Antiproton annihilation on light nuclei at very low energies

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Abstract

The recent experimental data obtained by the OBELIX group on $\bar{\text{p}}$D and $\bar{\text{p}}^4$He total annihilation cross sections are analyzed. The combined analysis of these data with existing antiprotonic atom data allows, for the first time, the imaginary parts of the S-wave scattering lengths for the two nuclei to be extracted. The obtained values are: Im $a_0^{sc} = [-0.62 \pm 0.02(\text{stat}) \pm 0.04(\text{sys})]$ fm for $\bar{\text{p}}$D and Im $a_0^{sc} = [-0.36 \pm 0.03(\text{stat})^{+0.19}_{-0.11}(\text{sys})]$ fm for $\bar{\text{p}}\,^4$He. This analysis indicates an unexpected behaviour of the imaginary part of the $\bar{\text{p}}$-nucleus S-wave scattering length as a function of the atomic weight A:

$$|\text{Im } a_0^{sc}|\ (\bar{\text{p}}\text{p}) > |\text{Im } \ a_0^{sc}|\ (\bar{\text{p}}\text{D}) > |\text{Im } \ a_0^{sc}|\ (\bar{\text{p}}^4\text{He}).$$

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1 Introduction

Recent data obtained in the experiments on $\bar{p}D$ and $\bar{p}^4He$ annihilation in flight at low energy [1] and on the measurement of the shift and the width of the $1S$ level of the $\bar{p}D$ atom [2] gave a first indication of a quite unusual phenomenon. Indeed, the imaginary part of the $\bar{p}D$ scattering length appears to be smaller than the $\bar{p}p$ one [3]. The performed experiments were the first investigations of the $\bar{p}D$ system at low energy; their realization was a very difficult task and the precision and accuracy obtained were limited.

The first aim of this work is to combine the data coming from these experiments in order to extract improved information on the imaginary part of the $\bar{p}D$ scattering length. As we have shown in our previous paper [4], devoted to the $\bar{p}p$ annihilation at low energies [5, 6], a successful analysis can be done within the scattering length approximation written for the systems with Coulomb attraction. These two approaches, in-flight annihilation and antiprotonic atom experiments, were shown to give coherent results and can be used as complementary. The second aim of this work is to perform a first phenomenological analysis of the antiproton annihilation on heavier nuclei in S-wave and in P-wave.

The values of the $\bar{p}D$ and $\bar{p}^4He$ annihilation cross sections used in this paper are reported in Tab. 1. These are the only $\bar{p}$-nucleus annihilation data available at incident momentum below 200 MeV/c. The data concerning shifts and widths of antiprotonic atoms will be reported in the text.

Table 1: Values of the $\bar{p}D$ and $\bar{p}^4He$ total annihilation cross sections used in this work, multiplied by the square of the incoming beam velocity β for different $\bar{p}$ incident momenta. In the data from [1], in addition to the quoted statistical and systematic errors, an overall normalization error of 2.5% has to be considered.

gaseous target (ref.)	$\bar{p}$ incident momentum (MeV/c)	$\beta^2\sigma^T_{ann}$ (mbarn)
D_2 [1]	69.6±1.5	3.45±0.08 (stat) ±0.15 (sys)
	45.7±3.5	2.12±0.06 (stat) ±0.33 (sys)
	36.3±5.1	1.96±0.08 (stat) ±0.55 (sys)
^4He [1]	70.4±1.3	4.63±0.10 (stat)±0.19 (sys)
	47.0±3.3	2.45±0.10 (stat)±0.35 (sys)
^4He [7]	45.0±5.0	3.1±0.7

The structure of the article is the following. In section 2 we present all the necessary scattering length formalism for systems with Coulomb attraction. In section 3 the last experimental data on the $\bar{p}p$ annihilation cross sections are analyzed and the $\bar{p}p$ low energy parameters, extracted both from in-flight annihilation and from atomic data, are shown to be in excellent agreement. Section 4 is devoted to the combined analysis of the $\bar{p}D$ data coming from in flight annihilation and atomic experiments; in this section the imaginary part of the $\bar{p}D$ scattering length is obtained. In section 5 we apply the analogous procedure to the $\bar{p}^4He$ annihilation data and we obtain the imaginary part of the $\bar{\mathrm{p}}^4$He scattering length. In addition, we examine the behaviour of $\bar{\mathrm{p}}$-nucleus scattering parameters as a function of the atomic weight A, for S-wave and P-wave, for different nuclei. Finally, the conclusions contain a brief summary of the results.

2 Scattering length approximation

The starting point to develop the scattering length approximation is the relation between the K-matrix for a given orbital momentum l and the strong interaction phase shift in presence of Coulomb forces δ_l^{sc} [8]:

$$\frac{1}{K_l^{sc}(q^2)} = g_l(\eta) q^{2l+1} [C_0^2(\eta) \cot\delta_l^{sc} - 2\eta h(\eta)], \tag{1}$$

where q is the center-of-mass momentum, B the Bohr radius, $\eta = 1/qB$

$$\begin{aligned} g_0(\eta) &= 1, \\ g_l(\eta) &= \prod_{m=1}^{l} \left(1 + \frac{\eta^2}{m^2}\right), \qquad l = 1, 2, \ldots \end{aligned}$$

$$C_0^2(\eta) = \frac{2\pi\eta}{1 - \exp(-2\pi\eta)}; \qquad h(\eta) = \frac{1}{2}\left[\Psi(i\eta) + \Psi(-i\eta)\right] - \frac{1}{2}\ln\left(\eta^2\right)$$

with the digamma function Ψ. The K-matrix is related to the S-matrix by

$$S(q) = \frac{1 + i g_l(\eta) q^{2l+1} w(\eta)^* K}{1 - i g_l(\eta) q^{2l+1} w(\eta) K}$$

with $w(\eta) = C_0^2(\eta) + 2i\eta h(\eta)$.

The scattering length approximation used at low energies is equivalent to the replacement of the K-matrix by a constant:

$$\frac{1}{K_l^{sc}(q^2)} = -\frac{1}{a_l^{sc}} + o(q^2).$$

Within this approximation, the annihilation cross section σ_{ann}^l for a given orbital momentum l takes the form [9, 10]:

$$q^2 \sigma_{ann}^l = (2l+1) 4\pi \frac{g_l(\eta) q^{2l+1} \, C_0^2(\eta) \, \mathrm{Im}\,(-a_l^{sc})}{|1 - i g_l(\eta) q^{2l+1} w(\eta) a_l^{sc}|^2}. \tag{2}$$

The scattering length approximation allows the low energy parameters to be obtained also from atomic data. Following Trueman [11], it is necessary to replace in (1) q by $-i\sqrt{2\mu E}$, μ being the reduced mass, $E = E_{nl} + \Delta E_{nl}$ the exact position of the Coulomb level E_{nl} shifted and broadened by the strong interaction ΔE_{nl}, and $\cot\delta_l^{sc}$ by i:

$$\frac{1}{a_l^{sc}} = -g_l(\eta) q^{2l+1} [C_0^2(\eta) i - 2\eta h(\eta)]. \tag{3}$$

Within this approximation (when one neglects effective range corrections), this expression is exact. If one supposes that $\Delta E_{nl}/E_{nl} \ll 1$ or, equivalently, $a_l^{sc}/B^{2l+1} \ll 1$ one obtains different approximate relations usually called Deser [12] (for S-wave in the first order) or Trueman [11] (for S- and P-wave in higher orders) formulas.

3 p̄p annihilation

To illustrate the agreement between the values of the low energy parameters extracted from the in-flight annihilation and from the atomic data, let's start from the p̄p system, where the experimental data at very low energy are more abundant and precise. This procedure was already applied to the p̄p system and is described in detail in [4]. In the present analysis, we add recently measured experimental points [13]. The results of the fit are presented in Fig. 1. As in the previous analysis [4], the experimental point at 43.6 MeV/c, which suffered from possible unknown systematics [13], was not used for the fit.

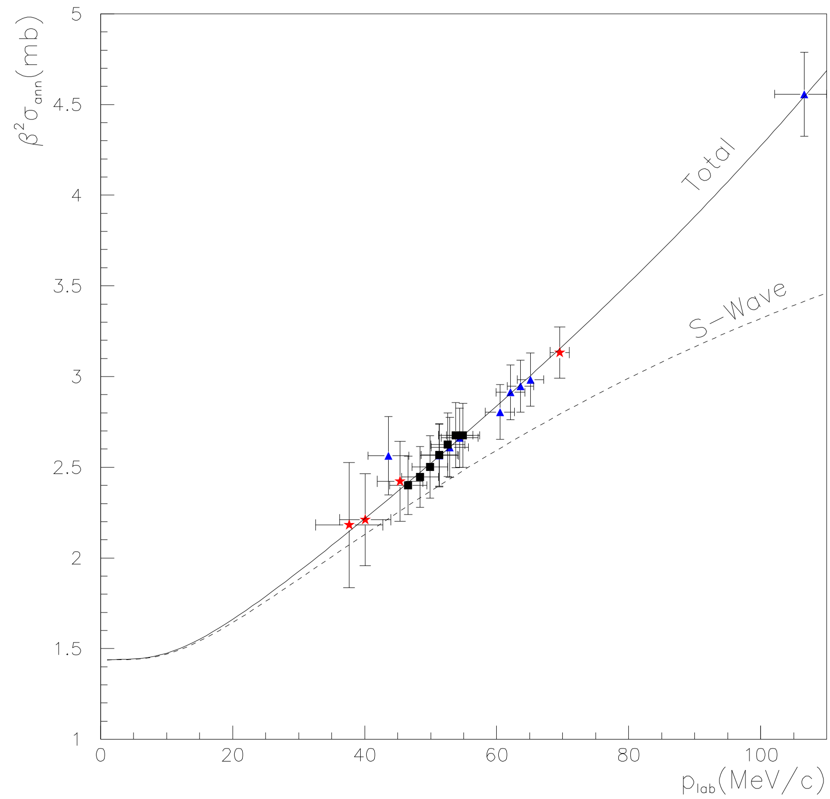

Figure 1: Values of the total p̄p annihilation cross section multiplied by the square of the incoming beam velocity. Experimental data are from [5] (▲), [6] (■), [13] (★). The error bars represent the quadratic addition of the statistical error and of the systematic error interval divided by $\sqrt{12}$. Moreover the data are affected by an overall normalization error: 3.4% for the data from [5, 6] and 2.5% for the data from [13]. The theoretical curves are the result of the present work: the full line is the total annihilation cross section, the dashed line represents the S-wave contribution.

The fit was performed by means of the MINUIT program [14] and provided the following best fit values for the imaginary parts of the p̄p scattering length (S-wave) and of the scattering volume (P-wave):

$$\begin{aligned} \text{Im } a_0^{sc} &= -[0.69 \pm 0.01(\text{stat}) \pm 0.03(\text{sys})] \text{ fm}; \\ \text{Im } a_1^{sc} &= -[0.75 \pm 0.05(\text{stat}) \pm 0.04(\text{sys})] \text{ fm}^3 \end{aligned} \tag{4}$$

with the value of $\chi^2 = 0.25$ per point. In the fitting procedure only the statistical errors were accounted for; their propagation produced the errors quoted as statistical in the values of the best fit parameters. The errors quoted as systematic come from the overall normalization error of the experimental data. Let us remind the reader that the following additional hypotheses were used in the fitting procedure: the parameters are the spin averaged ones, $|\text{ Re } a_0^{sc}| = |\text{ Im } a_0^{sc}|$ and the real part of the scattering volume can be neglected.

It is very instructive to compare the values obtained for the two parameters with the ones obtained from the atomic data. The last world averaged value for the shift and the width of the $1S$ atomic level of the p̄p atom is [16]:

$$\Delta E_{1S} + i\frac{\Gamma_{1S}}{2} = [-(0.721 \pm 0.014) + i(0.548 \pm 0.021)] \text{ keV}$$

which gives, by means of the Trueman formula, the imaginary part of the scattering length:

$$\text{Im } a_0^{sc} = -(0.694 \pm 0.027) \text{ fm},$$

in excellent agreement with the result obtained from the in-flight annihilation data (4).

The imaginary part of the P-wave scattering volume can be extracted from the width of the $2P$ atomic level of the p̄p atom. The majority of the atomic experiments obtain this value indirectly through the intensity balance procedure which gives, actually, the lower limit for this parameter (see discussion in [4, 16, 17]). The world averaged value obtained by this method is [15]:

$$\Gamma_{2P} = (32.5 \pm 2.1) \text{ meV}$$

which corresponds to:

$$\text{Im } a_1^{sc} = -(0.66 \pm 0.04) \text{ fm}^3.$$

This value is smaller than the value obtained from the in-flight annihilation data (4).

However, the unique direct measurement of the width of the $2P$ atomic level, which has been recently performed [16], gives:

$$\Gamma_{2P} = (38.0 \pm 2.8) \text{ meV}$$

which corresponds to:

$$\text{Im } a_1^{sc} = -(0.77 \pm 0.06) \text{ fm}^3$$

in excellent agreement with (4).

The main conclusion of this analysis is that the results obtained from these different experimental approaches are in quite good agreement; therefore these experiments can be considered as complementary.

4 $\bar{\text{p}}$D annihilation

The first measurement of the $1S$ level of the $\bar{\text{p}}$D atom [2] gave a very surprising result: the width of this level seems to have the same size as the corresponding one of the $\bar{\text{p}}$p atom. The shift and the width of the $1S$ atomic level were found to be:

$$\Delta E_{1S} + i\frac{\Gamma_{1S}}{2} = [-(1.05 \pm 0.25) + i(0.55 \pm 0.37)] \text{ keV},$$

which give the following scattering length for the $\bar{\text{p}}$D system:

$$a_0^{sc} = [(0.7 \pm 0.2) - i(0.4 \pm 0.3)] \text{ fm}.$$

Unfortunately, the experimental errors are too large to allow for an unambiguous comparison with the $\bar{\text{p}}$p scattering length.

The precision of the imaginary part of the scattering length can be improved significantly if the information coming from atomic measurements are combined with the data on $\bar{\text{p}}$D annihilation in flight. The general idea is quite simple: the P-wave parameters are fixed from the atomic data (which are quite precise in this case), the real part of the scattering length is taken from the atomic data too (the annihilation cross section is not very sensitive to this parameter). Thus we can perform a fit to the annihilation cross section data with only one free parameter: the imaginary part of the S-wave scattering length.

The shift and the width of the $2P$ atomic level were measured directly with approximately ten percent accuracy [16]:

$$\Delta E_{2P} + i\frac{\Gamma_{2P}}{2} = [-(243 \pm 26) + i(245 \pm 15)] \text{ meV}$$

which correspond to the following scattering volume:

$$a_1^{sc} = [(3.3 \pm 0.3) - i(3.18 \pm 0.18)] \text{ fm}^3.$$

The results of the fitting procedure are presented in Fig. 2; the procedure provided the following best fit value for the imaginary part of the scattering length:

$$\text{Im } a_0^{sc} = -[0.62 \pm 0.02(\text{stat}) \pm 0.05(\text{sys})] \text{ fm}$$

with a rather large value of the χ^2, which amounts to 8.3 per point.

In the fitting procedure only the statistical errors were accounted for; their propagation produced the error quoted as statistical in the value of the best fit parameter. The quoted systematic error originates from two main sources of systematics. The first one is the error on the determination of the imaginary part of the scattering volume (giving 0.03 fm) and the second one is connected to the overall normalization error of the data (giving 0.04 fm). These two errors are added quadratically. On the contrary, the imaginary part of the scattering length is practically insensitive to the variation of its real part, within the experimental errors taken from the atomic experiment.

The agreement of the theoretical curve with the experimental data, within the statistical errors, is not very good, as shown by the large value of the χ^2. This discrepancy could

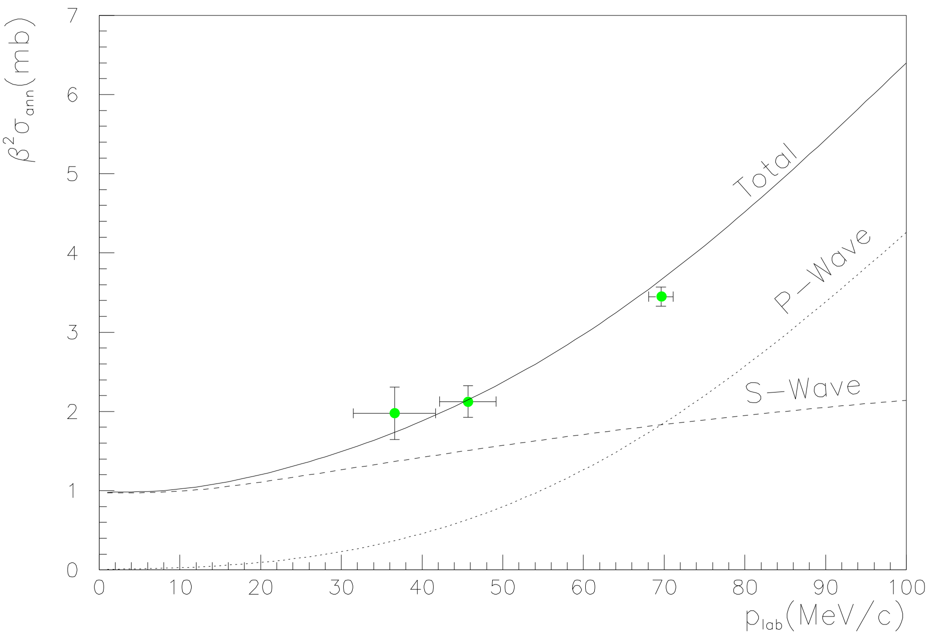

Figure 2: Values of the total $\bar{p}D$ annihilation cross section multiplied by the square of the incoming beam velocity. Experimental data are from [1] (•). The error bars represent the quadratic addition of the statistical error and of the systematic error multiplied by $\sqrt{12}$. Moreover the data are affected by an overall normalization error of 2.5%. The theoretical curves are the result of the present work: the full line is the total annihilation cross section, the dashed line represents the S-wave contribution, the dotted line the P-wave contribution.

be due to at least two reasons. The experimental points, especially at the lowest values of the incident momentum, are affected by large individual systematic errors, due to the difficulty of separating annihilations coming from the different momentum components of the antiproton beam [1]. Moreover, at the lowest value of the incident momentum, the spread of the projectile momentum distribution is quite large (see Tab. 1); as the annihilation cross section, in this momentum range, is rapidly increasing with the lowering of the incident momentum, the measured value of the cross section should be considered as an average value over the momentum interval, rather than the cross section value at the center of the interval.

In the error bars reported in Fig. 2 the two effects previously mentioned are acconted for and the theoretical predictions appear in fair agreement with the experimental data. Therefore, in spite of the large value of the χ^2 of the fit, we consider that the result obtained for the best fit parameter could be confidently accepted.

In conclusion, the combined analysis of the in-flight annihilation data and atomic data allows the knowledge of the imaginary part of the p̄D scattering length to be improved significantly. Moreover, the indication obtained from the atomic experiments that this value does not exceed the p̄p one is confirmed.

As a final comment, let us remark that, within a naive geometrical approach to annihilation, one could expect the imaginary part of the p̄D scattering length to be approximately equal to the sum of the p̄n and p̄p scattering lengths:

$$\text{Im } a_0(\bar{\text{p}}\text{D}) \approx \text{Im } a_0(\bar{\text{p}}\text{n}) + \text{Im } a_0(\bar{\text{p}}\text{p}).$$

These results show that this vision is completely wrong especially if one takes into account the quite large value of the imaginary part of the n̄p (or equivalently p̄n) scattering length

$$\text{Im } a_0(\bar{\text{n}}\text{p}) = -[0.83 \pm 0.07(\text{stat})] \text{ fm},$$

which was obtained from the data on n̄p annihilation [18].

5 $\bar{\text{p}}^4$He annihilation and P-wave in different nuclei

The ground state of the $\bar{\text{p}}^4$He atom is experimentally unaccessible. Therefore one cannot obtain any information about the S-wave scattering parameters from atomic experiments. Nevertheless, it is possible to evaluate the imaginary part of the $\bar{\text{p}}^4$He scattering length if one combines the atomic information with the data on in-flight annihilation [1].

The P-wave scattering volume:

$$a_1^{sc} = [-(3.4 \pm 0.4) - i(4.4 \pm 0.5)] \text{ fm}^3$$

as well as the imaginary part of the D-wave scattering parameter :

$$\text{Im } a_2^{sc} = -(1.42 \pm 0.06) \text{ fm}^5$$

can be extracted from the atomic data [19]. Unfortunately, there is no experimental information about $\bar{\text{p}}^4$He interaction in S-wave. To perform the fit, we choose the value of the real part of the S-wave scattering length Re $a_0^{sc} = 1.0$ fm, which is only slightly higher than the corresponding parameter for the p̄p and p̄D systems. Given the large arbitrariness on this parameter, we assume an error of 50% on its value (±0.5 fm).

Thus, we can perform a fit of the available $\bar{\text{p}}^4$He annihilation cross section data with only one free parameter: the imaginary part of the scattering length. The result of the fit is presented in Fig. 3; the experimental data considered are from [1] and from [7], a previous low statistical measurement of the $\bar{\text{p}}^4$He annihilation cross section performed at LEAR with a streamer chamber.

The fit to the annihilation cross section data provided the following best fit value for the imaginary part of the scattering length:

$$\text{Im } a_0^{sc} = [-0.36 \pm 0.03(\text{stat})^{+0.19}_{-0.11}(\text{sys})] \text{ fm}.$$

with a value of χ^2=1.8 per point. In the fitting procedure only the statistical errors were accounted for; as in the previous cases their propagation produced the error quoted

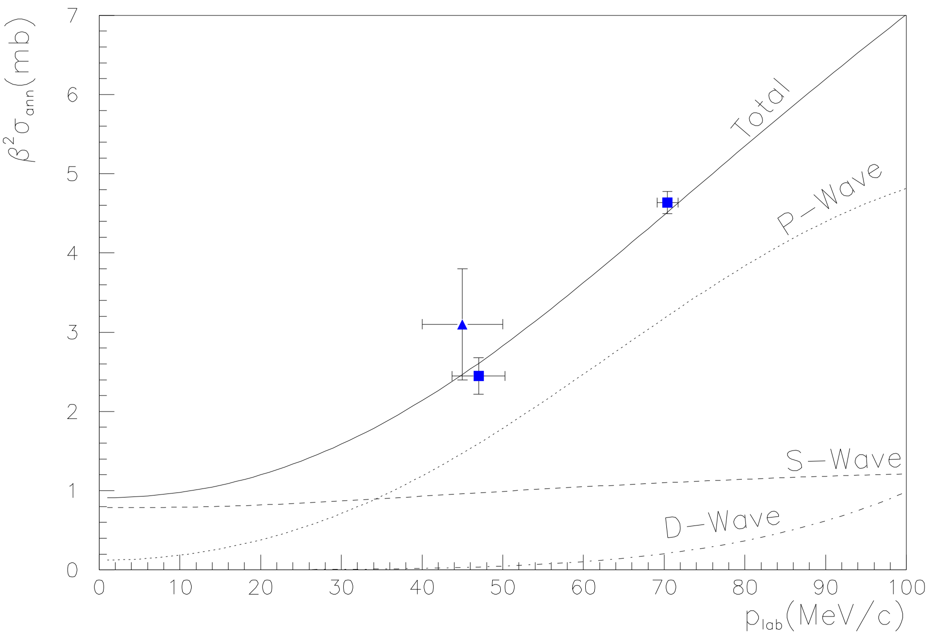

Figure 3: Values of the total $\bar{p}^4$He annihilation cross section multiplied by the square of the incoming beam velocity. Experimental data are from [1] (■) and [7] (▲). The theoretical curves are the result of the present work: the full line is the total annihilation cross section, the dashed line represents the S-wave contribution, the dotted line the P-wave contribution, the dashed-dotted line the D-wave contribution.

as statistical in the value of the fit parameter. Here the systematic error contains two contributions. The first comes from the error on the imaginary part of the scattering volume and from the uncertainty on the real part of the scattering length; the second comes from the overall normalization error of the data. The contribution coming from the imaginary part of the D-wave scattering parameter is negligible. As a final remark, let us emphasize that this is the first experimental evaluation of the imaginary part of the $\bar{\text{p}}^4$He S-wave scattering length.

Figure 4 summarizes the values of the imaginary part of the p̄-nucleus scattering length as a function of the atomic weight A. This function seems to be a decreasing one.

It is interesting to compare this unusual behaviour with the behaviour of the imaginary part of the p̄-nucleus P-wave scattering volume as a function of the atomic weight. For p̄p, the information is obtained from atomic and annihilation experiments, as described in section 3. For heavier nuclei (D, He, Li) the data come from the atomic experiments [16, 19, 20], where the shifts and the widths of the $2P$ levels are measured directly. The experimental widths as well as the imaginary parts of the scattering volumes, calculated

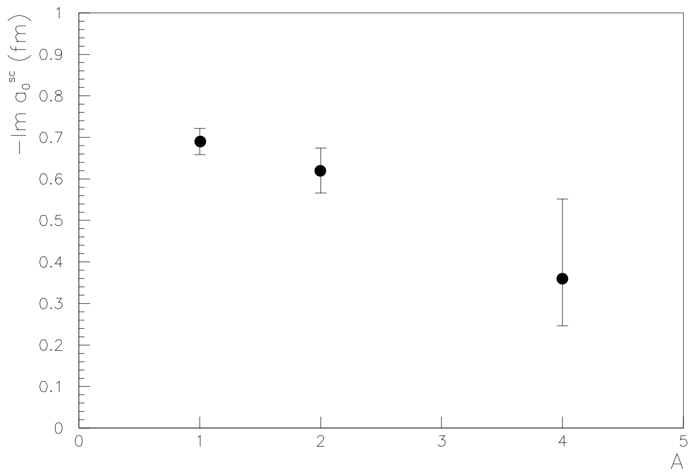

Figure 4: Absolute values of the imaginary part of the antiproton-nucleus scattering length as a function of atomic weight A. Statistical and systematic errors are added quadratically.

through the first order of the Trueman formula, are presented in Tab. 2.

Table 2: $\bar{\mathrm{p}}$-nucleus $2P$ level widths and Im a_1^{sc} calculated in the first order of the Trueman formula.

System	ref.	Γ_{2p} (meV)	$-$Im a_1^{sc} (fm^3)
$\bar{\mathrm{p}}$H	[16]	38.0 ± 2.8	0.77 ± 0.06
$\bar{\mathrm{p}}$D	[16]	489 ± 30	3.18 ± 0.20
$\bar{\mathrm{p}}^3$He	[19]	$(25 \pm 9) \cdot 10^3$	3.1 ± 1.1
$\bar{\mathrm{p}}^4$He	[19]	$(45 \pm 5) \cdot 10^3$	4.4 ± 0.5
$\bar{\mathrm{p}}^6$Li	[20]	$(444 \pm 210) \cdot 10^3$	4.3 ± 2.0
$\bar{\mathrm{p}}^7$Li	[20]	$(456 \pm 190) \cdot 10^3$	4.1 ± 1.7

These values, as a function of the atomic weight, are presented in Fig. 5. Unfortunately, the large experimental errors do not allow for an unambiguous conclusion about the form of the functional dependence. Nevertheless, this function seems to have, at least, a saturation-like behaviour.

An analogous question can be put forward for the D-wave. The experimental atomic information for heavy nuclei is quite abundant (see for review [21]). However, the scattering length approximation used here to extract the scattering parameter a_l^{sc} is no longer valid. When one works quite far from the threshold, it is necessary to take into account higher terms in the development of the K-matrix and to use an effective range approxi-

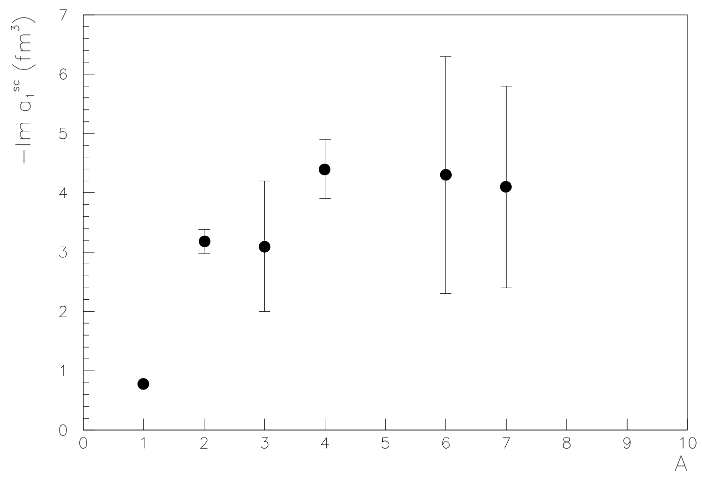

Figure 5: Absolute values of the imaginary part of the antiproton-nucleus scattering volume as a function of the atomic weight A.

mation instead of the scattering length one. Thus, the K-matrix must be written as:

$$\frac{1}{K_l^{sc}(q^2)} = -\frac{1}{a_l^{sc}} + \frac{1}{2} r_l^{sc} q^2 + o(q^4).$$

Here q is the momentum of the corresponding Coulomb level: $q = iq_B = i/nB$. For light nuclei, like hydrogen or helium, for which B = 57.6 fm and B = 18 fm respectively, a correction coming from the second term is negligible, if one supposes that r_l^{sc} is of the order of 1.0 fm. For heavier nuclei, for instance ^{19}F, $B = 3.37$ fm and this value is of the same order as all other parameters in this expression. Therefore, the development has no sense and the scattering parameters cannot be extracted from the results of atomic experiments. Note that the experiments measuring annihilation cross sections have no such problem because, in principle, any value of q can be chosen.

6 Conclusions

The proposed combined analysis of the recent p̄-light nucleus annihilation data and antiprotonic atom data allows our knowledge of the low energy parameters in these systems to be improved. The imaginary part of the spin-averaged p̄D scattering length can be obtained:

$$\mathrm{Im}\ a_0^{sc}(\bar{\mathrm{p}}\mathrm{D}) = -[0.62 \pm 0.02(\mathrm{stat}) \pm 0.05(\mathrm{sys})]\ \mathrm{fm}.$$

Moreover, for the first time, it becomes possible to evaluate the imaginary part of the spin-averaged $\bar{p}^4$He scattering length:

$$\text{Im } a_0^{sc}(\bar{p}^4\text{He}) = [-0.36 \pm 0.03(\text{stat})^{+0.19}_{-0.11}(\text{sys})] \text{ fm}.$$

Compared to the the imaginary part of the spin-averaged $\bar{p}p$ scattering length

$$\text{Im } a_0^{sc}(\bar{p}p) = -[0.69 \pm 0.01(\text{stat}) \pm 0.03(\text{sys})] \text{ fm},$$

obtained from annihilation data and in agreement with antiprotonic atom experiments, these values indicate the presence of a quite unexpected phenomenon: the absolute value of the scattering length seems to be a decreasing function of the atomic weight.

A naive geometrical vision of $\bar{p}$-nucleus annihilation would suggest a value of the $\bar{p}$-nucleus scattering length increasing with the nucleus size. This naive picture is also wrong when one analyses the imaginary part of the P-wave scattering volume: the function seems to have, at least, a saturation-like behaviour.

To confirm this unusual phenomenon involving low energy $\bar{p}$-nucleus interactions, it would be necessary to perform new measurements with higher statistics. This experimental information, expanded to heavier nuclei, could be obtained with the new AD facility at CERN.

References

[1] A. Zenoni et al., *Phys. Lett.* **B461** (1999) 413.

[2] M. Augsburger et al., *Phys. Lett.* **B461** (1999) 417.

[3] K.V. Protasov, Workshop on Hadron Spectroscopy, March 8-12, 1999, LNF Frascati, (Italy).

[4] J. Carbonell, K.V. Protasov, A. Zenoni, *Phys. Lett.* **B397** (1997) 345.

[5] A. Bertin et al., *Phys. Lett.* **B369** (1996) 77.

[6] A. Benedettini et al., *Nucl. Phys. B* (Proc. Suppl.) **56A** (1997) 58.

[7] F. Balestra et al., *Phys. Lett.* **B230** (1989) 36.

[8] V.D. Mur, V.S. Popov, *Sov. J. Nucl. Phys.* **42** (1985) 930.

[9] J. Carbonell, K.V. Protasov, *Hyp. Int.* **76** (1993) 327.

[10] J. Carbonell, K.V. Protasov, *Nucl. Phys. B* (Proc. Suppl.) **56** (1997) Addendum.

[11] T.L. Trueman, *Nucl. Phys.* **26** (1961) 57.

[12] S. Deser, M.L. Goldberger, K. Baumann and W. Thirring, *Phys. Rev.* **96** (1954) 774.

[13] A. Zenoni et al., *Phys. Lett.* **B461** (1999) 405.

[14] MINUIT, CERN Program Library, D506

[15] M. Augsburger et al., submitted to *Nucl. Phys.* **A**;
Nucl. Phys. **A655** (1999) 305c.

[16] D. Gotta et al., submitted to *Nucl. Phys.* **A**.

[17] C.J. Batty, *Rep. Progr. Phys.* **52** (1989) 1165.

[18] G.S. Mutchler et al., *Phys. Rev.* **D 38** (1988) 742.

[19] M. Schneider et al., *Z. Phys.* **A 338** (1991) 217.

[20] R. Guigas et al., *Phys. Lett.* **137 B** (1984) 323.

[21] C.J. Batty, E. Friedman, A. Gal, *Nucl. Phys.* **A 592** (1995) 487.