A window to mirror world : the deuteron anapole moment

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A Window to Mirror World: The deuteron anapole moment

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We calculate the deuteron anapole moment within the nonrelativistic formalism. Relevant operators at the leading order are obtained and the matrix elements are calculated with zero-range-approximated wave functions. The result is checked against the one obtained from the multipole expansion formalism. Numerical values are compared to those obtained with the Argonne $v18$ phenomenological model. Gauge invariance of the net result as well as separate contributions is examined.

1 Introduction

The subject of this paper is to understand the weak interactions between hadrons by exploiting an electric property of the deuteron. Despite of the success of Weinberg-Salam-Glashow model [1], despite of the discovery of Z^0 boson in huge accelerator facilities, the weak interactions of hadrons remain the most elusive area in the physics world. For example, people do not know yet how weak this interaction is. There have been only a few experimental challenges that try to uncover this problem but we are still far away from grasping something definitive. Nevertheless the progress of the experimental apparatus and techniques makes the elusive body more and more definite. Now with the experimental data on one hand and with the theoretical grounds on the other, the access to the problem becomes more and more realistic.

Why is the weak interaction interesting?

Symmetries that the nature possesses open many possibilities to understand it in the methodology of the theoretical physics, which was originated from Newton's pioneering work of universal gravity. Lorentz, chiral, parity, time reversal, charge conjugation, and many other symmetries make it possible to construct theories in well-defined and compact ways. The standard model, which is believed to be the most fundamental theory of nature, is built on the basis of these symmetries. Sometimes, however, nature hints to us that some symmetries are broken. The most prominent case may be the broken symmetry of parity for the weak interaction. Even though a process accompanied by the weak interaction, β-decay, was already discovered in the early 20th century, people didn't know that the weak interaction occurs differently in the mirror world until 1957. In 1956, Lee and Yang [2] suggested that the mirror image of the weak interaction is not the same as the original one, i.e., parity symmetry is broken. In the next year it was verified experimentally [3]. To the knowledge reported so far, weak interaction is the only interaction that breaks the parity symmetry among the four fundamental interactions. Imagine that there are four kinds of mirror in the nature: strong, weak, electromagnetic, and gravitational ones. When you reflect yourself on the mirror, you will find one different image of you in the weak mirror, e.g., only left portion of your body will be reflected. This asymmetry is a peculiar property of the weak interaction and this makes it interesting.

Why is the weak interaction difficult?

In reality one cannot tear apart the four interactions. In fact there is only one mirror, which reflects the four interactions simultaneously. In few cases one can isolate specific interactions and look at them: leptons do not interact strongly. However for the hadrons which are the major interest of nuclear physics, such a separation is impossible in practice. In order to see the weak interaction on the mirror, one should tell the asymmetric image from the symmetric one. But the difficulty is that this asymmetry is very faint. Conventionally, strength of weak interaction is about 10^{-5} times weaker than the electromagnetic interaction and about 10^{-7} times weaker than the strong interaction. Detecting weak effects is similar to observing something, which is as bright as Venus in the early evening or morning within the sun with naked eyes. Actually, no one can discern this small light from Sun's huge brightness. Discriminating the weak effect from the strong or electromagnetic back ground is a formidable challenge and it will require high-technology instruments as well as great patience. This is why the observation of weak interaction is difficult.

Weak interaction of the hadron

On the theoretical side, there is one important reason that makes the weak-interaction analysis difficult. It was remarked in the former paragraph that one should distinguish a faint weak image from the very bright image made by strong interactions. In the picture of the standard model, the nature is made of quarks, gluons, leptons, photons, and weak bosons. In the constituent quark model, hadrons are described as bound states of quarks. One great difficulty embedded here is that the standard model can hardly connect to constituent quark models quantitatively. In order to do so, one should evaluate infinite series whose elements do not converge to zero. For that reason, the standard model can hardly help nuclear physics in practical calculations. In order to overcome this problem, the simplest choice is to rely on theories or models which treat hadrons as basic degrees of freedom. The strong interaction theories or models based on meson-exchange picture achieved a great many success in many problems and areas in nuclear physics. The same strategy – weak interaction of hadrons via meson exchange – was especially developed by one of the authors together with Donoghue and Holstein in early 80's [4].

The meson-exchange model of hadronic weak interactions is now a general method in theoretical nuclear physics and we will investigate a specific problem on this basis.

Weak interaction at the atomic level

The study of weak interactions at the atomic level started in mid-70's and the first decisive observation was achieved in Novosibirsk in 1978. In the atomic bismuth vapor, it was observed that the plane of polarization of photon prefers, say, left to right. Several months later, the same parity-nonconserving phenomenon was observed in a reaction of deep inelastic scattering of longitudinally polarized electrons on protons and deuterons.

Even though the existence of charged and neutral weak bosons were confirmed in CERN in 1983, the success of Novosibirsk opened a branch of nuclear weak interactions. It is widely recognized that the atomic measurements are important tests of the standard model. The comparison between precision measurements at atomic level and accelerator energies could make possible extensions beyond the standard model. One of the challenges in the field has been the experimental determination of the various spin and isospin dependent effects to the low-energy weak hadronic interactions.

Anapole moment

Shortly after the experimental discovery of parity nonconservation in 1957, Zel'dovich [5] noted a new type of electromagnetic moment, the anapole moment. The anapole moment is an electromagnetic property of systems with non-zero spin. While the well-known magnetic dipole is P- and T-even, the anapole is a P-odd and T-even operator. Such a parity-odd effect can be probed only by virtual photons. Thus the effect of the anapole moment can be, for example, measured in electron-nucleon scattering but cannot be measured through direct interactions of the electromagnetic field and the nucleon.

The parity-nonconserving neutral weak interactions between electrons and nucleons are dominated by spin-independent Z^0's axial-coupling to electrons and vector-coupling to nucleons. However this interaction is transparent to hadronic weak interaction which is our primary interest. Neutral hadronic weak interaction can be probed in a process which includes axial Z^0 coupling to a nucleon and vector coupling to an electron. The vector Z^0 coupling to electron is suppressed by a factor of $(4\sin^2\theta_w - 1)/2 \sim -0.05$. In this case, radiative corrections can amount to the leading Z^0 exchange contribution. The anapole effect, which is in general a higher order correction to the leading order, did not draw people's attention until Flambaum and Khriplovich showed that its effect can be dominant for heavy nuclei [6]. They showed that the anapole effect of a nucleus is proportional to $A^{2/3}$. For $A \geq 20$, the anapole effect dominates over the spin-dependent Z^0 exchange or other radiative corrections. Since then many elaborate calculations have been done with the anapole moment of heavy nuclei [7–10] and its existence was observed in 1997 by the Colorado group [11].

Deuteron

In the calculation of the anapole moment of heavy nuclei, a variety of potentials such as Woods-Saxon or phenomenological ones have been used in obtaining wave functions. In order to avoid the complexity in deriving relevant current operators, current conservation is assumed or Siegert's theorem [12] is used. Without the exact knowledge of strong interactions of nucleons in a nucleus, some uncertainty, small or large, is inevitable in nuclear calculations. One critical drawback is that the magnitude of uncertainty or error is hard to be estimated. For example, one can say that a simple potential model has some error bar larger than more exact phenomenological calculations but one cannot suggest the uncertainty of the phenomenological models with definite numbers. A most promising alternative will be a very simple bound system which can be free from the uncertainty of many-body systems. The most famous and the best-understood system that satisfies this requirement may be the deuteron. Its wave function shows good coincidence among numerous phenomenological models. Being the two-body system, exchange currents are well-defined and well-ordered in the effective field theory. For these reasons, we pick up the deuteron as the object of our study.

2 Definition of the Anapole Moment

One can find a few papers [6, 10, 13] where the anapole moment is defined in different ways. Basically, the anapole moment is the parity-nonconserving (PNC) spin-dependent electromagnetic property of a system. Therefore, it always contains the spin operator of the system.

The anapole moment is an electric multipole and thus it can be defined from the multipole expansion of the vector potential. This one, $\vec{A}(\vec{x})$, generated by a source current $\vec{j}$ reads

$$\vec{A}(\vec{x}) = \int d\vec{x}' \frac{\vec{j}(\vec{x}')}{|\vec{x} - \vec{x}'|}. \tag{1}$$

The leading term of the expansion in $|\vec{x}|$ is the magnetic monopole moment term and the second order term is the magnetic dipole term. The next order is divided into the sum of magnetic quadrupole and anapole terms. The anapole moment derived in this way reads

$$\vec{a} \equiv \frac{2\pi}{3} \int d\vec{x}\, \vec{x} \times \left(\vec{x} \times \vec{j}(\vec{x}) \right). \tag{2}$$

We should note that $\vec{j}(\vec{x})$ in this definition is the matrix element of a current density operator.

The general expression of the matrix element of a conserved four-current for a spin-$\frac{1}{2}$ particle in momentum space is written as

$$\begin{aligned} \langle j^\mu(q) \rangle = \Big\langle F_1(q^2)\, \gamma^\mu - i F_2(q^2)\, \sigma^{\mu\nu} q_\nu \\ + a(q^2)\, (\not{q}\, q^\mu - q^2 \gamma^\mu) \gamma_5 - i d(q^2)\, \sigma^{\mu\nu} q_\nu \gamma_5 \Big\rangle . \end{aligned} \tag{3}$$

$F_1(q^2)$ and $F_2(q^2)$ are electric and magnetic form factors, respectively. The axial form factors are $a(q^2)$ and $d(q^2)$ that are the anapole and electric dipole terms, respectively. In the nonrelativistic limit, the anapole term reduces to

$$a(q^2)\, \vec{q}^{\,2} (\vec{\sigma} - \hat{q}\, \vec{\sigma} \cdot \hat{q}), \tag{4}$$

which also satisfies current conservation.

An alternative way to obtain the anapole moment is to expand the interaction Lagrangian,

$$\mathcal{L}_{\rm int} = \vec{j} \cdot \vec{A} \tag{5}$$

in powers of q. Matrix element of spin-dependent term at q^2 order is identified as the anapole moment. One advantage of this approach is that in contrast to the definition from the conserved four-current, current conservation is not assumed. Gauge invariance or its breakdown can be shown explicitly from this approach. In this work, we calculate the anapole moment from the interaction Lagrangian. Spin-dependent matrix elements at q^2 are calculated with zero-range-approximated (ZRA) wave function. The results will be compared with those we obtained recently with the definition from the multipole expansion [15].

3 Parity-Admixed Wave Function

Figure 1 shows the weak interaction of nucleons through the exchange of mesons; π, ρ, ω, and etc. The vertex marked with $\times$ represents PNC nucleon-meson coupling. Since there are one parity-conserving (PC) and one PNC vertices, the potential that the diagram stands for is parity-odd as a whole. This PNC interaction generates

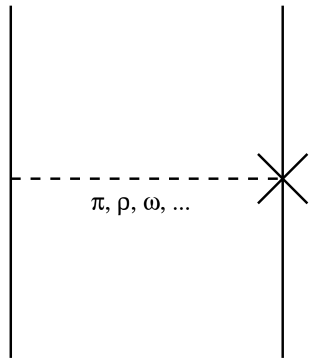

Figure 1. PNC meson-exchange potential.

parity-admixed wave function. The role of mesons varies widely from process to process. In the capture of polarized thermal neutron by a proton, $\vec{n}+p\to d+\gamma$, or in the deuteron-electron scattering, the exchange of π dominates the PNC interaction. In contrast, the PNC effects in p-p scattering or $n+p\to d+\vec{\gamma}$ is dominated by the vector-meson exchanges. In our problem, where electron scatters with deuteron, PNC interaction by pion-exchange is the most dominant. One-pion-exchange PNC potential reads

$$
\begin{aligned}
V_{\rm pnc}(\vec{r}) &= i\frac{g_A h_{\pi NN}}{\sqrt{2}f_\pi}(\vec{\tau}_1\times\vec{\tau}_2)^z\,\vec{I}\cdot\left[\frac{\vec{p}_1-\vec{p}_2}{2},\frac{e^{-m_\pi r}}{4\pi r}\right] \\
&= \frac{g_A h_{\pi NN}}{\sqrt{2}f_\pi}(\vec{\tau}_1\times\vec{\tau}_2)^z\,\vec{I}\cdot\hat{r}\,\frac{d}{dr}y_0(r),
\end{aligned}
\tag{6}
$$

where $\vec{r}\equiv\vec{r}_1-\vec{r}_2$, $r\equiv|\vec{r}|$, $\vec{I}\equiv\frac{1}{2}(\vec{\sigma}_p+\vec{\sigma}_n)$, and $y_0(r)\equiv e^{-m_\pi r}/(4\pi r)$. The constants, g_A and f_π, are given the values 1.267 and 92.4 MeV respectively. The parity-even component is mostly dominated by the $^3S_1$ state and the probability of $^3D_1$ is only 5.7%. In the ZRA calculation, we take into account the $^3S_1$ state only. The contribution of the D state is discussed in the numerical results. The PNC potential (6) operating on the parity-even component of the deuteron wave function produces a component in P channel. Only a $^3P_1$ state is possible from the parity-even components which are in $^3S_1$-$^3D_1$ state. The parity-admixed wave function contains the components in $^3S_1$, $^3D_1$, and $^3P_1$ channels and can be generally written as

$$
\psi_d(\vec{r}) = \frac{1}{\sqrt{4\pi}r}\left[\left(u(r)+S_{12}(\hat{r})\frac{w(r)}{\sqrt{8}}\right)\zeta_{00}-ih_{\pi NN}\sqrt{\frac{3}{2}}\,\vec{I}\cdot\hat{r}\,v(r)\,\zeta_{10}\right]\chi_{1J_z}, \tag{7}
$$

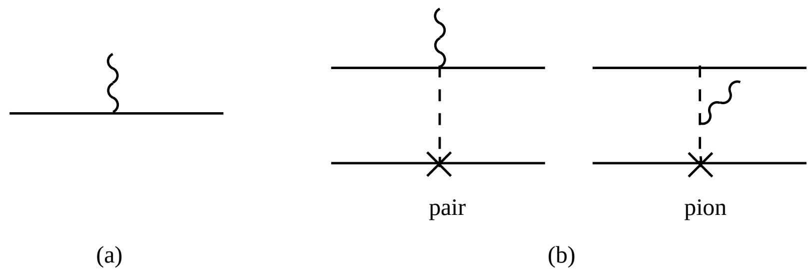

Figure 2. Diagrams representing one- and two-body electromagnetic contributions.

where $S_{12}(\hat{r}) \equiv 3\,\vec{\sigma}_1\cdot\hat{r}\,\vec{\sigma}_2\cdot\hat{r}-\vec{\sigma}_1\cdot\vec{\sigma}_2$ and χ and ζ represent a spinor and an isospinor, respectively.

4 Transition Operators

Figure 2 shows the diagrams that we consider. The one-body current [Fig. 2(a)] includes spin and convection currents:

$$\vec{j}_{\rm spin}(\vec{x}) = e\sum_{i=1}^{2}\frac{\mu_N^i}{2m_N}\vec{\nabla}_x\times\left(\vec{\sigma}_i\,\delta^{(3)}(\vec{x}-\vec{r}_i)\right), \tag{8}$$

$$\vec{j}_{\rm conv}(\vec{x}) = e\sum_{i=1}^{2}\frac{1+\tau_i^z}{4m_N}\left\{\vec{p}_i,\delta^{(3)}(\vec{x}-\vec{r}_i)\right\}. \tag{9}$$

The corresponding interaction Lagrangian reads

$$\mathcal{L}_{\rm spin} = \vec{j}_{\rm spin}\cdot\vec{A} = e\sum_{i=1}^{2}\frac{\mu_N^i}{2m_N}\,\vec{\varepsilon}\cdot(\vec{\sigma}_i\times\vec{\nabla}_{r_i})\,e^{i\vec{q}\cdot\vec{r}_i}, \tag{10}$$

$$\mathcal{L}_{\rm conv} = \vec{j}_{\rm conv}\cdot\vec{A} = e\sum_{i=1}^{2}\left\{\frac{\vec{p}_i}{2\,m_N}\cdot\vec{\varepsilon},\,e^{i\,\vec{q}\cdot\vec{r}_i}\,\frac{1+\tau_i^z}{2}\right\}. \tag{11}$$

The PNC two-body currents are shown diagramatically in Fig. 2(b). The Kroll-Ruderman and pion-pole terms are called as ‘pair’ and ‘pion’ terms, respectively. The pair term can be derived from the minimal coupling, $\vec{p}\to\vec{p}-e\vec{A}$, to the PNC potential of Eq. (6). The current density operators of pair and pion terms read

$$\vec{j}_{\rm pair}(\vec{x}) = -e\frac{g_A h_{\pi NN}}{2\sqrt{2}f_\pi}(\vec{\tau}_1\cdot\vec{\tau}_2-\tau_1^z\tau_2^z)\,y_0(r_{12})\sum_{i=1}^{2}\vec{\sigma}_i\,\delta^{(3)}(\vec{x}-\vec{r}_i), \tag{12}$$

$$\vec{j}_{\rm pion}(\vec{x}) = -e\,\frac{g_A h_{\pi NN}}{2\sqrt{2}f_\pi}\,(\vec{\tau}_1\cdot\vec{\tau}_2-\tau_1^z\tau_2^z)$$
$$\times\left(\vec{\sigma}_1\cdot\vec{\partial}_1-\vec{\sigma}_2\cdot\vec{\partial}_2\right)\left(\vec{\partial}_1-\vec{\partial}_2\right)y_0(r_{1x})\,y_0(r_{2x}), \tag{13}$$

where $r_{12}\equiv|\vec{r}_1-\vec{r}_2|$, $r_{ix}\equiv|\vec{r}_i-\vec{x}|$. And μ_N^i is defined as

$$\mu_N^i\equiv\frac{1}{2}\left(\mu_S+\tau_i^z\mu_V\right), \tag{14}$$

with $\mu_S=0.88$ and $\mu_V=4.71$. The interaction Lagrangians of the pair and pion terms are

$$\begin{aligned}\mathcal{L}_{\rm pair} &= \vec{j}_{\rm pair}\cdot\vec{A}\\ &= -\frac{g_A\,h_{\pi NN}}{2\sqrt{2}f_\pi}\,(\vec{\tau}_1\cdot\vec{\tau}_2-\tau_1^z\,\tau_2^z)\left(\vec{\sigma}_1\cdot\vec{\varepsilon}\,e^{i\vec{q}\cdot\vec{r}_1}+\vec{\sigma}_2\cdot\vec{\varepsilon}\,e^{i\vec{q}\cdot\vec{r}_2}\right)\frac{e^{-m_\pi r_{12}}}{4\pi r_{12}}, \qquad (15)\\ \mathcal{L}_{\rm pion} &= \vec{j}_{\rm pion}\cdot\vec{A}\\ &= -\frac{g_A\,h_{\pi NN}}{2\sqrt{2}f_\pi}\,(\vec{\tau}_1\cdot\vec{\tau}_2-\tau_1^z\,\tau_2^z)\left(\vec{\sigma}_1\cdot\vec{\partial}_1-\vec{\sigma}_2\cdot\vec{\partial}_2\right)\\ &\quad\times\int d\vec{r}_3\,\frac{e^{-m_\pi r_{13}}}{4\pi r_{13}}\left(\vec{\partial}_3\cdot\vec{\varepsilon}\,e^{i\vec{q}\cdot\vec{r}_3}+e^{i\vec{q}\cdot\vec{r}_3}\,\vec{\varepsilon}\cdot\vec{\partial}_3\right)\frac{e^{-m_\pi r_{23}}}{4\pi r_{23}}. \qquad (16)\end{aligned}$$

Now let us consider the operators in the small q values and expand them in powers of q. Moving to the center of mass frame, we discard all the variables related with the center of mass coordinate or its motion.

4.1 Operators in q^0

$$\mathcal{L}_{\rm conv}(q^0) = \left\{\frac{\vec{p}_1-\vec{p}_2}{4m_N}\cdot\vec{\varepsilon},\frac{\tau_1^z-\tau_2^z}{2}\right\}, \tag{17}$$

$$\mathcal{L}_{\rm pair}(q^0) = -\frac{g_A\,h_{\pi NN}}{\sqrt{2}f_\pi}\,(\vec{\tau}_1\cdot\vec{\tau}_2-\tau_1^z\tau_2^z)\;\vec{I}\cdot\vec{\varepsilon}\;\frac{e^{-m_\pi r_{12}}}{4\pi r_{12}}, \tag{18}$$

$$\mathcal{L}_{\rm pion}(q^0) = \frac{g_A\,h_{\pi NN}}{\sqrt{2}f_\pi}(\vec{\tau}_1\cdot\vec{\tau}_2-\tau_1^z\tau_2^z)\left(\vec{I}\cdot\vec{\varepsilon}-\vec{I}\cdot\vec{r}\;\vec{r}\cdot\vec{\varepsilon}\;\frac{1+m_\pi r}{r^2}\right)\frac{e^{-m_\pi r_{12}}}{4\pi r_{12}}. \tag{19}$$

In obtaining the pion term at $O(q^0)$, the following relations are useful:

$$\int d\vec{r}_3\,\frac{e^{-m_\pi r_{13}}}{4\pi r_{13}}\,\frac{e^{-m_\pi r_{23}}}{4\pi r_{23}} = \frac{e^{-m_\pi r_{12}}}{4\pi(2m_\pi)}, \tag{20}$$

$$\vec{\varepsilon}\cdot\int d\vec{r}_3\,\frac{e^{-m_\pi r_{13}}}{4\pi r_{13}}\,\left(\vec{\partial}_3+\vec{\partial}_3\right)\frac{e^{-m_\pi r_{23}}}{4\pi r_{23}} = \vec{\varepsilon}\cdot(\vec{\partial}_1-\vec{\partial}_2)\,\frac{e^{-m_\pi r_{12}}}{4\pi(2m_\pi)}$$
$$= -\vec{\varepsilon}\cdot\vec{r}_{12}\,\frac{e^{-m_\pi r_{12}}}{4\pi r}. \qquad (21)$$

The convection and two-body terms are summed up to give

$$\begin{aligned}
&\mathcal{L}_{\rm conv}(q^0)+\mathcal{L}_{\rm pair}(q^0)+\mathcal{L}_{\rm pion}(q^0)\\
&= \left\{\frac{\vec{p}_1-\vec{p}_2}{4m_N}\cdot\vec{\varepsilon},\frac{\tau_1^z-\tau_2^z}{2}\right\}\\
&\quad -\frac{g_A\,h_{\pi NN}}{\sqrt{2}f_\pi}(\vec{\tau}_1\cdot\vec{\tau}_2-\tau_1^z\,\tau_2^z)\,\vec{I}\cdot\vec{r}\;\vec{r}\cdot\vec{\varepsilon}\;\frac{1+m_\pi r}{r^2}\;\frac{e^{-m_\pi r_{12}}}{4\pi r_{12}}\\
&= \frac{i}{2}\left[\frac{-(\vec{\partial}_r)^2}{m_N}+V_{\rm pnc}\,,\,\vec{\varepsilon}\cdot\vec{r}\,\frac{\tau_1^z-\tau_2^z}{2}\right]\\
&= \frac{i}{2}\left[H\,,\,\vec{\varepsilon}\cdot\vec{r}\,\frac{\tau_1^z-\tau_2^z}{2}\right]. \qquad (22)
\end{aligned}$$

Taken between eigenstates of the interaction, the last expression gives zero as expected from gauge invariance when $E_i = E_f$. Notice that when the energy is not conserved, one has to also take into account an electromagnetic interaction involving the time component of the photon field, ε^0, which combines with the above one to provide the more general gauge-invariant combination, $(q^0\,\vec{\varepsilon}-\vec{q}\,\varepsilon^0)\,\vec{r}\ldots$.

4.2 Operators in q^2

$$\begin{aligned}
\mathcal{L}_{\rm conv}(q^2) &= \left\{\frac{\vec{p}_1-\vec{p}_2}{4m_N}\cdot\vec{\varepsilon},\,\left[-\frac{1}{2}\left(\vec{q}\cdot\frac{1}{2}(\vec{r}_1-\vec{r}_2)\right)^2\right]\,\frac{\tau_1^z-\tau_2^z}{2}\right\}\\
&= -\frac{1}{16m_N}\left\{-i\vec{\partial}_{r_{12}}\cdot\vec{\varepsilon},\,(\vec{q}\cdot\vec{r}_{12})^2\,\frac{\tau_1^z-\tau_2^z}{2}\right\}\\
&= -\frac{1}{48m_N}\left[-i(\vec{\partial}_{r_{12}})^2\,,\,\vec{\varepsilon}\cdot\vec{r}_{12}\,(\vec{q}\cdot\vec{r}_{12})^2\,\frac{\tau_1^z-\tau_2^z}{2}\right]\\
&\quad -\frac{1}{24m_N}\left\{-i\vec{\partial}_{r_{12}}\,,\,\vec{\varepsilon}\,(\vec{q}\cdot\vec{r}_{12})^2\,-\vec{q}\,(\vec{\varepsilon}\cdot\vec{r}_{12})(\vec{q}\cdot\vec{r}_{12})\,\frac{\tau_1^z-\tau_2^z}{2}\right\}, \qquad (23)\\
\mathcal{L}_{\rm pair}(q^2) &= -\frac{g_A\,h_{\pi NN}}{2\sqrt{2}f_\pi}(\vec{\tau}_1\cdot\vec{\tau}_2-\tau_1^z\,\tau_2^z)\,\vec{I}\cdot\vec{\varepsilon}\,\left[-2\,\frac{1}{2}\left(\vec{q}\cdot\frac{1}{2}(\vec{r}_1-\vec{r}_2)\right)^2\right]\,y_0(r_{12})\\
&= \frac{g_A\,h_{\pi NN}}{\sqrt{2}f_\pi}(\vec{\tau}_1\cdot\vec{\tau}_2-\tau_1^z\,\tau_2^z)\,\vec{I}\cdot\vec{\varepsilon}\,\left[\frac{1}{8}\left(\vec{q}\cdot\vec{r}_{12}\right)^2\right]\,y_0(r_{12}), \qquad (24)
\end{aligned}$$

$$
\begin{aligned}
\mathcal{L}_{\rm pion}(q^2) &= -\frac{g_A\, h_{\pi NN}}{2\sqrt{2} f_\pi}\, (\vec{\tau}_1\cdot\vec{\tau}_2 - \tau_1^z\,\tau_2^z)\,\vec{I}\cdot\left(\vec{\partial}_1-\vec{\partial}_2\right) \\
&\quad \times \vec{\varepsilon}\cdot(\vec{\partial}_1-\vec{\partial}_2)\,\int d\vec{r}_3\, y_0(r_{13})\,\left\{-\frac{1}{2}\left[\vec{q}\cdot\left(\vec{r}_3-\frac{1}{2}(\vec{r}_1+\vec{r}_2)\right)^2\right]\right\}\, y_0(r_{23}) \\
&= -\frac{1}{2}\,\frac{g_A\, h_{\pi NN}}{\sqrt{2} f_\pi}\,(\vec{\tau}_1\cdot\vec{\tau}_2 - \tau_1^z\,\tau_2^z)\,\vec{I}\cdot\left(\vec{\partial}_1-\vec{\partial}_2\right) \\
&\quad \times\left(-\frac{1}{12}\right)\,\left(\frac{\vec{\varepsilon}\cdot\vec{q}\,\vec{q}\cdot\vec{r}_{12}-q^2\,\vec{\varepsilon}\cdot\vec{r}_{12}}{m_\pi}-\frac{\vec{\varepsilon}\cdot\vec{r}_{12}}{2r_{12}}(\vec{q}\cdot\vec{r}_{12})^2\right)\, y_0(r_{12}) \\
&= \frac{g_A\, h_{\pi NN}}{\sqrt{2} f_\pi}\,(\vec{\tau}_1\cdot\vec{\tau}_2 - \tau_1^z\,\tau_2^z)\,\frac{1}{12}\,\vec{I}\cdot\left[\frac{\vec{r}_{12}}{r_{12}}\,\left(q^2\,(\vec{\varepsilon}\cdot\vec{r}_{12})-(\vec{\varepsilon}\cdot\vec{q})\,(\vec{q}\cdot\vec{r}_{12})\right)\right. \\
&\quad +\frac{\vec{\varepsilon}\;(\vec{q}\cdot\vec{r}_{12})-\vec{q}\;(\vec{\varepsilon}\cdot\vec{r}_{12})}{r_{12}}\,(\vec{q}\cdot\vec{r}_{12})+\frac{\vec{q}\;(\vec{\varepsilon}\cdot\vec{q})-\vec{\varepsilon}\,q^2}{m_\pi} \\
&\quad \left.+\vec{r}_{12}\,\frac{(\vec{\varepsilon}\cdot\vec{r}_{12})\,(\vec{q}\cdot\vec{r}_{12})^2}{2\,r_{12}^3}\,(1+m_\pi r_{12})-3\vec{\varepsilon}\,\frac{(\vec{q}\cdot\vec{r}_{12})^2}{2r_{12}}\right]\, y_0(r_{12}). \qquad (25)
\end{aligned}
$$

In the last part, we gathered terms that give evidence of current conservation, while the last two have to be combined with the convection current and the pair one to fulfill this relation. Another expression that is useful at the second order in q is the following (a plane wave for the center of mass motion is accounted for):

$$
\begin{aligned}
&\vec{\varepsilon}\cdot(\vec{\partial}_1-\vec{\partial}_2)\,\int d\vec{r}_3\,\frac{e^{-m_\pi r_{13}}}{4\pi r_{13}}\,\left\{-\frac{1}{2}\left[\vec{q}\cdot\left(\vec{r}_3-\frac{1}{2}(\vec{r}_1+\vec{r}_2)\right)\right]^2\right\}\,\frac{e^{-m_\pi r_{23}}}{4\pi r_{23}} \\
&\quad =\vec{\varepsilon}\cdot(\vec{\partial}_1-\vec{\partial}_2)\,\left[-\frac{1}{24}\frac{e^{-m_\pi r_{12}}}{4\pi}\left(q^2\frac{1+m_\pi r_{12}}{m_\pi^3}+\frac{1}{2m_\pi}(\vec{q}\cdot\vec{r}_{12})^2\right)\right] \\
&\quad =-\frac{1}{12}\,\vec{\varepsilon}\cdot\left[\left(-\frac{\vec{r}_{12}}{m_\pi}\,q^2+\frac{\vec{q}}{m_\pi}\,\vec{q}\cdot\vec{r}_{12}\right)-\frac{\vec{r}_{12}}{2r_{12}}(\vec{q}\cdot\vec{r}_{12})^2\right]\,\frac{e^{-m_\pi r_{12}}}{4\pi}. \qquad (26)
\end{aligned}
$$

To get the above expression, the Yukawa functions have been expressed as integrals over momentum variables $\vec{k}_1$ and $\vec{k}_2$. The factor, $\vec{r}_3-\frac{1}{2}(\vec{r}_1+\vec{r}_2)$, is made to appear by deriving with respect to the $\vec{k}$ variables. Then the integral over $\vec{r}_3$ provides a δ-function involving $\vec{k}_1$ and $\vec{k}_2$ variables. It remains to integrate over the remaining $\vec{k}$ variable. It is noticed that the derivative with respect to the $\vec{r}_{12}$ variable removes factors m_π at the denominator. At the last line, the first combination of terms between parentheses can be checked to be gauge invariant.

When summing up the three contributions, it is seen that terms which do not conserve the current individually cancel out. Only remains the contributions that are explicitly gauge invariant. Then the total interaction reads

$$
\begin{aligned}
&\mathcal{L}_{\rm conv}(q^2)+\mathcal{L}_{\rm pair}(q^2)+\mathcal{L}_{\rm pion}(q^2)\\
=&\ \frac{1}{24m_N}\left\{i\vec{\partial}_{r_{12}}\,,\ \left(\vec{\varepsilon}\,(\vec{q}\cdot\vec{r}_{12})^2-\vec{q}\,(\vec{\varepsilon}\cdot\vec{r}_{12})(\vec{q}\cdot\vec{r}_{12})\right)\,\frac{\tau_1^z-\tau_2^z}{2}\right\}\\
&+\frac{g_A\,h_{\pi NN}}{\sqrt{2}f_\pi}\,(\vec{\tau}_1\cdot\vec{\tau}_2-\tau_1^z\,\tau_2^z)\,\frac{1}{12}\,\vec{I}\cdot\left\{\frac{\vec{r}_{12}}{r_{12}}\left[q^2\,(\vec{\varepsilon}\cdot\vec{r}_{12})-(\vec{\varepsilon}\cdot\vec{q})\,(\vec{q}\cdot\vec{r}_{12})\right]\right.\\
&\left.+\frac{\vec{\varepsilon}\,\,(\vec{q}\cdot\vec{r}_{12})-\vec{q}\,\,(\vec{\varepsilon}\cdot\vec{r}_{12})}{r_{12}}\,(\vec{q}\cdot\vec{r}_{12})+\frac{\vec{q}\,\,(\vec{\varepsilon}\cdot\vec{q})-\vec{\varepsilon}\,q^2}{m_\pi}\right\}\,\frac{e^{-m_\pi r_{12}}}{4\pi}. \qquad (27)
\end{aligned}
$$

5 Calculation of Matrix Elements with the ZRA Wave Function

5.1 ZRA wave function

The radial component of the $^3S_1$ deuteron wave function in the ZRA reads

$$\psi_d(r)=\sqrt{\frac{\alpha}{2\pi}}\frac{e^{-\alpha r}}{r}, \qquad (28)$$

where $\alpha\equiv\sqrt{E_d\,m_N}$ with the deuteron binding energy $E_d=2.2246$ MeV. The $^3P_1$ component of the parity admixed wave function is, modulo $\vec{I}\cdot\hat{r}$,

$$\tilde{\psi}_d(r)=\frac{g_A\,h_{\pi NN}}{4\sqrt{2}\pi f_\pi}\int dr'\,r'^2G(r,\,r')\frac{e^{-m_\pi r'}}{r'^2}(1+m_\pi r')\,\psi_d(r'), \qquad (29)$$

with the Green's function projected on the $\ell=1$ space,

$$G(r,\,r')=-m_N\frac{e^{-\alpha r}}{r^2}\,(1+\alpha r)\frac{e^{-\alpha r'}\,(1+\alpha r')-e^{\alpha r'}\,(1-\alpha r')}{2\alpha^3 r'^2}\theta(r-r'). \qquad (30)$$

5.2 Matrix elements at q^0

To make the results to converge, we replace in the PNC potential $\exp(-m_\pi r)$ by $\exp(-m_\pi r)-\exp(-\Lambda r)$.

Convection term

$$
\begin{aligned}
\langle\mathcal{L}_{\rm conv}(q^0)\rangle &= \int d\vec{r}\,\langle\,|\frac{\vec{p}_p}{m_N}\cdot\vec{\varepsilon}|\,\rangle\\
&= \int d\vec{r}\,\langle\,|\,\frac{i}{2}\,\left[(p^2+\alpha^2),\frac{\vec{r}}{m_N}\cdot\vec{\varepsilon}\right]\,|\,\rangle
\end{aligned}
$$

$$
\begin{aligned}
&= -2\,\frac{i}{2}\,(2\alpha)\int d\vec{r}\,\left\langle \frac{e^{-\alpha r}}{r}\,V_{\rm pnc}\,\vec{r}\cdot\vec{\varepsilon}\,\frac{e^{-\alpha r}}{r}\right\rangle \\
&= 2\,\frac{g_A\,h_{\pi NN}}{\sqrt{2}f_\pi}\,\vec{I}\cdot\vec{\varepsilon}\;\frac{2\alpha}{3}\int dr\,e^{-2\alpha r}\,r\,\frac{d}{dr}\left(\frac{e^{-m_\pi r}}{r}-\frac{e^{-\Lambda r}}{r}\right) \\
&= -2\,\frac{g_A\,h_{\pi NN}}{\sqrt{2}f_\pi}\,\vec{I}\cdot\vec{\varepsilon} \\
&\quad\times\frac{2\alpha}{3}\left[\log\frac{\Lambda+2\alpha}{m_\pi+2\alpha}-2\alpha\left(\frac{1}{m_\pi+2\alpha}-\frac{1}{\Lambda+2\alpha}\right)\right].
\end{aligned}
\tag{31}
$$

Pair term

$$
\begin{aligned}
\langle\mathcal{L}_{\rm pair}(q^0)\rangle &= 2\,\frac{g_A\,h_{\pi NN}}{\sqrt{2}f_\pi}\,\vec{I}\cdot\vec{\varepsilon}\int d\vec{r}\,\left[\frac{u(r)}{r}\right]^2\left(\frac{e^{-m_\pi r}}{4\pi r}-\frac{e^{-\Lambda r}}{4\pi r}\right) \\
&= 2\,\frac{g_A\,h_{\pi NN}}{\sqrt{2}f_\pi}\,\vec{I}\cdot\vec{\varepsilon}\,(4\pi)\int dr\,u^2(r)\left(\frac{e^{-m_\pi r}}{4\pi r}-\frac{e^{-\Lambda r}}{4\pi r}\right) \\
&= 4\alpha\,\frac{g_A\,h_{\pi NN}}{\sqrt{2}f_\pi}\,\vec{I}\cdot\vec{\varepsilon}\,\log\frac{\Lambda+2\alpha}{m_\pi+2\alpha}.
\end{aligned}
\tag{32}
$$

Pionic term

$$
\begin{aligned}
\langle\mathcal{L}_{\rm pion}(q^0)\rangle &= -\frac{g_A\,h_{\pi NN}}{\sqrt{2}f_\pi}\int d\vec{r}\,\left[\frac{u(r)}{r}\right]^2\vec{I}\cdot(\vec{\partial}_1-\vec{\partial}_2)\;\vec{\varepsilon}\cdot\vec{r}\left(\frac{e^{-m_\pi r}}{4\pi r}-\frac{e^{-\Lambda r}}{4\pi r}\right) \\
&= -8\pi\,\frac{g_A\,h_{\pi NN}}{\sqrt{2}f_\pi}\int dr\,u^2(r)\,\vec{I}\cdot\vec{\varepsilon}\left(1+\frac{1}{3}\frac{d}{dr}\right)\left(\frac{e^{-m_\pi r}}{4\pi r}-\frac{e^{-\Lambda r}}{4\pi r}\right) \\
&= -4\alpha\,\frac{g_A\,h_{\pi NN}}{\sqrt{2}f_\pi}\,\vec{I}\cdot\vec{\varepsilon}\,\left\{\log\frac{\Lambda+2\alpha}{m_\pi+2\alpha}\right. \\
&\quad\left.-\frac{1}{3}\left[\log\frac{\Lambda+2\alpha}{m_\pi+2\alpha}-2\alpha\left(\frac{1}{m_\pi+2\alpha}-\frac{1}{\Lambda+2\alpha}\right)\right]\right\}.
\end{aligned}
\tag{33}
$$

It can be checked that the sum of the three terms is zero, in agreement with the expectation from current conservation. In calculating some integrals, we used

$$
\int dr\,\left(\frac{e^{-m_\pi r}}{r}-\frac{e^{-\Lambda r}}{r}\right)=\log\left(\frac{\Lambda}{m_\pi}\right).
\tag{34}
$$

5.3 Matrix elements at q^2

Spin term

$$
\begin{aligned}
\langle \mathcal{L}_{\rm spin}(q^2)\rangle &= -\pi\,\frac{\mu_p-\mu_n}{\sqrt{6}m_N}\int dr\, r\, u(r)\, v(r)\, \vec{I}\cdot\left[q^2\,\vec{\varepsilon}-\vec{q}\,(\vec{q}\cdot\vec{\varepsilon})\right] \qquad (35)\\
&= -2\,\frac{g_A\, h_{\pi NN}}{\sqrt{2}f_\pi}\,\frac{(\mu_p-\mu_n)\pi}{4\pi}(2\,\alpha)\\
&\quad\times\frac{m_N}{12\alpha}\left(\frac{1}{m_\pi+2\alpha}+\frac{m_\pi}{(m_\pi+2\alpha)^2}\right)\vec{I}\cdot\left[q^2\,\vec{\varepsilon}-\vec{q}\,(\vec{q}\cdot\vec{\varepsilon})\right]\\
&= -\frac{g_A\, h_{\pi NN}}{6\sqrt{2}f_\pi}\,\mu_V\,\frac{m_\pi+\alpha}{(m_\pi+2\alpha)^2}\,\vec{I}\cdot\left[q^2\,\vec{\varepsilon}-\vec{q}\,(\vec{q}\cdot\vec{\varepsilon})\right]. \qquad (36)
\end{aligned}
$$

This result is the same with the one in [14] and also becomes equivalent to the one obtained in [13] when the overall factor $m_N^2/(4\pi)$ is corrected. In the calculation, we used the following relations:

$$
\int d\vec{r}\,\frac{e^{-\alpha r}}{r}\,\vec{r}\,G(\vec{r},\vec{r}\,') = -\frac{m_N}{4\alpha}\,\vec{r}\,'e^{-\alpha r'},
$$

$$
-\int d\vec{r}\,'\frac{m_N}{4\alpha}\,\vec{r}\,'\,e^{-\alpha\,r'}\,\vec{I}\cdot\vec{\partial}_{r'}\left(\frac{e^{-m_\pi r'}}{4\pi r'}\right)\frac{e^{-\alpha r'}}{r'} = \frac{m_N}{12\alpha}\left(\frac{1}{m_\pi+2\alpha}+\frac{m_\pi}{(m_\pi+2\alpha)^2}\right)\vec{I}. \qquad (37)
$$

Convection term

$$
\begin{aligned}
\langle \mathcal{L}_{\rm conv}(q^2)\rangle &= \sqrt{\frac{3}{2}}\Bigg\{\frac{4\pi}{24m_N}\int dr\, r\, u(r)\, v(r)\,\vec{I}\cdot\vec{\varepsilon}\,q^2\\
&\quad+\frac{4\pi}{24m_N}\int dr\, r^3\,\frac{1}{5}\left[u(r)\,v'(r)-u'(r)\,v(r)-\frac{u(r)\,v(r)}{r}\right]\\
&\quad\times\vec{I}\cdot\left[\vec{\varepsilon}\,q^2+2\vec{q}\,(\vec{\varepsilon}\cdot\vec{q})\right]\Bigg\}\\
&= \sqrt{\frac{3}{2}}\frac{4\pi}{24m_N}\Bigg\{\int dr\, r\,\frac{2}{3}\,u(r)\,v(r)\,\vec{I}\cdot\left[\vec{\varepsilon}\,q^2-\vec{q}\,(\vec{\varepsilon}\cdot\vec{q})\right]\\
&\quad-\frac{1}{2}\int dr\, r^3\,\frac{2}{15}\left[u(r)\,v''(r)-u''(r)\,v(r)-2\,\frac{u(r)\,v(r)}{r^2}\right]
\end{aligned}
$$

$$\times \vec{I}\cdot\left[\vec{\varepsilon}\, q^2 + 2\vec{q}\,(\vec{\varepsilon}\cdot\vec{q})\right]\Big\} \tag{38}$$

$$\begin{aligned}
&= \frac{1}{3}\,\frac{g_A\, h_{\pi NN}}{\sqrt{2} f_\pi}\,\frac{\pi}{6}\;\frac{m_\pi+\alpha}{(m_\pi+2\alpha)^2}\,\vec{I}\cdot\left[q^2\,\vec{\varepsilon} - \vec{q}\,(\vec{q}\cdot\vec{\varepsilon})\right] \\
&\quad + \frac{1}{180}\,\frac{g_A\, h_{\pi NN}}{\sqrt{2} f_\pi}\;\left(\frac{2\alpha}{(m_\pi+2\alpha)^2} + \frac{2\alpha\cdot 2m_\pi}{(m_\pi+2\alpha)^3}\right) \\
&\quad \times \vec{I}\cdot\left[\vec{\varepsilon}\, q^2 + 2\vec{q}\,(\vec{\varepsilon}\cdot\vec{q})\right].
\end{aligned} \tag{39}$$

The second term in Eq. (38) can be rewritten as

$$\begin{aligned}
&-\sqrt{\frac{3}{2}}\,\frac{4\pi}{48 m_N}\int dr\, r^3\;\frac{2}{15}\left(u(r)\, v''(r) - u''(r)\, v(r) - 2\,\frac{u(r)v(r)}{r^2}\right) \\
&= -\sqrt{\frac{3}{2}}\,\frac{4\pi}{48 m_N}\int dr\, r^2\;\frac{2}{15}\left(-3\,u(r)\, v'(r) + 3\,u'(r)\, v(r) - 2\,\frac{u(r)v(r)}{r}\right).
\end{aligned} \tag{40}$$

Pair term

$$\begin{aligned}
\langle \mathcal{L}_{\rm pair}(q^2)\rangle &= -\frac{g_A\, h_{\pi NN}}{\sqrt{2} f_\pi}\;\frac{1}{12}\int dr\; r\; u^2(r)\; e^{-m_\pi r}\,\vec{I}\cdot\vec{\varepsilon}\, q^2 \\
&= -\frac{g_A\, h_{\pi NN}}{\sqrt{2} f_\pi}\;\frac{2\,\alpha}{12}\;\frac{1}{(m_\pi+2\alpha)^2}\,\vec{I}\cdot\vec{\varepsilon}\, q^2.
\end{aligned} \tag{41}$$

Pion term

$$\begin{aligned}
\langle \mathcal{L}_{\rm pion}(q^2)\rangle &= \frac{g_A\, h_{\pi NN}}{\sqrt{2} f_\pi}\int dr\; u^2(r)\;\left\{\frac{1}{6}\left(\frac{1}{m_\pi} - \frac{2r}{3}\right)\,\vec{I}\cdot\left[\vec{\varepsilon}\, q^2 - \vec{q}(\vec{\varepsilon}\cdot\vec{q})\right]\right. \\
&\qquad \left. - \frac{r}{180}\,(1+m_\pi r)\vec{I}\cdot\left[\vec{\varepsilon}\, q^2 + 2\vec{q}\,(\vec{\varepsilon}\cdot\vec{q})\right] + \frac{r}{12}\vec{I}\cdot\vec{\varepsilon}\, q^2\right\} e^{-m_\pi r} \\
&= \frac{g_A\, h_{\pi NN}}{\sqrt{2} f_\pi}\frac{2\alpha}{6}\left(\frac{1}{m_\pi(m_\pi+2\alpha)} - \frac{2}{3}\,\frac{1}{(m_\pi+2\alpha)^2}\right)\,\vec{I}\cdot\left[\vec{\varepsilon}\, q^2 - \vec{q}(\vec{\varepsilon}\cdot\vec{q})\right] \\
&\quad - \frac{1}{180}\,\frac{g_A\, h_{\pi NN}}{\sqrt{2} f_\pi}\left(\frac{2\alpha}{(m_\pi+2\alpha)^2} + \frac{2\alpha\cdot 2m_\pi}{(m_\pi+2\alpha)^3}\right)\,\vec{I}\cdot\left[\vec{\varepsilon}\, q^2 + 2\vec{q}\,(\vec{\varepsilon}\cdot\vec{q})\right] \\
&\quad + \frac{g_A\, h_{\pi NN}}{\sqrt{2} f_\pi}\;\frac{2\alpha}{12}\,\frac{1}{(m_\pi+2\alpha)^2}\,\vec{I}\cdot\vec{\varepsilon}\, q^2.
\end{aligned} \tag{42}$$

Summing up all the contributions from convection, pair, and pion terms, one gets, omitting the factor $\vec{I}\cdot\left[\vec{\varepsilon}\,q^2-\vec{q}(\vec{\varepsilon}\cdot\vec{q})\right]$,

$$a_d = \frac{g_A\,h_{\pi NN}}{\sqrt{2}f_\pi}\;\frac{m_\pi(m_\pi+\alpha)+6\alpha\,(m_\pi+2\alpha)-4\alpha m_\pi}{18m_\pi(m_\pi+2\alpha)^2}$$
$$= \frac{g_A\,h_{\pi NN}}{\sqrt{2}f_\pi}\frac{m_\pi^2+3\alpha m_\pi+12\alpha^2}{18m_\pi(m_\pi+2\alpha)^2}. \tag{43}$$

This result is the same with the one in [13] modulo the overall factor $m_N^2/(4\pi)$. The spin term is gauge-invariant by itself, which can be verified easily by replacing the spin polarization $\vec{\varepsilon}$ with $\vec{q}$ in the overall factor $\vec{I}\cdot\left[\vec{\varepsilon}\,q^2-\vec{q}(\vec{\varepsilon}\cdot\vec{q})\right]$. Convection and pion terms are separated into gauge-invariant and gauge-variant parts. The pair term does not satisfy gauge invariance. In the sum of the three terms, gauge-variant terms cancel out and only gauge-invariant contributions remain. As a result, the anapole moment we obtain satisfies gauge invariance and this is the first time to be shown explicitly in the present case.

6 Checking the Consistency

In this Section we derive the anapole moment with the definition from the mutipole expansion (2). With the current operators in Eqs. (8), (9), (12), and (13) together with the wave function (7), we obtain

$$\vec{a}_{\rm spin} = -\vec{I}\,h_{\pi NN}\,\mu_V\frac{\pi}{\sqrt{6}m_N}\int dr\,r\,u(r)v(r), \tag{44}$$

$$\vec{a}_{\rm conv} = \vec{I}\,h_{\pi NN}\,\frac{1}{3}\frac{\pi}{\sqrt{6}m_N}\int dr\,r\,u(r)v(r), \tag{45}$$

$$\vec{a}_{\rm pair}+\vec{a}_{\rm pion} = \vec{I}\,h_{\pi NN}\,\frac{g_A}{6\sqrt{2}f_\pi}\int dr\,u^2(r)\left(\frac{1}{m_\pi}-\frac{2}{3}r\right)e^{-m_\pi r}. \tag{46}$$

In the above calculation, we take into account only the central part of the parity-even component in the wave function to make a direct comparison with ZRA results.

The spin term can be shown to be gauge-invariant by taking divergence of the spin current operator in Eq. (8) and its result is equal to Eq. (35). The expression of the convection term is equal to the part that satisfies the current conservation in Eq. (39). In contrast to Lagrangian calculation where there is gauge-variant contributions, a multipole calculation has only a gauge-invariant term. In the multipole calculation, $\vec{a}_{\rm pair}$ and $\vec{a}_{\rm pion}$ are different from those obtained in the Lagrangian calculation. However the part that satisfies the gauge invariance in the two-body terms, the first integrand in Eq. (43) is equivalent to the matrix element of multipole expansion. This implies that when a term is not gauge-invariant by itself, its expression depends

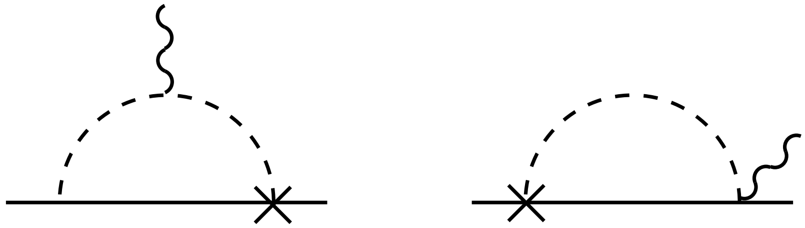

Figure 3. The diagrams of the leading order nucleonic anapole moment. Solid, dashed, and wavy lines represent the nucleon, pion, and photon, respectively.

on how it is calculated but the gauge-invariant result is not affected by the choice of definitions. A similar observation was made in the calculation of the anapole moment of heavy nuclei [10]. The definition of multipole moment, Eq. (2), assures gauge invariance of the result if the relevant operators are taken into consideration properly.

7 Numerical Values

In discussing the magnitude of the deuteron anapole moment, one should be careful not to omit some significant contributions other than the current terms considered in the previous Sections. Identifying such terms can be done systematically with the counting rule of the heavy-baryon-chiral-perturbation theory. If the momentum transfer in a process is Q, then the magnitude of the process is specified as $(Q/\Lambda)^{\nu}$ where Λ is the chiral symmetry breaking scale and the order ν is given as

$$\nu = 2L - 2C + 1 + \sum_i \nu_i. \tag{47}$$

Here L is the number of loops, C is the number of disconnected nucleon lines, and ν_i is given as

$$\nu_i = d_i + \frac{n_i}{2} + e_i - 2, \tag{48}$$

where d_i, n_i, and e_i are the numbers of derivatives, nucleon lines, and external gauge fields at the vertex i, respectively. Applying this counting rule, one can easily verify that the 1- and 2-body currents we consider are at the same order, $\nu = -2$ [a]. An isolated nucleon can also have an anapole moment. The leading contributions to the nucleon anapole moment are represented diagramatically in Fig. 3. In this case,

[a] We assume that the weak πNN vertex has $\nu_i = -1$.

one can again easily verify that the diagrams have the order $\nu = -2$. The nucleon anapole moment at leading and sub-leading orders is calculated in several works [16–19]. The magnitude at leading order is

$$\begin{aligned} \vec{a}_N &= -\frac{g_A}{6\sqrt{2} f_\pi m_\pi} \, e\vec{I} h_{\pi NN} \\ &= -0.46\, e\vec{I} h_{\pi NN}. \end{aligned} \tag{49}$$

The deuteron anapole moment at leading order can be obtained by summing the current and nucleon contributions. With the ZRA wave function, the numerical values of the gauge-invariant contributions read

$$\vec{a}^{\,\mathrm{ZRA}}_{\mathrm{spin}} = -1.03\, e\,\vec{I}\, h_{\pi NN}, \tag{50}$$

$$\vec{a}^{\,\mathrm{ZRA}}_{\mathrm{conv}} + \vec{a}^{\,\mathrm{ZRA}}_{\mathrm{pair}} + \vec{a}^{\,\mathrm{ZRA}}_{\mathrm{pion}} = 0.18\, e\,\vec{I}\, h_{\pi NN}. \tag{51}$$

The deuteron anapole moment is then

$$\begin{aligned} \vec{a}^{\,\mathrm{ZRA}}_{d} &= \vec{a}^{\,\mathrm{ZRA}}_{\mathrm{spin}} + \vec{a}^{\,\mathrm{ZRA}}_{\mathrm{conv}} + \vec{a}^{\,\mathrm{ZRA}}_{\mathrm{pair}} + \vec{a}^{\,\mathrm{ZRA}}_{\mathrm{pion}} + \vec{a}_N \\ &= -1.31\, e\,\vec{I}\, h_{\pi NN}. \end{aligned} \tag{52}$$

In a recent paper [15], we have calculated the deuteron anapole moment at the leading order with a realistic NN interaction model. We adopted the definition of multipole expansion, Eq. (2), and used the wave functions from the Argonne $v18$ potential. We obtained about 30% reduction in the magnitude of the deuteron anapole moment. This amount is slightly larger than the estimated uncertainty, 20% in [14]. However the amount of reduction varies widely from term to term. The spin term which is the most dominant contribution reduces by about half, $\vec{a}^{\,Av18}_{\mathrm{spin}} = -0.53\, e\,\vec{I}\, h_{\pi NN}$. The contribution of the central part ($^3S_1$ channel) to this value is $-0.72\, e\,\vec{I}\, h_{\pi NN}$. This reduction stems purely from the realistic treatment of the deuteron wave function in which the effect of a short-range repulsion in S-states as well as the known repulsive character of the NN interaction in the $^3P_1$ channel are taken into account properly. In addition to the central part, we calculated the contribution of the tensor part ($^3D_1$ channel) thoroughly. In contrast to small admixture of D state in the deuteron wave function (5.76%), its destructive contribution amounts to 27% of the central value. The combination of realistic phenomenology and tensor force reduces the largest contribution by about 48%.

The reduction of the remaining convection, pair, and pion terms is more drastic. The sum of these terms is only $-0.04\, e\vec{I} h_{\pi NN}$ for the $Av18$ model. The magnitude is less than one quarter of the ZRA value. As a result, the total magnitude of the deuteron anapole moment for the $Av18$ model is

$$\vec{a}^{\,Av18}_{d} = \vec{a}^{\,Av18}_{\mathrm{spin}} + \vec{a}^{\,Av18}_{\mathrm{conv}} + \vec{a}^{\,Av18}_{\mathrm{pair}} + \vec{a}^{\,Av18}_{\mathrm{pion}} + \vec{a}_N = -0.91\, e\,\vec{I}\, h_{\pi NN}. \tag{53}$$

8 Discussions

The conventional electron-nucleon PNC Hamiltonian is parameterized as

$$H_{\rm PNC}^{e\text{-}N} = \frac{G_\mu}{\sqrt{2}}\left(C_{1N}\bar{u}_e\gamma_\mu\gamma_5 u_e\ \bar{u}_N\gamma^\mu u_N + C_{2N}\bar{u}_e\gamma_\mu u_e\ \bar{u}_N\gamma^\mu\gamma_5 u_N\right), \tag{54}$$

where $G_\mu = 1.66\times 10^{-5}\,\mathrm{GeV}^{-2}$ is the muon decay constant. The part that has the coefficient C_{2N} is the nucleon-spin-dependent term. Measurement of C_{2N} would directly indicate the higher-order effects such as radiative corrections or anapole contributions. Radiative corrections for the deuteron were calculated in [20], whose result reads

$$C_d^r \equiv C_{2p} + C_{2n} \simeq 0.014 \pm 0.003. \tag{55}$$

Now we can evaluate the C_d^{ana} from the result we obtained. Noting that the sign of interaction term changes from Lagrangian to Hamiltonian, we have

$$C_d^{ana} = \frac{\sqrt{2}\,\alpha}{G_\mu\, e} a_d^{Av18} = 0.145\, h_{\pi NN} \times 10^5. \tag{56}$$

With the reasonable range of $h_{\pi NN}$ suggested in [4], the magnitude of anapole contribution ranges

$$0 \le C_d^{ana} \le 0.017 \tag{57}$$

and at the best value of $h_{\pi NN}$, we have

$$C_d^{ana} = 0.007. \tag{58}$$

The above estimation shows that the higher order correction to the PNC *e-d* scattering can be sensitive to the magnitude of $h_{\pi NN}$. Precise measurement of the effect will be very important in narrowing the wide region that $h_{\pi NN}$ occupies in the parameter space.

Our calculations and results show that the result is gauge-invariant. At the leading order, the largest contribution, spin term is gauge-invariant by itself and the net result is insensitive to terms that do not satisfy the gauge invariance. However the argument is valid only when the magnitude of $h_{\pi NN}$ is as large as the best value in [4] or the one obtained from the anapole moment of ^{133}Cs [11]. If $h_{\pi NN}$ is as small as the one from the ^{18}F experiment [21] or in the soliton model calculation [22], the dominance of pion-exchange potential becomes suspected and the role of ρ or ω mesons can be comparable to π. In that case, contribution of ρ or ω exchanges should be calculated explicitly and gauge invariance can be an important criterion to interpret the calculation steps as well as the results.

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