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Abstract. We present microscopic calculations of low energy scattering observables in all possible four nucleon systems: n-^{3}H, p-^{3}He and p-^{3}H. Results were obtained by solving Faddeev-Yakubovski equations in configuration space, appropriately modified to include Coulomb and three-nucleon forces.

1 Introduction

Three- and four-nucleon systems are the testing ground for studying the nuclear interaction. If modern NN potentials have reached a very high degree of accuracy in describing the two-nucleon data, they fail already to account for the binding energies of the lightest nuclei. The use of three-nucleon forces (3NF) is – with some exceptions [1, 2] – mandatory. By adjusting its parameters one can obtain a satisfactory description of nuclear binding energies up to A=10 [5].

Apart the effect of rescaling nuclear thresholds, low energy three-nucleon scattering observables are quite insensitive to 3NF. Furthermore, since 3N spectra does not contain any narrow resonances, three-nucleon dynamics is relatively rigid once deuteron and triton binding energies are fixed. The four nucleon continuum is a challenging few-body nuclear problem. Its interest lies not only in the natural progression that it represents, towards a systematic description of nuclear systems with increasing complexity, but also in the richness of the $A = 4$ nuclear chart itself (see Fig. 1). Four nucleon problem, we believe, implies a qualitative jump in respect to the $A = 3$ case.

2 Theoretical tools

We use the Faddeev-Yakubovski (FY) equations in configuration space to tackle the four-particle problem. However in their original form these equations are applicable only for systems interacting via short-range pairwise forces. An elegant

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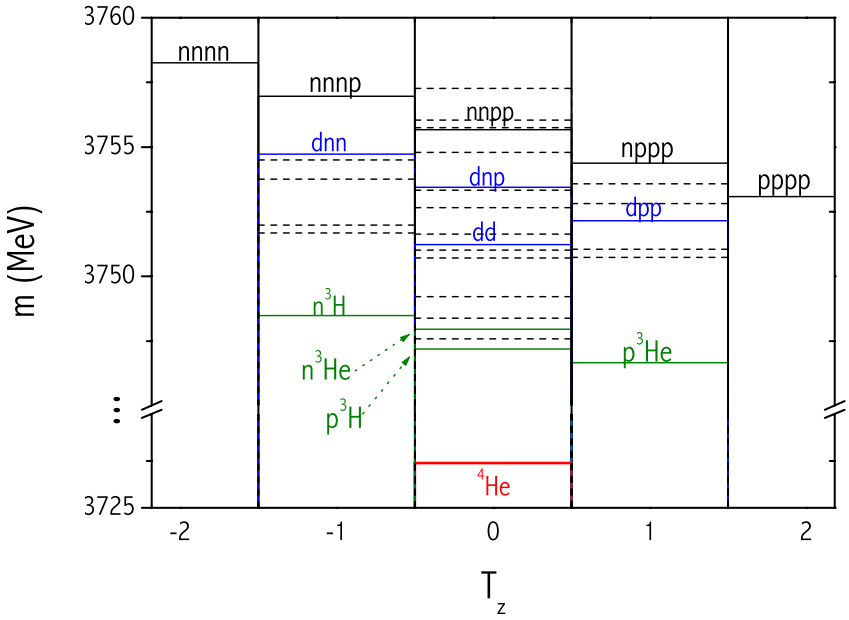

Figure 1. Four nucleon energy spectra. Single lines indicate particle thresholds, whereas dashed lines represent particle decay unstable excited states (resonances).

way to include 3NF into FY equations was suggested in [3] for four identical particles. These equations reads:

$$\begin{aligned} (E - H_0 - V_{12})\, K^4_{12,3} &= V_{12}(P^+ + P^-)\left[(1+Q)K^4_{12,3} + H^{34}_{12}\right] + V_{12,3}\Psi \\ (E - H_0 - V_{12})\, H^{34}_{12} &= V_{12}\tilde{P}\left[(1+Q)K^4_{12,3} + H^{34}_{12}\right] \end{aligned} \quad (1)$$

with P^+, P^-, $\tilde{P}$ and Q being permutation operators:

$$P^+ = (P^-)^- = P_{23}P_{12}; \quad Q = \varepsilon P_{34}; \quad \tilde{P} = P_{13}P_{24} = P_{24}P_{13}. \quad (2)$$

and

$$\Psi = \left[1 + (1 + P^+ + P^-)\right] Q(1 + P^+ + P^-)K^4_{12,3} + (1 + P^+ + P^-)(1 + \tilde{P})H^{34}_{12}$$

the total wave function.

Equation (1) becomes however non appropriate once long range interaction, in particular Coulomb, is present. In fact, FY components remain coupled even in far asymptotics, thus making the implementation of correct boundary conditions hardly possible. In order to circumvent this problem we have split Coulomb potential (V_C) into long ($V_C^{(l)}$) and short ($V_C^{(sh)}$) range parts by means of some arbitrary cut-off function $\chi(x,y,z)$, similarly as it was done by Merkuriev for 3-body equations [4]:

$$V_C(x) = V_C^{(sh)}(x,y,z) + V_C^{(l)}(x,y,z); \qquad \begin{aligned} V_C^{(sh)}(x,y,z) &= \chi(x,y,z)V_C(x) \\ V_C^{(l)}(x,y,z) &= \left[1 - \chi(x,y,z)\right] V_C(x) \end{aligned} \quad (3)$$

$\chi(x,y,z)$ is a smooth function which equals 1 for small x values or when $x \ll (y,z)$ and vanishes for $x \gg (y,z)$. Then the 4-body FY equations can be rewritten in the form:

$$\begin{aligned}(E - H_0 - V_{12}^{(sh)} - \sum V_C^{(l)})K^4_{12,3} &= V_{12}^{(sh)}(P^+ + P^-)\left[(1+Q)K^4_{12,3} + H^{34}_{12}\right] + V_{12,3}\Psi \\ (E - H_0 - V_{12}^{(sh)} - \sum V_C^{(l)})H^{34}_{12} &= V_{12}^{(sh)}\tilde{P}\left[(1+Q)K^4_{12,3} + H^{34}_{12}\right] \end{aligned} \quad (4)$$

We use for short the following notations:

$$V_{12}^{(sh)} = \left(V_{(N)} + V_C^{(sh)}\right)_{12}. \quad (5)$$

In this form FY equations become assymptoticaly uncoupled and appropriate boundary conditions can be easily implemented.

Equations (4) are solved by making partial wave decomposition of amplitudes $K^4_{12,3}$ and H^{34}_{12}:

$$K_i(\boldsymbol{x}_i, \boldsymbol{y}_i, \boldsymbol{z}_i) = \sum_{LST} \frac{\mathcal{K}_i^{LST}(x_i, y_i, z_i)}{x_i y_i z_i} \left[L(\hat{x}_i, \hat{y}_i, \hat{z}_i) \otimes S_i \otimes T_i\right] \quad (6)$$

$$H_i(\boldsymbol{x}_i, \boldsymbol{y}_i, \boldsymbol{z}_i) = \sum_{LST} \frac{\mathcal{H}_i^{LST}(x_i, y_i, z_i)}{x_i y_i z_i} \left[L(\hat{x}_i, \hat{y}_i, \hat{z}_i) \otimes S_i \otimes T_i\right] \quad (7)$$

The partial components $\mathcal{K}_i^{LST}$ and $\mathcal{H}_i^{LST}$are expanded in the basis of three-dimensional splines. Thus integro-differential equation (4) is converted into system of linear equations. For detailed discussion on the equations and method used one can refer to [10].

3 Results

3.1 n-^{3}H elastic scattering

The elastic scattering of neutrons on tritium is the simplest 4N reaction. Treatment of this system doesn't require calculation of the costly integrals due to the Coulomb terms in eq. (4). Nevertheless this system contains two, spin degenerated, narrow resonances at low energy ($E_{cm} \approx 3$ MeV). Ability of nuclear interaction models to describe scattering cross sections in this resonance region is still an open question [6, 8].

	MT I-III		Av. 14		Av. 18+UIX	
	$J^\pi = 0^+$	$J^\pi = 1^+$	$J^\pi = 0^+$	$J^\pi = 1^+$	$J^\pi = 0^+$	$J^\pi = 1^+$
n-^{3}H	4.10	3.63	4.28	3.81	4.04	3.60
p-^{3}He	11.5	9.20	12.7	-	11.3	-
p-^{3}H	-63.1	5.50	-13.9	5.77	-16.5	5.39

Table 1. 4N scattering lengths calculated using different interaction models.

Semi-realistic MT I-III potential was shown to be very successful in describing total as well as differential cross sections [7] at the energies bellow n-n-d

threshold. Situation is less obvious for realistic potentials, which require much larger partial wave basis (PWB) to obtain converged results. Recently we have managed to considerably enlarge our PWB compared to [8]. Despite of having some effect on the negative parity phase shifts, the total cross section near the resonance peak ($E_{cm} = 3\ MeV$) has not been improved (see Fig. 3). We should notice, however, that the $\mathcal{J}^{\Pi} = 2^-$ phase shifts, the most relevant due to its statistical factor, are not fully converged yet. They still show some trend to increase, however we estimate that it can not suffice to reproduce the experimental data.

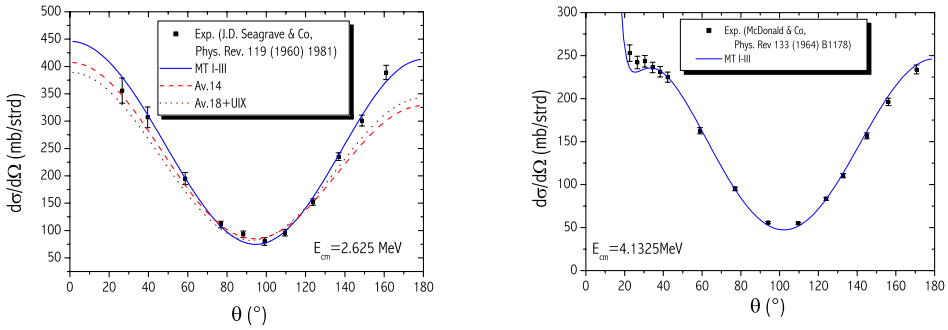

Figure 2. Differential cross sections versus centre-of-mass angle for n+^{3}H at E$_{cm}$ = 2.625 MeV (figure on the left) and for p+^{3}He at E$_{cm}$ = 4.1325 MeV (figure on the right)

By including UIX-3NF we have managed to reproduce the experimental zero-energy cross sections (see Table 1), which are overestimated by realistic NN interaction without 3NF. However their effect near the peak remains very small. Other smaller discrepancy exists near the minima of total cross sections ($E_{cm} \approx 0.4 MeV$). In this region, determined by positive parity states, both MT I-III and Av.18+UIX overestimate experimental data. This suggest that low energy parameters (0^+ and 1^+ scattering lengths and effective ranges) are not described sufficiently well.

3.2 p-^{3}He elastic scattering

This system is an isospin partner of n-^{3}H. Yet calculations were done uniquely with MT I-III model, except of $\mathcal{J}^{\Pi} = 0^+$ scattering lengths for which our Av.18 and Av.18+UIX predictions agree very well with the results of Pisa group [9]. Like in the n-^{3}H case, MT I-III was found to be successful in describing differential cross sections up to d+p+p threshold (see Fig. 2).

3.3 p+^{3}H scattering at very low energies

^{4}He continuum is the most complex 4N system, since its spectrum contains numerous resonances (see Fig. 1). Calculations of p+^{3}H scattering are furthermore complicated by the existence of the first $\mathcal{J}^{\Pi} = 0^+$ excitation of ^{4}He in its thresholds vicinity. This resonance located at $E_R \approx 0.4\ MeV$ above p+^{3}H threshold covers with its width $\Gamma \approx 0.5\ MeV$ almost the entire region below n+^{3}He. In order to separate n+^{3}He and p+^{3}H channels one should properly treat Coulomb

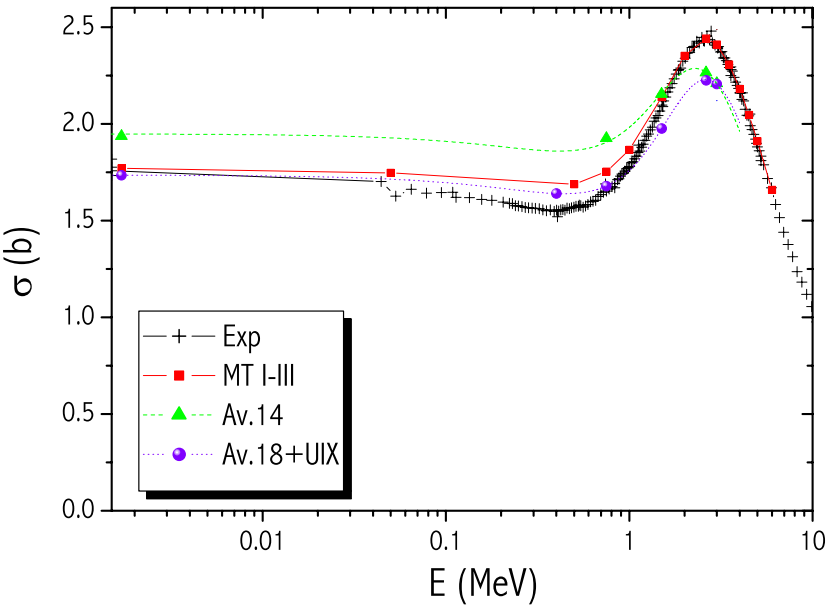

Figure 3. Calculated n+^{3}H total cross sections compared with experimental data [11].

interaction, the task is being furthermore burden since both thresholds are described by the same isospin quantum numbers.

When ignoring Coulomb interaction, as was a case in the most of the calculations performed until now, n+^{3}He and p+^{3}H thresholds coincide. 0^+ resonant state moves below the joint threshold and becomes a bound state for all NN potentials models we use. Former fact is reflected in low energy scattering observables: on one hand N+NNN scattering length in 0^+ state is found positive, on the other hand excitation function decreases smoothly with incident particles energy and does not show any resonant behavior.

By properly taking Coulomb interaction into account, thus separating n+^{3}He and p+^{3}H thresholds, we have placed the ^{4}He virtual state in between. However unlike in the other 4N systems, MT I-III predictions for p-^{3}H singlet ($\mathcal{J}^\Pi = 0^+$) scattering length as well as the excitation function – $\frac{d\sigma}{d\Omega}(E)\big|_{\theta=120^\circ}$ – are in disagreement with experimental data. In this model ^{4}He virtual state is obtained too close to the p-^{3}H threshold and therefore has very small width. Nevertheless out of resonance region the excitation function approaches experimental data points, which is not a case in the calculations without Coulomb interaction considered. Our calculations with realistic potentials are still limited in PWB. Nevertheless, they provide very promising results displayed in Fig. 4. Pure 2NF models predict too large singlet scattering length, thus placing the virtual state too far from threshold. On the other hand, by implementing UIX-3NF in conjunction with Av.18 NN model one obtains singlet scattering length as well as the excitation function $\frac{d\sigma}{d\Omega}(E)\big|_{\theta=120^\circ}$ in agreement with experimental data [12]. A detailed analysis of these calculations is in progress [10, 13] as well as their extension above the n-3He threshold.

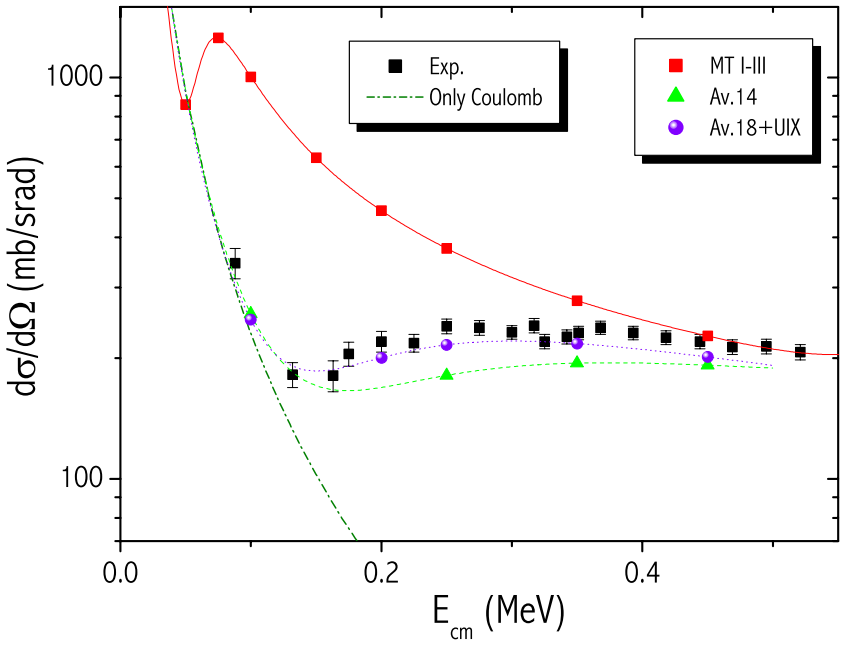

Figure 4. Energy dependence of p+^{3}He elastic differential cross sections at 120°: experimental results are compared with our calculation.

References

1. P. Doleschal: Nucl. Phys. **A602** (1996) 60
2. F. Gross, A. Stadler: Phys. Rev. Lett. **78** (1997) 26
3. D. Hüber, H. Witała, A. Nogga et al.: Few-Body Syst. **22** (1997) 107
4. S.P. Merkuriev: Ann. Phys. (N.Y.) **130** (1980) 395
5. S. Pieper et. al.: Phys. Rev. **C66** (2002) 044310
6. A. C.Fonseca: Phys. Rev. Lett. **83** (1999) 4021
7. J. Carbonell: Few-Body Syst. Suppl. **12** (2000) 439
8. F.Ciesielski, J. Carbonell, C. Gignoux: Phys. Lett. **B447** (1999) 199
9. M. Viviani, S. Rosati, A. Kievski: Phys. Rev. Lett. **81** (1998) 1580
10. R. Lazauskas: PhD Thesis, Université Joseph Fourier, Grenoble (2003)
11. T.W. Phillips, B.L. Berman and J.D. Seagrave: Phys. Rev. **C22** (1980) 384
12. N. Jarmie, M.G. Silbert et al.: Phys. Rev. Rev. **130** (1963) 5
13. R. Lazauskas and J. Carbonell: to be published