }}(\lambda)+\mathcal{P}_{\text {tag,flav }}^{\text {wrong }} \mathcal{P}_{\text {dec,flav }}^{\text {wrong }}(\lambda)\right],
\end{gathered}
$$

where $R_{\text {flav }}$ is the fraction of the corresponding flavour in hadronic events. For $u / d, s, c$ flavours, the $\mathcal{P}_{\text {flav }}$ variables correspond to the $\mathcal{P}_{\text {cat }}$ previously defined; for the $b$ flavour, $\mathcal{P}_{\text {flav }}$ is obtained by summing over the four B hadron categories, weighted by their respective fractions.

| Category | $\begin{aligned} & \hline \hline \mathcal{P}_{\text {dec }}^{\text {right }} \\ & \mathcal{P}_{\text {deo }}^{\text {derog }} \end{aligned}(l)$ | $\begin{array}{ll} \hline \hline \mathcal{P}_{\text {dec }}^{\text {right }}(\mathrm{K}) \\ \mathcal{P}_{\text {dec }}^{\text {derog }} \end{array}(\mathrm{K})$ |
| :---: | :---: | :---: |
| $u / d$ | 0.0003(00) | $0.0073(01)$ |
|  | $0.0002(00)$ | $0.0048(01)$ |
| $s$ | 0.0003(00) | $0.0085(02)$ |
|  | $0.0002(00)$ | $0.0075(02)$ |
| c | 0.0005(00) | $0.0212(03)$ |
|  | $0.0019(00)$ | $0.0101(02)$ |
| $\mathrm{B}^{\mp}$ | 0.0289(05) | $0.0864(08)$ |
|  | $0.0045(02)$ | $0.0286(05)$ |
| $\overline{\mathrm{B}}_{d}^{0}$ (no oscill.) | $0.0274(05)$ | $0.0784(08)$ |
|  | $0.0068(02)$ | $0.0282(05)$ |
| $\mathrm{B}_{d}^{0}$ (oscill.) | 0.0081(03) | $0.0291(05)$ |
|  | $0.0262(05)$ | $0.0775(08)$ |
| $\mathrm{B}_{s}^{0}$ | $0.0165(07)$ | $0.0608(13)$ |
|  | $0.0161(07)$ | $0.0489(12)$ |
| $b$-baryons | $0.0245(14)$ | $0.0614(22)$ |
|  | $0.0025(05)$ | $0.0465(20)$ |

Table 7: Probabilities of right/wrong sign in the decay hemisphere. The numbers in parentheses are the 1-standard deviation statistical uncertainties only on the last digits.

### 5.4 Fitting procedure

An unbinned maximum likelihood method is applied to the set of tagged events where $\lambda$ is measured and greater than $\lambda_{\text {min }}$ and the parameter $\Delta m_{d}$ is fitted by minimizing the following function using the MINUIT [18] program:

$$
\mathcal{L}=-\sum_{\text {like sign evts }} \ln \left(\mathcal{P} \mathcal{t a g , \text { sec }}_{\text {like sign }}\left(\lambda, \Delta m_{d}\right)\right)-\sum_{\text {unlike sign evts }} \ln \left(\mathcal{P}_{\text {tag,dec }}^{\text {unlike sign }}\left(\lambda, \Delta m_{d}\right)\right) .
$$

Unless the tagging is done with the jet charge, each hemisphere can be used to measure the decay distance as well. As a consequence, some events occur twice in the fitting sample (namely $11 \%$ of the whole sample). However, the two entries correspond to independent values of $\lambda$, so they do not give redundant information. We can exclude a
significant underestimate of the statistical error through the following test: when suppressing completely the double counting (by excluding randomly one of the two entries for the same event), the statistical error is increased by about $10 \%$; the effect of double counting is necessarily smaller because this procedure corresponds to a loss of information in the sample.

The same procedure is applied to simulated events to check its consistency.

### 5.5 Results and study of systematic errors

The fit has been performed on real data with $\lambda_{\min }=2 \mathrm{~mm}$. Table 8 gives the number of entries available for each combination of sign indicators. The result is:

$$
\Delta m_{d}=0.568_{-0.046}^{+0.050}(\text { stat. }) \mathrm{ps}^{-1}
$$

| decay sign $\downarrow$ tag sign $\rightarrow$ | $j+l$ | $j+\mathrm{K}$ | $j+b$ tag |
| :---: | :---: | :---: | :---: |
| $l$ | 1383 | 1347 | 7350 |
| K | 2264 | 4494 | 12575 |

Table 8: Number of entries for each sign combination with $\lambda>2 \mathrm{~mm}$.
The same procedure, applied on a simulated sample generated with $\Delta m_{d}=0.50 \mathrm{ps}^{-1}$ and with similar statistics as the data, gives

$$
\Delta m_{d}=0.470 \pm 0.039 \mathrm{ps}^{-1}
$$

in good agreement with the input value.
Figure 9 gives the ratio:

$$
\rho=\frac{N_{\text {unlike sign }}-N_{\text {like sign }}}{N_{\text {unlike sign }}+N_{\text {like sign }}}
$$

for the data as a function of $\lambda$ (summing the results from all sign indicators) together with the curve corresponding to the fitted value of $\Delta m_{d}$ (a), the same without smearing the distribution of $\lambda$ (see sect. 4) (b), the curve obtained with a time-independent mixing with a probability $\chi_{d}=0.17$ (c) and the curve expected if no mixing occurs (d). At small values of $\lambda$ the effect of the smearing (difference between (a) and (b) ) is comparable to the difference induced by the oscillation itself, so that the fit could be dominated by systematic errors. However this effect is small in the range used in the fit ( $\lambda>2 \mathrm{~mm}$ ) where the data disagree with the no-mixing and time-independent-mixing hypotheses by 12 and 6 standard deviations respectively.

The likelihood fit was redone separately for each of the six combinations of tagging methods and results have been summarized in Figure 10 (after correction for a systematic effect due the multiplicities of charged kaons in B decays, as described in the next section). The results are all compatible with the final combination, particularly if the systematic errors which are specific to kaons or leptons are taken into account. As an example, these contributions are completely uncorrelated in the ( $l+j$ versus $l$ ) and ( $\mathrm{K}+j$ versus K ) subsamples: they are quadratically added to the statistical errors to give the dashed lines in Figure 10.

### 5.5.1 Sources of systematic uncertainty

The relative ratio of like and unlike sign correlations (see Figure 11) is expected to be a combination of:

- an almost balanced like/unlike contribution from $u \bar{u}, d \bar{d}$ background
- an $s \bar{s}$ contribution slightly favoring unlike signs (because of leading kaons having the sign of the initial quark)
- a more complex $c \bar{c}$ contribution where leptons and kaons from a $c$ have opposite signs, resulting in opposite and same sign correlations when considering the sign indicators on both sides; the simulation indicates that they almost cancel each other.
- a non-oscillating $b$ component, giving a large constant ratio favoring unlike sign pairs
- the oscillating component.

The probabilities of the light quark components decrease rapidly with $\lambda$ whereas the $c \bar{c}$ probability varies less rapidly, but the non- $b$ component is small after the cut on $\lambda$ at 2 mm .

As the oscillation is seen over less than one period, the fitted value of $\Delta m_{d}$ is mainly sensitive to the average of the ratio $\rho$ from $\lambda_{\min }$ to $\infty$, weighted by the corresponding event rates. Hence it depends on the fraction of background and of the non-oscillating component (the shape of the oscillation is not constraining enough to allow a fit with freely varying fractions and lifetimes).

| Parameter | Value | Variation | $\begin{gathered} \hline \text { Effect on } \Delta m_{d} \\ \left(\mathrm{ps}^{-1}\right) \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| $\mathrm{B}_{d}^{0}$ fraction | 0.392 | $\pm 0.022$ | $\mp 0.005$ |
| $\mathrm{B}_{s}^{0}$ fraction | 0.100 | $\pm 0.022$ | $\mp 0.015$ |
| global B lifetime | 1.567 ps | $\pm 0.020 \mathrm{ps}$ | $\mp 0.001$ |
| $\mathrm{B}_{d}^{0}$ lifetime | 1.57 ps | $\pm 0.05 \mathrm{ps}$ | $\mp 0.006$ |
| $b$ fragmentation | $\left.<p_{B} / p_{\text {beam }}\right\rangle=0.70$ | $\pm 0.02$ | $\pm 0.003$ |
| $\mathrm{BR}(b \rightarrow l)$ | 11.0\% | $\pm 10 \%$ | $\pm 0.010$ |
| $\mathrm{BR}(b \rightarrow c \rightarrow l)$ | 8.1\% | $\pm 10 \%$ | $\mp 0.013$ |
| $\mathrm{K}^{+} / \mathrm{K}^{-}$mult. in B decays $Q_{\text {iet }}$ | $\begin{gathered} -0.043 \text { see te } \\ \text { see text } \end{gathered}$ |  | $\begin{aligned} & \pm 0.042 \\ & \pm 0.040 \end{aligned}$ |
| electron effic. and contamination | see text |  | $\pm 0.008$ |
| muon effic. and contamination | see text |  | $\pm 0.008$ |
| kaon effic. and contamination | see text |  | $\pm 0.032$ |
| cut on ( $\left\|\cos \theta_{\text {thrust }}\right\|$ ) | 0.7 | $0.7 \rightarrow 0.5$ | < 0.020 |
| parametrization | see text |  | $\pm 0.011$ |
| total |  |  | $\pm 0.078$ |

Table 9: Systematic errors on $\Delta m_{d}$.
The following effects have been taken into account to evaluate the systematic uncertainty on $\Delta m_{d}$.

- Fractions: the fractions of the different quark flavors produced from $Z^{0}$ decays are known with enough precision to give a negligible contribution. Larger effects can be expected from the uncertainties on the production rates of the various $B$ hadrons. $\mathrm{B}_{d}^{0}$ and $\mathrm{B}_{s}^{0}$ production rates have been varied following the criteria detailed in section 4.3.
- Lifetimes: the simulation was done with the same lifetime for all B-hadrons. A common variation of all B lifetimes ( $\pm 0.02 \mathrm{ps}$ ) gives, as it is expected with low udsc background, a small variation on $\Delta m_{d}\left( \pm 0.001 \mathrm{ps}^{-1}\right)$; on the contrary, a difference between $\tau_{B_{d}^{0}}$ and the other lifetimes makes the fraction of $\mathrm{B}_{d}^{0}$ depend on $\lambda$, and then the ratio (like sign)/(unlike sign) is distorted. If only the simulated $\mathrm{B}_{d}^{0}$ lifetime varies by $\pm 0.05 \mathrm{ps}$ the variation of $\Delta m_{d}$ is $\mp 0.006 \mathrm{ps}^{-1}$. The difference between the values of the $\mathrm{B}_{d}^{0}$ lifetimes used in the simulation and in Table 1 gives a correction of $+0.006 \mathrm{ps}^{-1}$ on $\Delta m_{d}$.
- Fragmentation: the B momentum spectrum was varied in such a way that $<p_{B} / p_{\text {beam }}>$ was $0.70 \pm 0.02$, giving a contribution of $\pm 0.003 \mathrm{ps}^{-1}$.
- Semileptonic branching ratios: the values of the semileptonic branching ratios used in the simulation were: $\operatorname{Br}(b \rightarrow l)=11.0 \%, \operatorname{Br}(b \rightarrow c \rightarrow l)=8.10 \%$. A relative variation of $\operatorname{Br}(b \rightarrow l)$ of $\pm 10 \%$ gives $\pm 0.010 \mathrm{ps}^{-1}$ on $\Delta m_{d}$, and $\pm 10 \%$ on $\operatorname{Br}(b \rightarrow$ $c \rightarrow l$ ) gives $\mp 0.013 \mathrm{ps}^{-1}$ on $\Delta m_{d}$.
- $\mathrm{K}^{+} / \mathrm{K}^{-}$multiplicities in B decays: the simulation used in this study differs significantly from the values measured in [16], so that the fitted value of $\Delta m_{d}$ is probably overestimated; to determine a correction, weights were applied to the simulated events, in order to reproduce each of the fractions $\mathrm{B}^{+} \rightarrow \mathrm{K}^{+} X, \mathrm{~B}^{+} \rightarrow \mathrm{K}^{-} X$, $\mathrm{B}_{d}^{0} \rightarrow \mathrm{~K}^{+} X, \mathrm{~B}_{d}^{0} \rightarrow \mathrm{~K}^{-} X$, measured in [16], and their uncertainties. As the effect of $\mathrm{B}^{ \pm}$was found to be predominant, it was not possible to take advantage of the fact that the fractions averaged over $\mathrm{B}^{ \pm}$and $\mathrm{B}_{d}^{0}$ have been measured with a better precision. Finally the correction on $\Delta m_{d}$ was evaluated to $-0.043 \pm 0.042 \mathrm{ps}^{-1}$ (with a quadratic addition of the uncertainties from the four $\mathrm{B} \rightarrow \mathrm{K}$ fractions)
- Jet charge (including detector effects): the probability $p$ of finding the right "production" sign from $Q_{j e t}$ has been cross checked on data by comparing the charges measured in opposite hemispheres. The fraction of events with unlike signs (expected to be $p^{2}+(1-p)^{2}$ if $p$ is the same for all flavors) was evaluated on both data and simulated hadronic events, with different $b$ enrichment; the results were found to be compatible within a margin corresponding to $\pm 0.005$ on $p$. However the extrapolation to pure $b \bar{b}$ samples depends on the assumed fraction of $\mathrm{B}_{s}^{0}$ and then the uncertainty on $p$ was estimated to 0.010 , giving a variation of $\pm 0.040 \mathrm{ps}^{-1}$ on $\Delta m_{d}$.
- Lepton identification: changing the electron identification efficiency by $\pm 5 \%$ changes the fitted value of $\Delta m_{d}$ by $\pm 0.008 \mathrm{ps}^{-1}$; changing by $\pm 10 \%$ the hadron misidentification probability gives $\pm 0.002 \mathrm{ps}^{-1}$ for $\Delta m_{d}$. For muons, the corresponding uncertainties induced on $\Delta m_{d}$ are $\pm 0.007$ and $\pm 0.005 \mathrm{ps}^{-1}$.
- Kaon identification: the uncertainty in the kaon identification efficiency of the RICH was estimated to be $\pm 10 \%$; the effect on $\Delta m_{d}$ is $\mp 0.016$. A margin of $\pm 30 \%$ on the contamination by pions gives $\pm 0.028 \mathrm{ps}^{-1}$ on $\Delta m_{d}$.
- Geometrical acceptance: to check possible edge effects in the acceptance of the RICH detector, the range on $\left|\cos \left(\theta_{\text {thrust }}\right)\right|$ was reduced in steps from 0.7 down to 0.5 ; the corresponding uncertainty on $\Delta m_{d}$ is estimated to be less than $0.020 \mathrm{ps}^{-1}$.
- Parametrization: there are statistical uncertainties on $\mathcal{P}_{\text {tag }}^{\text {right /urong }}$ and $\mathcal{P}_{\text {dec }}^{\text {right/urong }}$, and on the parametrizations of $\mathcal{P}_{c a t}(\lambda)$ and $\mathcal{P}_{\text {osc }}(\lambda)$. By splitting the simulated events sample into ten subsamples and computing with each of them an independent set of coefficients, the observed dispersion of the fitted $\Delta m_{d}$ values was $0.024 \mathrm{ps}^{-1}$. The uncertainty for the total sample is then estimated to be $0.008 \mathrm{ps}^{-1}$.
The smearing introduced in the resolution function of the distance estimation (see sect. 4) modifies the parametrization of the functions $\mathcal{P}_{\text {cat }}(\lambda)$. The fitted value of
$\Delta m_{d}$ is then moved by $0.007 \mathrm{ps}^{-1}$ : this quantity has been added to the systematic error.
- Choice of $\lambda_{\text {min }}$ : the fit was redone with different values of $\lambda_{\min }$ between 1 and 4 mm : the result is stable. The dispersion of results is compatible with the statistical fluctuations, and the systematic error is estimated to be less than $0.020 \mathrm{ps}^{-1}$.

Table 9 summarizes the contributions to the total systematic uncertainty on $\Delta m_{d}$.
Taking into account the corrections due to differences in lifetimes values and in the charged K rates in B meson decays between data [16] and the simulation, the measured value of $\Delta m_{d}$ is then:

$$
\Delta m_{d}=0.531_{-0.046}^{+0.050}(\text { stat. }) \pm 0.078 \text { (syst.) } \mathrm{ps}^{-1}
$$

## 6 Conclusions

The time-dependent $\mathrm{B}_{d}^{0}-\overline{\mathrm{B}_{d}^{0}}$ oscillations have been measured from the distributions of high $p_{t}$ lepton impact parameter obtained separately for same sign and opposite sign dilepton events:

$$
\Delta m_{d}=0.47 \pm 0.08(\text { stat. })_{-0.08}^{+0.09}(\text { syst. }) \mathrm{ps}^{-1}
$$

The analysis has been extended to include charged kaons from secondary vertices and the mean jet charge to tag the produced and decaying $B$ mesons giving:

$$
\Delta m_{d}=0.531_{-0.046}^{+0.050}(\text { stat. }) \pm 0.078(\text { syst. }) \mathrm{ps}^{-1}
$$

or equivalently:

$$
\Delta m_{d}=\left(3.49_{-0.30}^{+0.33} \pm 0.52\right) 10^{-4} \mathrm{eV} / c^{2}
$$

Using the averaged value of $\tau_{B_{d}^{0}}=1.57 \pm 0.050 \mathrm{ps}[10]$ this gives:

$$
x_{d}=\Delta m_{d} / \Gamma=0.834_{-0.074}^{+0.080} \pm 0.125 .
$$

The data deviate from the hypothesis of a time-independent mixing (with a probability $\chi_{d}=0.17$ ) by 6 standard deviations.

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Figure 1: Ratio $R_{++/+-}$of the $t_{\delta}$ distributions obtained from same sign and opposite signs dilepton events when both leptons have a $p_{t}$ greater than $1 \mathrm{GeV} / c$. Black circles correspond to data events, triangles to simulated events with $\Delta m=0.47 \mathrm{ps}^{-1}$. The dotted histogram corresponds to the hypothesis of time independent mixing.


Figure 2: Fraction $F_{++}$of same sign correlations obtained when both leptons have a $p_{t}$ greater than $1 \mathrm{GeV} / c$, with the contribution from the various sources according to the simulation.


Figure 3: $\Delta m_{d}$ value measured for different $p_{t}$ cuts.


Figure 4: Flight distance estimator $\lambda$ versus the simulated distance in $b$ hemispheres.


Figure 5: Comparison of the decay distance $(\lambda)$ distribution in real and simulated data after having applied the additional smearing as explained in section 5.2.


Figure 6: Distribution of the flight distance estimator $\lambda$ for different $b$ categories; the curve represents the parametrization for $\lambda>2 \mathrm{~mm}$.


Figure 7: Distribution of the flight distance estimator $\lambda$ for $u / d, s, c$ categories; the curve represents the parametrization for $\lambda>2 \mathrm{~mm}$.

## Fraction of reversed BOd



Figure 8: Probability of $\mathrm{B}_{d}^{0}$ oscillation as a function of the flight distance estimator $\lambda$. The curves show the result of the parametrization described in the text for different values of $\Delta m_{d}$.


Figure 9: Ratio $\left(N_{\text {unlike sign }}-N_{\text {like sign }}\right) /\left(N_{\text {unlike sign }}+N_{\text {like sign }}\right)$ as a function of $\lambda$. The curves are the expected shapes for, (a) the fitted oscillation frequency (full line), (b) the same without smearing the distribution of $\lambda$ (open circles), (c) a time-independent mixing with $\chi_{d}=0.17$ (dashed), and (d) without mixing.


Figure 10: Results on $\Delta m_{d}$ for different combinations of the sign indicators. The dashed error bars show typical results after adding the systematic contribution in quadrature. In the two examples shown the errors are totally uncorrelated.


Figure 11: Contributions of the different categories are given as functions of $\lambda$. In the upper plot, the highest level is the expected value of $\rho$ (see section 5.5) if only charged B mesons would appear in both hemispheres ( $\mathrm{B}^{+} \mathrm{B}^{-}$), the other curves show the contributions of different $B$ hadrons in the decay hemisphere (averaging over all $B$ flavors in the tagging hemisphere), and of light quark pairs. The lower plot shows the abundances of the different flavors in the fitting sample (according to the simulation), as functions of $\lambda$.


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