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PARAMETRIZATION OF THE DEUTERON WAVE FUNCTION OF THE PARIS N–N POTENTIAL

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We present a convenient analytical parametrization, in both configuration and momentum spaces, of the deuteron wavefunction calculated with the Paris potential.

The deuteron wavefunction of the recently published Paris potential [1] reproduces quite well the known low energy properties and the electromagnetic form factor $A(q^2)$ of the deuteron. In particular the predicted value of the asymptotic D- to S-wave ratio η, 0.02608, is in excellent agreement with the recent high precision measurements of ref. [2], 0.0259 ± 0.0007, and ref. [3], 0.02649 ± 0.00043. It has been shown furthermore by Klarsfeld et al. [4] that for internucleon distances $r \gtrsim 0.6$ fm, this wavefunction satisfies rigorous bounds imposed by the experimental deuteron data and now improved by the new experimental values of η [2,3].

In view of the use of this wavefunction in low and medium energy nuclear reaction problems, numerical values have been given in table VI of ref. [1]. We have also found that a cubic spline interpolation of this table reproduces very accurately the original wavefunction. The numerical values and the cubic spline interpolation subroutine can be obtained on request. However, since in some cases an analytical form is useful, we present here an algebraic parametrization of the Paris-potential deuteron wavefunction, in both configuration and momentum spaces.

It is appropriate to parametrize the radial wavefunction components $U(r)/r$ and $W(r)/r$ as a discrete superposition of Yukawa-type terms since this was done for the potential itself. Therefore $U(r)$ and $W(r)$ of table VI of ref. [1] are parametrized as follows

$$U_a(r) = \sum_{J=1}^{n} C_J \exp(-m_J r),$$
$$W_a(r) = \sum_{J=1}^{n} D_J \exp(-m_J r)(1 + 3/m_J r + 3/m_J^2 r^2), \quad (1)$$

where the subscript a refers to "analytical".

The corresponding momentum space wavefunction components are

$$U_a(p)/p = (2/\pi)^{1/2} \sum_{J=1}^{n} C_J/(p^2 + m_J^2),$$
$$W_a(p)/p = (2/\pi)^{1/2} \sum_{J=1}^{n} D_J/(p^2 + m_J^2). \quad (2)$$

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The boundary conditions $U_a(r) \to r$ and $W_a(r) \to r^3$ as $r \to 0$ imply

$$\sum_{J=1}^{n} C_J = 0, \quad \sum_{J=1}^{n} D_J/m_J^2 = \sum_{J=1}^{n} D_J = \sum_{J=1}^{n} D_J m_J^2 = 0\,. \tag{3}$$

In order to satisfy these constraints with accuracy the last coefficient of $U_a(r)$ and the last three of $W_a(r)$ are to be computed with the following formulae

$$C_n = -\sum_{J=1}^{n-1} C_J\,,$$

$$D_{n-2} = \frac{m_{n-2}^2}{(m_n^2 - m_{n-2}^2)(m_{n-1}^2 - m_{n-2}^2)} \times \left(-m_{n-1}^2 m_n^2 \sum_{J=1}^{n-3} \frac{D_J}{m_J^2} + (m_{n-1}^2 + m_n^2) \times \sum_{J=1}^{n-3} D_J - \sum_{J=1}^{n-3} D_J m_J^2\right) \tag{4}$$

and two other relations deduced by circular permutation of $n-2$, $n-1$ and n. The masses m_J are chosen to be $m_J = \alpha + (J-1)m_0$, with $m_0 = 1\ \mathrm{fm}^{-1}$ and

$$\alpha = (2m_R|E_D|)^{1/2}/\hbar = 0.23162461\ \mathrm{fm}^{-1}\,,$$

where m_R and E_D are the neutron proton reduced mass and the deuteron binding energy of ref. [1]. This choice ensures the correct asymptotic behavior.

The values of the parameters C_J and D_J are listed in table 1. The accuracy of this parametrization is illustrated by the magnitude of

$$I_S = \left(\int_0^\infty \mathrm{d}r[U(r) - U_a(r)]^2\right)^{1/2} = 3.5 \times 10^{-4}$$

Table 1
Coefficients of the parametrized deuteron wavefunction components. The last C_J and the last three D_J are to be computed from eq. (4). The notation $a \pm n$ stands for $a \times 10^{\pm n}$.

$C_J\ (\mathrm{fm}^{-1/2})$	$D_J\ (\mathrm{fm}^{-1/2})$
0.88688076 + 00	0.23135193 − 01
−0.34717093 + 00	−0.85604572 + 00
−0.30502380 + 01	0.56068193 + 01
0.56207766 + 02	−0.69462922 + 02
−0.74957334 + 03	0.41631118 + 03
0.53365279 + 04	−0.12546621 + 04
−0.22706863 + 05	0.12387830 + 04
0.60434469 + 05	0.33739172 + 04
−0.10292058 + 06	−0.13041151 + 05
0.11223357 + 06	0.19512524 + 05
−0.75925226 + 05	see eq. (4)
0.29059715 + 05	see eq. (4)
see eq. (4)	see eq. (4)

and

$$I_D = \left(\int_0^\infty \mathrm{d}r[W(r) - W_a(r)]^2\right)^{1/2} = 4.0 \times 10^{-4}\,.$$

We have also checked that this parametrization reproduces very well the deuteron parameters and form factor obtained in ref. [1].

References

[1] M. Lacombe et al., Phys. Rev. C21 (1980) 861.
[2] W. Grüebler, V. König, P.A. Schmelzbach, B. Jenny and F. Sperisen, Phys. Lett. 92B (1980) 279.
[3] K. Stephenson and W. Haeberli, Phys. Rev. Lett. 45 (1980) 520.
[4] S. Klarsfeld, J. Martorell and D.W.L. Sprung, Nucl. Phys. A352 (1981) 113;
S. Klarsfeld, private communication.