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Weak Radiative Decays of Beauty Baryons

Pedagogical Approach

Z.J.Ajaltouni[1], G.Pakhlova[2], P.Robbe[3]

[1] *Laboratoire de Physique Corpusculaire de Clermont-Ferrand*
IN2P3/CNRS Université Blaise Pascal
F-63177 Aubière Cedex FRANCE
[2] *Institute of Theoretical and Experimental Physics (ITEP), Moscow, RUSSIA*
3 *Université Paris-Sud, LAL, Orsay, FRANCE*

Abstract

Weak decays of beauty baryons $\mathcal{B}_b = \Lambda_b, \Omega_b$ offer the opportunity to perform some tests of CP symmetry and time-reversal invariance TR. In this note, the *radiative* two-body decay channel is studied and emphasis is put on the angular distributions of final particles and on the importance of the polarization of the $\mathcal{B}_b$ baryon.

1 Introduction

Weak decays of beauty baryons represent a wide investigation field for checking symmetry laws like **CP** or **TR** (Time -Reversal) symmetry [1]. In the case of weak radiative decay (WRD) like $\Lambda_b \to \Lambda + \gamma$, interesting tests of Λ polarization can be performed in order to cross-check the Standard Model predictions. Same kind of tests can be done in the case of three body decays like $\Lambda_b \to \Lambda\ \ell^+\ \ell^-$ where the lepton pairs could come either from a vector-meson like J/ψ or a virtual photon. Furthermore, because of the Λ final polarization, TR invariance can be checked with observables built from the kinematics parameters of the final particles.

On the phenomenological side, the WRD of any baryon $\mathcal{B}_1 \to \mathcal{B}_2 + \gamma$ is described by the phenomenological lagrangian [2] :

$$\mathcal{L} = \bar{\psi}_1 (C + D\gamma_5)\sigma_{\mu\nu} F^{\mu\nu} \psi_2 \quad + h.c.$$

where coefficients C and D are respectively the Parity Conserving (PC) and the Parity Violating (PV) parts. From this lagrangian, the matrix element describing the radiative transition can be computed and the corresponding radiative width is given by :

$$\Gamma(\mathcal{B}_1 \to \mathcal{B}_2 + \gamma) = \frac{1}{8\pi} \frac{({M_1}^2 - {M_2}^2)}{M_1} (|C|^2 + |D|^2)$$

M_i being the mass of the baryon $\mathcal{B}_i$.

Furthermore, coefficients C and D are also related to asymmetries arising from the final state decays and help to probe Parity and CP symmetries in baryonic weak decays. [2]

2 Angular Momentum Analysis

Performing angular momentum analysis is an important task to cross-check the validity of symmetry laws, especially *CP symmetry*, in baryonic weak decays. We will focus on the channel $\Lambda_b \to \Lambda + \gamma$ followed by the decay $\Lambda \to p + \pi^-$ which, at the quark level, corresponds to the penguin electroweak process $b \to s + \gamma$.

Let $\vec{J}$ be the spin of the decaying baryon, $\vec{L}$ is the orbital angular momentum of the $\gamma\Lambda$ system and $\vec{S}$ is their *total spin*. Due to *the massless nature* of the photon, it could not state that $\vec{S} = \vec{s}_1 + \vec{s}_2$, $\vec{s}_1$ and $\vec{s}_2$ being respectively the spin of the photon and the final baryon. Indeed, the spin angular momentum of the photon cannot be disentagled from its orbital one and, according to the Landau-Lifschitz approach [3], it can be set that the parity of the final $\gamma\Lambda$ system is given by :

$$P(\gamma\Lambda) \quad = \quad (-1)^{(\ell+1)} P(\Lambda)$$

where ℓ is an L eigenvalue with two possible values $\ell = 0, 1$.
While $P(\Lambda_b) = P(\Lambda) = +1$, the following relations can be inferred :

$$\ell = 0 \Rightarrow P(\gamma\Lambda) = -1 \implies \text{Parity Violation}$$

$$\ell = 1 \Rightarrow P(\gamma\Lambda) = \ +1 \ \Longrightarrow \text{Parity Conservation}$$

In the remaining study, the Λ_b rest-frame will be used :

- The z-axis is given by the Λ_b momentum in the laboratory frame, $\vec{z} \mid\mid \vec{p_0} = \ \vec{p}_{\Lambda_b}/p_{\Lambda_b}$ It is chosen as the *quantization axis* in the helicity formalism.
- The y-axis is given by the direction of the vector product $\vec{p_0} \times \vec{p_\Lambda}$
- Finally the x-axis is obtained as $\vec{x} = \ \vec{y} \times \vec{z}$

$\vec{p}_\Lambda$ being the Λ momentum in the Λ_b rest-frame.

Helicity configurations

The spin of the initial decaying particle being 1/2 and the photon helicity having only *two values*, $\lambda_\gamma = \ \pm 1$, the helicity of the final state according to the (Δ)-axis where $(\Delta) \mid\mid \vec{p_\Lambda}$ is well constrained :

$$\lambda_f \ = \ \lambda_\Lambda - \lambda_\gamma \ = \ \pm\, 1/2$$

So, two configurations may arise :

$$(\lambda_\Lambda, \lambda_\gamma) \ = \ (1/2, 1) \Longrightarrow \lambda_f = \ -1/2$$

$$(\lambda_\Lambda, \lambda_\gamma) \ = \ (-1/2, -1) \Longrightarrow \lambda_f = \ +1/2$$

Departing from these two helicity states and from the initial spin component of the Λ_b, $m = s_z(\Lambda_b)$, the decay amplitude can be computed. The different computational steps are given below :

• Let $m, \lambda_\Lambda, \lambda_\gamma$ and $\vec{p}_\Lambda = \ (p, \theta, \phi)$ be fixed. According to the Jackson convention [4] for the Wigner rotation matrices, the decay amplitude can be written as :

$$A_m(\hat{p}, \lambda_\Lambda, \lambda_\gamma) \ \propto < jm; \lambda_\Lambda \lambda_\gamma | S | jm > \ D^{j\star}_{m\lambda}(\phi, \theta, 0)$$

where $D^j{}_{m\lambda}(\phi, \theta, 0)$ is the representation of the rotation matrix given by $D^j{}_{m\lambda}(\phi, \theta, 0) = \ \exp(-im\phi)\ d^j{}_{m\lambda}(\theta)$ and $\lambda = \lambda_f$

In a more compact form, the above amlitude can be written as :

$$A_m \propto \ M(\lambda_\Lambda, \lambda_\gamma) D^{j\star}{}_{m\lambda}(\phi, \theta, 0)$$

•• In a second step, we must take into account the *polarization* of the initial baryon Λ_b when this latter is produced in the proton-proton collisions. The most suitable way is to introduce the *polarization density-matrix* of the Λ_b whose expression is :

$$\begin{pmatrix} \rho_{++} & \rho_{+-} \\ \rho_{-+} & \rho_{--} \end{pmatrix}$$

where indice $+$ is related to the positive value of the spin projection $m = +1/2$ and indice $-$ indicates the negative one, $m = -1/2$; $\rho_{++} = \rho_{\frac{1}{2},\frac{1}{2}}$, etc..

The density-matrix (DM) is *hermitean* , $\rho_{ij} = \rho^{\star}_{ji}$ and its trace verifies $\mathrm{Tr}\rho = 1$.

So, a general expression of the final angular distributions $W(\theta, \phi)$ must take account of the following physical requirements :

1) Instead of the ordinary squared matrix element $|A_m|^2$, a summation over the DM elements must be performed :

$$\Sigma_{m,m'} \rho_{mm'} A_m A^{\star}_{m\prime} \ \text{ with } \ m, m\prime = \pm 1/2$$

2) The helicities of the final particles γ and Λ being *not measured*, a summation over $\lambda_1 = \lambda_\gamma$ and $\lambda_2 = \lambda_\Lambda$ must be done too. These last ones appear in the *final helicity* according to the Δ-axis which takes only two values, $\lambda = \lambda_1 - \lambda_2 \ = \ \pm 1/2$.

Thus the final angular distribution will be given by the expression :

$$\begin{aligned} W(\theta, \phi) &\propto \Sigma_{\lambda_1,\lambda_2} \left(\Sigma_{m,m'} \rho_{mm'} A_m A^{\star}{}_{m\prime} \right) \\ &\propto \Sigma_{m,m\prime,\lambda} \rho_{mm'} M(\lambda_1, \lambda_2) M(\lambda_1, \lambda_2)^{\star} D^{j\star}_{m\lambda} D^{j}{}_{m'\lambda} \end{aligned}$$

The above relation contains only *8 terms* instead of 16 and if we replace $D^j_{m\lambda}$ by their standard expression, the final angular distribution will be :

$$W(\theta, \phi) = N \ \Sigma_{m,m\prime,\lambda} |M(\lambda)|^2 \exp(i(m - m')\phi) d^{j}{}_{m\lambda}(\theta) d^{j}{}_{m'\lambda}(\theta) \tag{1}$$

N being a normalization factor. Among the 8 terms, only *two different* values of M arise, $M(++) = M(1/2, 1)$ and $M(--) = M(-1/2, -1)$. Expressing this relation according to the DM elements ρ_{ij}, we get :

$$\begin{aligned} W(\theta, \phi) &= |M(++)|^2 (\rho_{++} \sin^2(\theta/2) + \rho_{--} \cos^2(\theta/2) - \Re(\rho_{+-} \exp(i\phi)) \sin\theta) \\ &+ |M(--)|^2 (\rho_{++} \cos^2(\theta/2) + \rho_{--} \sin^2(\theta/2) + \Re(\rho_{+-} \exp(i\phi)) \sin\theta) \end{aligned}$$

Due to the algebraic properties of the DM, there are *three unknown parameters* :

$$\rho_{++} \ , \ \Re(\rho_{+-}) \ \text{ and } \ \Im(\rho_{+-})$$

It is worth noticing that the matrix elements $M(++)$ and $M(--)$ correspond respectively to the right-handed photon and to the left-handed one and they will be written as M_R and M_L respectively.

3 Final Angular Distributions

The final step consists in averaging over the azimuthal angle ϕ around the z-axis. Consequently, the interference terms represented by the non-diagonal matrix element ρ_{+-} *disappear* and a simple expression is obtained for the *polar* angle distribution θ :

$$\begin{aligned} R(\theta) = \frac{dN}{d\cos\theta} & \propto \ |M_R|^2(\rho_{++}\sin^2(\theta/2) + \rho_{--}\cos^2(\theta/2)) \\ & + \ |M_L|^2(\rho_{++}\cos^2(\theta/2) + \rho_{--}\sin^2(\theta/2)) \end{aligned}$$

At this stage, new physical parameters can be introduced :

i) The initial polarization of the Λ_b baryon, $\mathcal{P} = P_{\Lambda_b} = \ \rho_{++} - \rho_{--}$
ii) The asymmetry parameter of the photon (due to its own polarization),

$$\alpha_\gamma \ = \ \frac{|M_L|^2 - |M_R|^2}{|M_L|^2 + |M_R|^2}$$

The final expression of $R(\theta)$ becomes :

$$R(\theta) = \frac{|M_R|^2 + |M_L|^2}{2}(1 + \alpha_\gamma \mathcal{P}\cos\theta) \tag{2}$$

Interesting remarks can be drawn from the expression above :

- This angular distribution is identical to the *standard one* describing the weak hyperon two-body decay like Ξ , $\Sigma \rightarrow \Lambda + \gamma$ and, as it is expected, this distribution violates parity symmetry because $R(\pi - \theta) \neq \ R(\theta)$ [5]
- To detect any parity violation in Λ_b decay, the photon must have different helicity weights, $|M_R|^2 \neq |M_L|^2 \ \Longrightarrow \alpha_\gamma \neq 0$
- To get evidence for a photon asymmetry even when this one is polarized, the initial baryon Λ_b must be polarized, $\rho_{++} \neq \rho_{--}$

So, knowing the polarization density-matrix of the Λ_b is an **essential ingredient** to perform any test of symmetry violation like CP or TR with the beauty baryon decays.

Simulation Method

The main point in simulating the decay process $\Lambda_b \rightarrow \Lambda + \gamma$ is the polar angle generation. By normalizing the $R(\theta)$ distribution, one can get the probability density function :

$$f(x) = \frac{1}{2}(1 + Ax) \quad \text{where } x = \ \cos\theta$$

with $A = \alpha_\gamma \mathcal{P}$ and $|A| \leq 1$

By taking a random variable Y uniformly generated in the interval $[0, 1]$, x is obtained from the following relation :

$$x = \ -1 + \sqrt{1 + A^2 + A(4Y - 2)}$$

4 How to generate $\Lambda_b \to \Lambda\gamma$ decay with EvtGen

Such decays $\Lambda_b \to \Lambda\gamma$ can be generated with EvtGen[6], the event generator used in the LHCb simulation software (Gauss)[7]. This section explains how to write the corresponding decay file and shows some results obtained with the generator.

4.1 Decay file

The decay model to use to generate this decay mode is `HELAMP`. In this case it needs 4 parameters that have to be given as arguments to the model. The parameters are the magnitudes and phases of the amplitudes $M(++)$ and $M(--)$ defined in Eq. (1).

The syntax of the decay file will be:

```
Decay Lambda_b0sig
1.000 Lambda0 gamma HELAMP |M(++)| arg[M(++)] |M(--)| arg[M(--)];
Enddecay
CDecay anti-Lambda_b0sig
End
```

where `Lambda_b0sig` is the alias to the signal Λ_b in the event.

It is important to note that Λ_b will be produced unpolarized by Pythia in the current Gauss software versions.

4.2 Some results obtained with EvtGen

Fig.1 shows the cosine of the decay angle θ as defined in section 2 obtained with a special production where the Λ_b were fully polarized, with parameters:

$$\begin{aligned} M(++) &= 1 \\ M(--) &= 0 \\ \rho_{++} &= 1 \\ \rho_{--} &= 0 \end{aligned}$$

As expected, the distribution follows Eq. (2) with $\alpha_\gamma = -1$ and $\mathcal{P} = +1$.

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[7] **Gauss User Guide and Reference Manual**

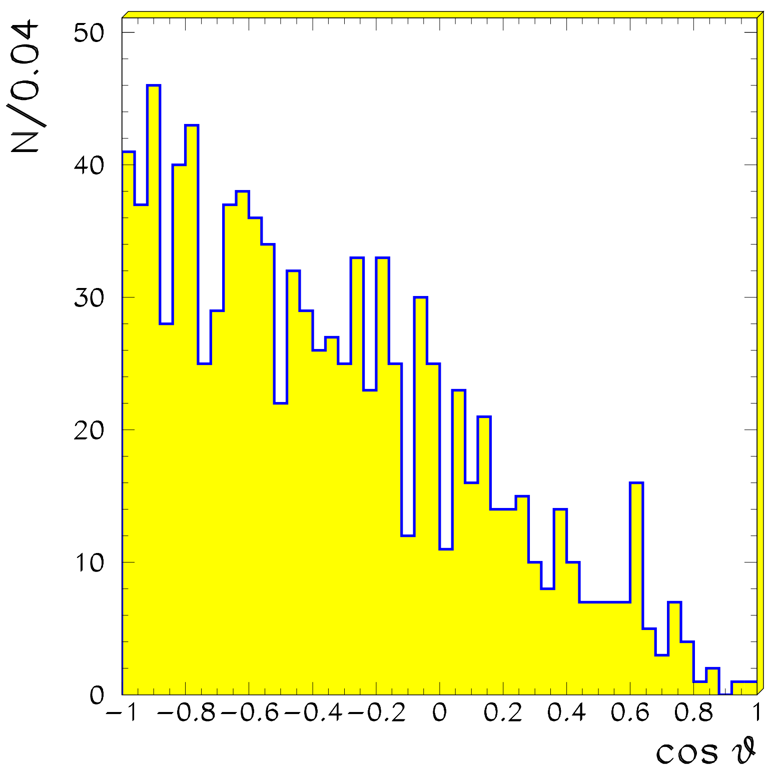

Figure 1: Helicity angle