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The analysis of color-superconducting two-flavor deconfined quark matter at moderate densities is extended to include a particular spin-1 Cooper pairing of those quarks which do not participate in the standard spin-0 diquark condensate. The generic properties of the anisotropic component can, however, be expected in any low-temperature many-fermion system of two relativistic species with appropriate interaction: (i) The relativistic spin-1 gap Δ' implies spontaneous breakdown of rotation invariance manifested in the form of the quasi-fermion dispersion law. (ii) The critical temperature of the anisotropic component is approximately given by the relation $T_c' \simeq \Delta'(T=0)/3$. (iii) For massless fermions the gas of anisotropic Bogolyubov-Valatin quasiquarks becomes effectively gapless and two-dimensional. Consequently, its specific heat depends quadratically on temperature. (iv) All collective Nambu-Goldstone excitations of the anisotropic phase have a linear dispersion law and the whole system remains a superfluid.

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Recent investigations suggest that the phase structure of QCD is very rich [1, 2]. At low temperatures and high densities strongly interacting matter is expected to be a color superconductor [3]. At asymptotically high densities, where the QCD coupling constant becomes small, this can be analyzed starting from first principles [4, 5], whereas at more moderate densities, present (presumably) in the interiors of neutron stars, these methods are no longer justified. In this region the low-energy dynamics of deconfined quark matter is often studied employing effective Lagrangians $\mathcal{L}_{\mathit{eff}}$ which contain local or non-local four-fermion interactions, most importantly interactions derived from instantons or on a more phenomenological basis [6–8]. The non-confining gluon $SU(3)_c$ gauge fields are then treated as weak external perturbations, and neglected in lowest approximation. Whereas the effective couplings which enter $\mathcal{L}_{\mathit{eff}}$ are rather uncertain at present, the general form of $\mathcal{L}_{\mathit{eff}}$ is partially constrained by symmetries. As the primary QCD Lagrangian at finite chemical potential it should be $SU(3)_c \times SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times O(3)$ invariant, where N_f is the number of (approximately) massless flavors, related to chiral symmetry.

In this letter we consider the case $N_f = 2$ which is most likely relevant at chemical potentials just above the deconfinement phase transition. On physical grounds it is then natural to assume that $\mathcal{L}_{\mathit{eff}}$ favors the spontaneous formation of spin-0 isospin singlet Cooper pair condensates [1, 9] $\delta = \langle \psi^T\, C\, \gamma_5\, \tau_2\, \lambda_2\, \psi \rangle$, where ψ is a quark field, C the matrix of charge conjugation, τ_2 a Pauli matrix which acts in flavor space, and λ_2 a Gell-Mann matrix which acts in color space. Due to the latter $SU(3)_c$ is broken down to $SU(2)_c$. This has the following consequences for the physical excitations of the system:

(i) Corresponding to the mixing of the colors 1 and 2 there are two Bogolyubov-Valatin quasiquarks for each flavor with the dispersion law

$$E_1^\pm(\vec{p}) = E_2^\pm(\vec{p}) \equiv E^\pm(\vec{p}) = \sqrt{(\epsilon_p \pm \mu)^2 + |\Delta|^2}\ . \quad (1)$$

The energy gap Δ is the solution of a selfconsistent gap equation and is found to be typically of the order ~ 100 MeV in model calculations [6–8]. $\epsilon_p = \sqrt{\vec{p}^{\,2} + M^2}$, where M is an effective Dirac mass, related to the chiral condensate $\langle\bar\psi\psi\rangle$ via a selfconsistency equation [8]. For each flavor there is an unpaired quark of color 3 with the dispersion law $\epsilon_3^\pm(\vec{p}) = \epsilon_p \pm \mu$.

(ii) Because of the spontaneous breaking of $SU(3)_c$ down to $SU(2)_c$ five of the eight gluons receive a mass (Meissner effect), whereas three remain massless [10]. Since no global symmetry is spontaneously broken there are no massless Goldstone bosons.

According to Cooper's theorem any attractive interaction leads to an instability at the Fermi surface. It is therefore rather unlikely, that the Fermi sea of color-3 quarks stays intact. As only quarks of a single color are involved, the pairing must take place in a channel which is symmetric in color. Assuming s-wave condensation in an isospin-singlet channel, a possible candidate is a spin-1 condensate [6]. This letter is devoted to a quantitative analysis of this possibility. To this end we consider the condensate

$$\delta' \;=\; \langle \psi^T\, C\, \sigma^{03}\, \tau_2\, \hat{P}_3^{(c)}\, \psi \rangle\ , \quad (2)$$

where $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$ and $\hat{P}_3^{(c)} = \frac{1}{3} - \frac{1}{\sqrt{3}}\lambda_8$ is the projector on color 3. Theoretical interest in δ' stems from the fact that it is a ground-state expectation value of a complex vector order parameter $\phi^{0n} \equiv \phi_n$ describing spin 1 and breaking spontaneously the rotational invariance of the system. In relativistic systems this is certainly not a very frequent phenomenon. It is possible only at finite chemical potential, which itself breaks Lorentz invariance explicitly. (Relativistic Cooper pairing into spin-1 with nonzero angular momentum was considered elsewhere, e.g., [9, 11].) Our investigation is potentially useful not only for understanding the behavior of a truly relativistic cold quark-gluon plasma but also in a non-relativistic many-fermion system provided it can be mapped onto a relativistic one [12]. Other examples are cold atomic systems with two different types of fermionic atoms or pairing in the ${}^3S_1$-channel in nuclear matter. Our numerical illustrations are, however, given solely in the QCD context.

For the quantitative analysis we have to specify the interaction. Guided by the structure of instanton-induced interactions (see, e.g., [7]) we consider a quark-antiquark term

$$\mathcal{L}_{q\bar{q}} = G\Big\{(\bar\psi\psi)^2 - (\bar\psi\vec\tau\psi)^2 - (\bar\psi i\gamma_5\psi)^2 + (\bar\psi i\gamma_5\vec\tau\psi)^2\Big\} \quad (3)$$

and a quark-quark term

$$\begin{aligned}\mathcal{L}_{qq} = & H_s\Big\{ (\bar\psi i\gamma_5 C\tau_2\lambda_A\bar\psi^T)(\psi^T Ci\gamma_5\tau_2\lambda_A\psi^T) \\ & - (\bar\psi C\tau_2\lambda_A\bar\psi^T)(\psi^T C\tau_2\lambda_A\psi^T) \quad \Big\} \\ & -H_t \;\; (\bar\psi\sigma^{\mu\nu}C\tau_2\lambda_S\bar\psi^T)(\psi^T C\sigma_{\mu\nu}\tau_2\lambda_S\psi^T)\ , \quad (4)\end{aligned}$$

where λ_A and λ_S are the antisymmetric and symmetric color generators, respectively. For instanton induced interactions the coupling constants fulfill the relation $G : H_s : H_t = 1 : \frac{3}{4} : \frac{3}{16}$, but for the moment we will treat them as arbitrary parameters. As long as they stay positive, the interaction is attractive in the channels giving rise to the diquark condensates δ and δ' as well as to the chiral condensate $\langle\bar\psi\psi\rangle$. It is straight forward to calculate the mean-field thermodynamic potential $\Omega(T,\mu)$ in the presence of these condensates:

$$\begin{aligned}\Omega(T,\mu) = & -4\sum_{i=1}^{3}\int\frac{d^3p}{(2\pi)^3}\Big[\frac{1}{2}(E_i^- + E_i^+) \\ & + T\ln\Big(1+e^{-E_i^-/T}\Big) + T\ln\Big(1+e^{-E_i^+/T}\Big)\Big] \\ & + \frac{1}{4G}(M-m)^2 + \frac{1}{4H_s}|\Delta|^2 + \frac{1}{16H_t}|\Delta'|^2\ , \quad (5)\end{aligned}$$

where m is the bare quark mass, $M = m - 2G\langle\bar\psi\psi\rangle$, $\Delta = -2H_s\delta$, and $\Delta' = 4H_t\delta'$. $E_{1,2}^{\pm}$ are given in Eq. (1), while the dispersion law for quarks of color 3 reads

$$E_3^{\mp}(\vec p) = \sqrt{(\sqrt{M_{\mathit{eff}}^2 + \vec p^{\,2}} \mp \mu_{\mathit{eff}})^2 + |\Delta'_{\mathit{eff}}|^2}\ , \quad (6)$$

where $\mu_{\mathit{eff}}^2 = \mu^2 + |\Delta'|^2\sin^2\theta$, $M_{\mathit{eff}} = M\mu/\mu_{\mathit{eff}}$, and

$$|\Delta'_{\mathit{eff}}|^2 = |\Delta'|^2\,(\cos^2\theta + \frac{M^2}{\mu_{\mathit{eff}}^2}\sin^2\theta)\ . \quad (7)$$

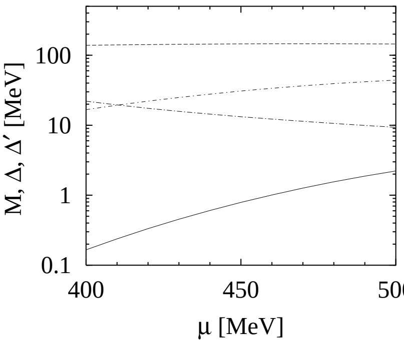

FIG. 1: M (dotted), Δ (dashed), and Δ' (solid) at $T = 0$ as functions of the quark chemical potential μ using parameter set 1 (see text). The dashed-dotted line indicates the result for Δ' taking parameter set 2.

Here $\cos\theta = p_3/|\vec p|$. Thus, as expected, for $\Delta' \neq 0$, $E_3^{\pm}(\vec p)$ is an anisotropic function of $\vec p$, clearly exhibiting the spontaneous breakdown of rotational invariance. For $M = 0$, the gap Δ'_{eff} vanishes at $\theta = \pi$. In general its minimal value is given by

$$\Delta'_0 = \frac{M|\Delta'|}{\sqrt{\mu^2 + |\Delta'|^2}}\ . \quad (8)$$

Expanding E_3^- around its minimum the low-lying quasiparticle spectrum takes the form

$$E_3^-(p_\perp, p_3) \approx \sqrt{\Delta_0'^2 + v_\perp^2(p_\perp - p_0)^2 + v_3^2 p_3^2}\ , \quad (9)$$

where $p_\perp^2 = p_1^2 + p_2^2$, and

$$v_\perp = \sqrt{1 - (\frac{\mu M}{\mu^2 + |\Delta'|^2})^2},\ v_3 = \frac{\Delta'_0}{M},\ p_0 = \frac{v_\perp}{v_3}|\Delta'|. \quad (10)$$

This leads to a density of states linear in energy:

$$N(E) = \frac{1}{2\pi}\frac{\mu^2 + |\Delta'|^2}{|\Delta'|}\ E\ \theta(E - \Delta'_0)\ . \quad (11)$$

The actual values for Δ, Δ' and M follow from the condition that the stable solutions correspond to the absolute minimum of Ω with respect to these quantities. Imposing $\partial\Omega/\partial\Delta'^* = 0$ we obtain the following gap equation for Δ':

$$\begin{aligned}\Delta' = 16H_t\Delta'\int\frac{d^3p}{(2\pi)^3}\Big\{ & (1 - \frac{\vec p_\perp^{\,2}}{s})\frac{1}{E_3^-}\tanh\frac{E_3^-}{2T} \\ & + (1 + \frac{\vec p_\perp^{\,2}}{s})\frac{1}{E_3^+}\tanh\frac{E_3^+}{2T}\Big\}\ , \quad (12)\end{aligned}$$

where $s = \mu_{\mathit{eff}}(\vec p^{\,2} + M_{\mathit{eff}}^2)^{1/2}$. Similarly one can derive gap equations for Δ and M by the requirements $\partial\Omega/\partial\Delta^* = 0$ and $\partial\Omega/\partial M = 0$, respectively. Together with Eq. (12), they form a set of three coupled equations, which have to be solved simultaneously. However, the equations for Δ and Δ' are not directly coupled, but only through their dependence on M.

In our numerical calculations we use a sharp 3-momentum cutoff Λ to regularize the divergent integrals. We then have five model parameters: m, Λ, G, H_s, and H_t. To get started we choose $m = 5$ MeV, $\Lambda = 600$ MeV, and $G\Lambda^2 = 2.4$ – leading to reasonable vacuum properties, $M = 393$ MeV and $\langle \bar{u}u\rangle = (-244\text{MeV})^3$ –, and the instanton relation to fix H_s and H_t ("parameter set 1"). The resulting values of M, Δ, and Δ' as functions of μ at $T = 0$ are displayed in Fig. 1. The chemical potentials correspond to baryon densities of about 4 - 7 times nuclear matter density. In agreement with earlier expectations [6] Δ' is small compared with Δ. However, it is strongly μ-dependent and rises considerably between $\mu = 400$ MeV and $\mu = 500$ MeV. Being a solution of a selfconsistency problem, Δ' is also extremely sensitive to the coupling constant H_t. As noted earlier, the effective interaction to be used at moderate densities is rather uncertain and hence the parameters listed above are by no means fixed. If we take H_t twice as large as before ("parameter set 2"), we arrive at the dashed-dotted line for Δ', which is then comparable to Δ. We also find that Δ' is very sensitive to the cutoff. This can be traced back to the factor $(1 - \vec{p}_\perp^{\,2}/s)$ in the gap equation (12). It is quite obvious then, that also the form of the regularization, i.e., sharp cutoff, form factor, etc., will have a strong impact on the results.

With increasing temperature both condensates, δ and δ', are reduced and eventually vanish in second-order phase transitions at critical temperatures T_c and T_c', respectively. It has been shown [5] that T_c is approximately given by the well-known BCS relation $T_c \simeq 0.57\Delta(T{=}0)$. In order to derive a similar relation for T_c' we inspect the gap equation (12) at $T = 0$ and in the limit $T \to T_c'$. Neglecting M (since $M \ll \mu$ this is valid up to higher orders in M^2/μ^2) and antiparticle contributions one gets

$$\int \frac{d^3p}{(2\pi)^3}\Big\{\Big[(1-\frac{\vec{p}_\perp^{\,2}}{s})\frac{1}{E_3^-(\vec{p})}\Big]_{\Delta'(T=0)} -(1-\frac{\vec{p}_\perp^{\,2}}{\mu\,|\vec{p}|})\frac{1}{|\mu-|\vec{p}||}\tanh\frac{|\mu-|\vec{p}||}{2T_c'}\Big\} \approx 0\ . \quad (13)$$

Since the integrand is strongly peaked near the Fermi surface, the $|\vec{p}|$-integrand must approximately vanish at $|\vec{p}| = \mu$, after the angular integration has been performed. From this condition one finds to lowest order in Δ'/μ:

$$T_c' \approx \frac{1}{3}\,\Delta'(T=0)\ . \quad (14)$$

The analogous steps would lead to $T_c/\Delta(T{=}0) \approx \frac{1}{2}$ instead of the textbook value of 0.57. This gives a rough idea about the quality of the approximation.

Numerical results for $\Delta(T)$ and $\Delta'(T)$ are shown in Fig. 2. The quantities have been rescaled in order to facilitate a comparison with the above relations for T_c

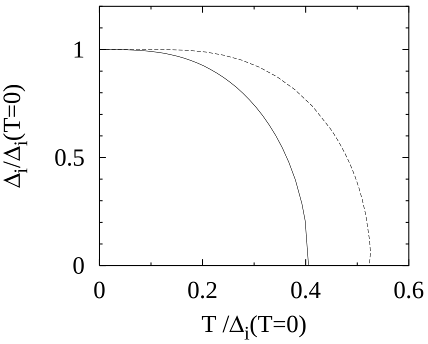

FIG. 2: $\Delta_i/\Delta_i(T{=}0)$ as function of $T/\Delta_i(T{=}0)$. Dashed: $\Delta_i = \Delta$. Solid: $\Delta_i = \Delta'$. The calculations have been performed at $\mu = 450$ MeV for parameter set 1.

and T_c'. Our results are in reasonable agreement with our estimates. These findings turn out to be insenstive to the actual choice of parameters.

The specific heat is given by $c_v = -T\partial^2\Omega/\partial T^2$ [19]. For $T \ll T_c$ it is completely dominated by quarks of color 3, since the contribution of the first two colors is suppressed by a factor $\mathrm{e}^{-\Delta/\mathrm{T}}$. Neglecting the T-dependence of M and Δ', and employing Eq. (11), one finds

$$c_v \approx \frac{12}{\pi}\frac{\mu^2+|\Delta'|^2}{|\Delta'|}T^2 \times \left[1+\frac{\Delta_0'}{T}+\frac{1}{2}\left(\frac{\Delta_0'}{T}\right)^2+\frac{1}{6}\left(\frac{\Delta_0'}{T}\right)^3\right]e^{-\frac{\Delta_0'}{T}}\ , \quad (15)$$

which should be valid for $T \ll T_c'$. In this regime c_v depends quadratically on T for $T \gtrsim \Delta_0'$, and is exponentially suppressed at lower temperatures.

To test this relation we evaluate $c_v(T)$ explicitly using Eq. (5). The results for fixed μ =450 MeV are displayed in Fig. 3. For numerical convenience we choose parameter set 2, leading to a relatively large $\Delta'(T{=}0) = 30.8$ MeV. The critical temperature is $T_c' \simeq 0.40\ \Delta'(T{=}0)$. For the energy gap we find $\Delta_0' = 0.074\ T_c'$. It turns out that Eq. (15), evaluated with constant values of Δ' and M, (dashed-dotted) is in almost perfect agreement with the numerical result (solid) up to $T \approx T_c'/2$. The phase transition, causing the discontinouity of c_v at $T = T_c'$, is of course outside the range of validity of Eq. (15). We also display c_v for $M = 0$ (dashed). Since Δ_0' vanishes in this case there is no exponential suppression, and c_v is proportional to T^2 down to arbitrarily low temperatures. However, even when M is included, the exponential suppression is partially cancelled by the extra-terms in square brackets of Eq. (15). For comparison we also show c_v for a system with $\Delta' = 0$, which exhibits a linear T dependence at low temperatures (dotted).

Our results show that, even though the magnitude of the gap parameter Δ' is strongly model dependent its relations to the critical temperature and the specific heat are quite robust. Thus, if we had empirical data, e.g.,

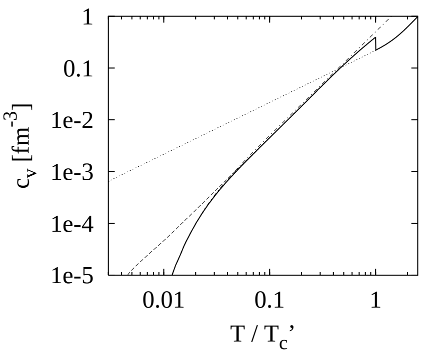

FIG. 3: Specific heat for parameter set 2 at $\mu = 450$ MeV as function of T/T_c'. Solid: full calculation, dashed: result for $M = 0$, dotted: without spin-1 condensate. The dashed-dotted line indicates the result of Eq. (15).

for the specific heat of dense quark matter, they could be used to extract information about the existence and the size of Δ'. In this context neutron stars and their cooling properties are the natural candidates to look at. In Ref. [1] it was suggested that the exponential suppresion of c_v related to the potential pairing of quarks of color 3 might have observable consequences for the neutrino emission of a neutron star. This argument has to be somewhat refined since, as seen above, $c_v(T)$ first behaves as T^2 and the exponential suppression sets in only at $T < \Delta_0'$. The relevance of c_v and the possible effect of diquark condensates on neutron star cooling was also discussed in Ref. [14]. On the other hand it has recently been argued [15], that the constraints imposed by charge and color neutrality might completely prohibit the existence of two-flavor color-superconducting matter in neutron stars.

Because of the spontaneously broken $U(1)\times O(3)$ symmetry in Eq. (2), for $\Delta' \neq 0$ there should be collective Nambu-Goldstone excitations in the spectrum. However, due to the Lorentz non-invariance of the system there can be subtleties [16–18]. The NG spectrum can be analyzed within an underlying effective Higgs potential

$$V(\phi) = -a^2\phi_n^\dagger\phi_n + \frac{1}{2}\lambda_1(\phi_n^\dagger\phi_n)^2 + \frac{1}{2}\lambda_2\phi_n^\dagger\phi_n^\dagger\phi_m\phi_m, \quad (16)$$

for the complex order parameter ϕ_n [17], with $\lambda_1+\lambda_2 > 0$ for stability. For $\lambda_2 < 0$ the ground state is characterized by $\phi_{vac}^{(1)} = (\frac{a^2}{\lambda_1})^{1/2}(0,0,1)$ which corresponds to our ansatz Eq. (2) for the BCS-type diquark condensate δ'. This solution has the property $\langle\vec{S}\rangle^2 = (\phi_{vac}^{(1)\dagger}\vec{S}\phi_{vac}^{(1)})^2 = 0$. The spectrum of small oscillations above $\phi_{vac}^{(1)}$ consists of 1+2 NG bosons, all with linear dispersion law: one zero-sound phonon and two spin waves [17]. Implying a finite Landau critical velocity, this fact is crucial for a macroscopic superfluid behavior of the system [18].

Note that for $\lambda_2 > 0$ there is a different solution $\phi_{vac}^{(2)} = (\frac{a^2}{2(\lambda_1+\lambda_2)})^{1/2}(1,i,0)$ with $\langle\vec{S}\rangle^2 = 1$. In this case the NG spectrum above $\phi_{vac}^{(2)}$ consists of one phonon with linear dispersion law and one spin wave whose energy tends to zero with momentum squared [17]. Clearly, there is no way of knowing without an explicit computation which one of the two anisotropic phases of deconfined quark matter is energetically favorable for a given interaction. Work in this direction is in progress.

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small [13].