Probing Dark Energy with Supernovae: Bias from the time evolution of the equation of state

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Abstract

Observation of thousands of type Ia supernovae should offer the most direct approach to probe the dark energy content of the universe. This will be undertaken by future large ground-based surveys followed by a space mission (SNAP/JDEM). We address the problem of extracting the cosmological parameters from the future data in a model independent approach, with minimal assumptions on the prior knowledge of some parameters. We concentrate on the comparison between a fiducial model and the fitting function and adress in particular the effect of neglecting (or not) the time evolution of the equation of state. We present a quantitative analysis of the bias which can be introduced by the fitting procedure. Such bias cannot be ignored as soon as the statistical errors from present data are drastically improved.

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1 Introduction

The general paradigm in cosmology is that we are living in a flat universe, which is dominated by a nearly homogeneous component with negative pressure. This component is often called Dark Energy (DE) and causes the expansion rate of the universe to accelerate.

The recent measurements of type Ia supernovae (thereafter denoted SN) are the most direct evidence of the presence of this component [1, 2, 3, 4]. It is also confirmed by the combination of results from the large-scale distribution of galaxies [6] and the most precise data on the cosmic microwave background (CMB) from the Wilkinson Microwave Anisotropie Probe (WMAP) [7]. A recent combined analysis is presented in [8].
Recently the detection of the late Integrated Sachs-Wolfe (ISW) effect has reinforced the case for DE [9]. All this is most frequently interpreted in the framework of the so-called "concordance Cosmology" [10].

A fundamental problem is the identification of the underlying nature of DE : cosmological constant, quintessence (for a review see [11]) or something else. The most common way is to measure its equation of state (EoS) defined as $w = p_X/\rho_X$ where p_X is the pressure and ρ_X the energy density of the DE. The ratio of the DE density to the critical density will be denoted Ω_X in a general model and Ω_Λ in the simplest case of a Cosmological Constant ($w = -1$). Ω_M is the corresponding parameter for (baryonic+cold dark) matter.

An ambitious SN program is now on the way. Important pieces of information will be provided by large ground-based surveys such as the Supernova Legacy Survey (SNLS) [12] and these investigations will culminate with a space mission as the Supernova Acceleration Probe (SNAP) instrument, part of the Joint Dark Energy Mission (JDEM), which aims at the discovery and follow-up of some 2000 SNIa per year in the redshift range $z = 0.2 - 1.7$ with very precise magnitude measurements [13]. Of course, the validity of the obtained precision will not depend on the size of the SN sample only, but also on the ability to control the systematic uncertainties at the same level [14]. These systematic errors are coming on one side from the instrument itself (calibration, etc, ...) and also from the SN astrophysical environment (evolution, lensing effects, ...). The SNAP collaboration proposes a global strategy to control such effects at the percent level [13].

In this context, it is necessary to analyze very carefully at which precision level it will be possible to draw any conclusions from the expected rich amount of data.

Authors have used the present SN data together with simulated sets to :

- evaluate the accuracy on the Ω_Λ parameter to validate the acceleration.
- measure the DE through the equation of state with w constant in redshift.
- evidence a possible redshift dependence $w(z)$ with various parametrizations.
- look at the model dependences and degeneracies and study the possible strategies to break the latter.

There is a large consensus that the future SN data *alone* will have difficulties to constrain an evolving equation of state and to break model degeneracies. It is mandatory to have a prior knowledge of the values of some parameters. In particular, a precise knowledge of Ω_M will be essential if one hopes to pick out the z dependence of w (see e.g. [15, 16, 17, 18, 19, 20, 21]) even in the simplest flat cosmology. However, to use some pieces of information from other sources than SN, it will be essential to combine the data in a coherent way, that is under the same hypothesis, in particular on the DE properties.

On the other hand, some potential difficulties have been pointed out by various authors. Indeed, most papers have been mainly interested in predicting, for each cosmological parameter, the errors around some fiducial value, close to the ones obtained from the present data as $\Omega_M = 0.3, \Omega_X = 0.7, w = -1, dw/dz = 0$. In this procedure, the framework is a particular fiducial model and the chosen fitting function is usually the one used to generate the data. This strategy is valuable for a first estimate but is too restrictive to pin down the underlying physics which is so far unknown.

For example, Maor et al. [16] and Gerke and Efstathiou [20] have stressed the problems which arise if the redshift dependence of the equation of state is neglected whereas this dependence is present. The multiple integral relation between the luminosity distance and w smears out information about w and its time-variation. Assuming in the fitting function that w is a constant, whereas it is not in the fiducial model, can lead to a very wrong estimate of the "constant" or even "effective" w value. At the same time, the central value of Ω_M (or Ω_X) is badly reconstructed.

It is unavoidable to get some ambiguities when trying to fit a particular fiducial cosmology with the "wrong" model ! In the following, we will call this problem the "bias problem". For present SN data, as will be shown, this question is not a concern as the statistical errors on model parameters stay large due to the limited statistics. In the perspective of much richer SN samples, this bias problem cannot be ignored. We address the problem in a more quantitative way than the authors of [22] which have only considered some specific classes of models. In this paper, we investigate the full parameter range, in a model independent way, to quantify where the 'bias' creates potential difficulties and where it can be ignored. We also evaluate the impact of priors when only SN measurements are used. We get a handle on the possible bias introduced in the analysis by looking at the different hypothesis which are done at the fitting function level. Results are presented in the parameter space of the fiducial models we have simulated.

This paper is organized as follows. In Section 2 we recall the theoretical description of the magnitude-redshift diagram for supernovae and we present the experimental situation. We identify the relevant parameters which enter as the parameters of the fiducial models and we define the bias which could be introduced by the fitting procedure.

In section 3, we focus on the extraction of the EoS parameter w_0. We illustrate the confusion created by the bias on examples. Then we analyze the impact of bias on simulated data : we consider future statistical samples corresponding to SNAP, SNLS and also a sample corresponding to present data. We try to find the safest way for extracting an effective w from the data with minimum bias and minimum priors. Our summary and conclusions are given in Section 4.

2 The experimental and theoretical framework

2.1 The experimental framework

The studies presented in this paper are performed with simulated supernovae samples with statistics equivalent to what we expect to have in the future. We concentrate on three sets of data which simulate the statistical power of the present and future data. We want to emphasize the importance of the future sample of supernovae taken on ground which are limited by the systematics inherent to this approach but are statistically one order of magnitude greater than the present sample.

- We reproduce the published data of Perlmutter et al. from [4]. We take directly the effective magnitude of the sample 3 of this paper, which corresponds to 60 SNIa, corrected from K corrections, extinction and stretch (the values are given in Table 3 of [4]). The errors on magnitude vary in the range [0.16-0.22].

- We simulate some future data coming from ground survey as the large SNLS survey at CFHT [5]. This survey has started in 2003 and the estimation after 5 years of running is to register a sample of 700 identified SNIa in the redshift range $0.3 < z < 1$. We simulate a sample as reported in Table 1 in agreement with the expected rate of [5]. We assume a magnitude dispersion of 0.15 for each supernova and constant in redshift after all corrections.

- We simulate data from a future space mission like SNAP, which plans to discover around 2000 identified SNIa, at redshift $0.2< z <1.7$ with very precise photometry and spectroscopy. The SN distribution is given in Table 1(from[14]). The magnitude dispersion is assumed to be constant and independent of the redshift at 0.15 for all SN's after correction. On some studies, we include the effect of adding a constant and uncorrelated in redshift systematic error of 0.02 on the magnitude.

A set of 300 very well calibrated SnIa at redshift < 0.1 should be measured by the incoming SN factory project [23]. This sample is needed to normalize the Hubble diagram and will be called in the following the "Nearby" sample.
A "SNAP" ("SNLS") sample means in this paper a simulation of the statistics expected from the SNAP like mission ("SNLS" survey) combined with the 300 nearby SN's. To make our point clear, we do not consider in the presented studies the experimental systematic errors, unless otherwise specified.

z	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7
SNLS	-	44	56	80	96	100	104	108	-	-	-	-	-	-	-	-
SNAP	35	64	95	124	150	171	183	179	170	155	142	130	119	107	94	80

Table 1: *number of simulated supernovae by bin of 0.1 in redshift for SNLS and SNAP respectively.*

2.2 The theoretical framework

In the standard Friedmann-Roberston-Walker metric, the apparent magnitude of astrophysical objects can be expressed as a function of the luminosity distance :

$$m(z) \;=\; 5\,log_{10}(D_L) + M_B - 5\,log_{10}(H_0/c) + 25 \;=\; M_s + 5\,log_{10}(D_L) \tag{1}$$

where M_B is the absolute magnitude of SNIa, M_S may be considered as a normalization parameter and $D_L(z) \equiv (H_0/c)\; d_L(z)$ is the H_0-*independent* luminosity distance to an object at redshift z. It is related to the comoving distance $r(z)$ by
$D_L(z) \;=\; (1+z)r(z)$, where

$$r(z) \;=\; \begin{cases} \frac{1}{\sqrt{-\Omega_k}}\sin(\sqrt{-\Omega_k}\,J), & \Omega_k < 0 \\ J, & \Omega_k = 0 \\ \frac{1}{\sqrt{\Omega_k}}\sinh(\sqrt{\Omega_k}\,J), & \Omega_k > 0 \end{cases} \tag{2}$$

$$\text{with }\; \Omega_k \;=\; 1 - \Omega_m - \Omega_X \equiv 1 - \Omega_T, \tag{3}$$

$$J \;=\; \int_0^z \frac{H_0}{H(z')} dz' \tag{4}$$

and

$$\left(\frac{H(z)}{H_0}\right)^2 \;=\; (1+z)^3\,\Omega_m + \frac{\rho_X(z)}{\rho_X(0)}\,\Omega_X + (1+z)^2\,\Omega_k, \tag{5}$$

with

$$\frac{\rho_X(z)}{\rho_X(0)} \;=\; \exp\left[3\int_0^z\; (1+w(z'))\; d\,\ln(1+z')\right] \tag{6}$$

Note that we have neglected the radiation component Ω_R.

2.2.1 Choice of a fitting function

A fitting function is an expression for $H(z)$ in terms of a number of parameters. $H(z)$ is usually expressed in term of the cosmological parameters contained in Eq.(5)[15, 16, 17, 18, 19, 20, 21] but other possibilities relying on geometrical parametrizations or arbitrary ansatz have been proposed [24]. In this paper, we adopt the first approach where the cosmological parameters have a direct physical interpretation within General Relativity. With such analysis, the constraints are aimed to be model independent and can be used for investigation of a large variety of DE models. However, we have to choose a parametrization for the time (or redshift) variation of the EoS.

The current parametrizations of $w(z)$ which can be found in the literature are the linear one [17]: $w(z) = w_0 + w_1 z$; the one advocated by Linder [25] with a better asymptotic behavior : $w(z) = w_0 + w_a z/(1+z)$; and also $w(z) = w_0 - \alpha\,\ln(1+z)$ from [20] which is advocated to describe a large sample of quintessence models characterized by a weakly varying EoS. We choose the linear one which allows to compare most easily to previous works. In this case, Eq.(6) becomes:

$$\rho_X(z) \;=\; \rho_X(0)\,e^{3w_1 z}\,(1+z)^{3(1+w_0-w_1)} \tag{7}$$

Nevertheless, our results are essentially independent of the choice of this parametrization since the redshift range of the SN data is limited to $z < 2$ and the sensitivity of the data on $w(z)$ concerns mainly the values taken by w at relatively low redshifts $z \simeq 0.2 - 0.4$ (the "sweet spot" of [18, 17, 21]) where all parametrizations reduce roughly to the linear one.

Finally, note that models with non-trivial variations of the EoS $w(z)$, like the pseudo-goldstone model of ref.[26], cannot be described by such parametrizations. Consequently, our analysis does not apply to this kind of models.

Therefore, five parameters have to be fitted in the most general procedure: $M_s, \Omega_M, \Omega_X, w_0, w_1$.

2.2.2 Choice of a fiducial model

We define fiducial models which depend on the parameters $M_s^F, \Omega_M^F, \Omega_X^F, w_0^F, w_1^F$.

In order to keep this paper clear, we concentrate on the case where the paradigm $\Omega_T = 1$ is verified by the fiducial models. This value is in agreement with the inflationary paradigm and it is supported by the analysis of the rich amount of data from the CMB [7]. Ω_X^F is then no longer free ($\Omega_X^F = 1 - \Omega_M^F$) and, if not stated otherwise, the parameters M_s^F and Ω_M^F are fixed at: $M_s^F = -3.6$ and $\Omega_M^F = 0.3$.

Our conclusions will not depend on variations of the M_s^F parameter. The variation due to different Ω_M^F values has been investigated. If $\Omega_T \neq 1$, it appears that our results can change significantly. Analysis of non-flat models will be presented elsewhere [27].

To focus on the dark energy measurement, we propose to scan a large variety of fiducial models as a function of the couple (w_0^F, w_1^F). We have scanned the plane (w_0^F, w_1^F) (hereafter denoted by $\mathcal{P}$) for the values $-2 < w_0^F < 0$ and $-2 < w_1^F < 2$. We pay particular attention to the reduced plane $\mathcal{P}_{\mathcal{R}}$ associated to the ranges $-1 < w_0^F < 0$ and $-1 < w_1^F < 1$, which represents a reasonable class of theoretical models. Indeed, in most of the models available in the literature, like the quintessence models [11], the weak energy condition $w_0^F > -1$ is satisfied (see e.g. [28]). They also possess in general a relatively weak z dependence for the EoS [20, 17]. However, we consider the full plane $\mathcal{P}$ in order to include in our analysis more exotic models which violate the weak energy condition or which are described by a modified Friedmann equation or by a modified theory of gravity (we refer to [29] for a recent list of models and references).

2.2.3 The fitting method

To analyse the (simulated) data, a minimization procedure has been used [30]. A standard Fisher matrix approach allows a fast estimate of the parameter errors. This method is however limited as it does not yield the central values of the fitted parameters. Then we adopt, unless specified, a minimization procedure based on a least square method. The least square estimators are determined by the minimum of the $\chi^2 = (\mathbf{m} - M(z,\Omega,w)^T \mathbf{V}^{-1}(\mathbf{m} - M(z,\Omega,w))$, where $\mathbf{m} = (m_1 ... m_n)$ is the vector of magnitude measurements, $M(z,\Omega,w)$ the corresponding vector of predicted values and $\mathbf{V}$ the covariance matrix of the measured magnitudes. The error on cosmological parameters is estimated at the minimum by using the first order error propagation technique: $\mathbf{U} = \mathbf{A}.\mathbf{V}.\mathbf{A^T}$ where $\mathbf{U}$ is the error matrix on the cosmological parameters and $\mathbf{A}$ the Jacobian of the transformation.

A full 5-parameter fit (5-fit) of the real or simulated data gives the central values and errors for the five parameters. In the following, the normalization parameter M_s is left free in the fit, unless specified, as its correlation with the cosmological parameters is strong [19]. To constrain accurately this parameter, which is fixed in most of the papers, we use the very low redshift simulated "Nearby" sample of 300 SN.

2.3 Choice of a fitting procedure

There are different ways to choose a fitting procedure. If one only wants to minimize the errors on the fitting parameters, a simple Fisher approach should be sufficient and should give conclusions depending on the initial fit hypothesis. Nevertheless, as we have already emphasized, this approach is fiducial model dependent and can lead to some bias that we want to quantify. Then, after a presentation of different Fisher results, we will expose our strategy to understand and explore the bias introduced by some fitting approach.

2.3.1 The Fisher approach

To perform a Fisher matrix analysis, a fiducial model is chosen and fixed. We take the "concordance model" version of the simplest flat ΛCDM model :

$$\Omega_M^F = 0.3,\ \Omega_X^F = 0.7,\ w_0^F = -1,\ w_1^F = 0 \text{ with } M_s^F = -3.6$$

We call this model 'Λ' in the following.

Each *fitting procedure* is defined by a particular choice of the parameters to be fitted: reducing the number of parameters will improve the parameter errors in the fitting function. Priors are introduced by fixing a parameter at a predefined value or inside a given range.

Table 2 gives the errors obtained with the Fisher matrix analysis for various fitting procedures with the SNLS and SNAP simulated data. The numbers are in agreement with the analysis of various authors [15, 16, 17, 18, 19, 20, 21].
Figure 1 gives the contours obtained in the plane (Ω_M, w_0) at 68.3 % CL with the SNAP statistics, for this fiducial model within the same fitting procedures.
As can be seen from the first line of Table 2 and from the black ellipse in Fig. 1, a complete 5 parameter-fit (5-fit)$(M_s, \Omega_M, \Omega_X, w_0, w_1)$, with no assumptions on the values of the cosmological parameters - including no constraint on the flatness of the universe- yields too large errors on the cosmological parameters to set any definite conclusions even with the high statistics sample expected for the SNAP+Nearby data.
Fig. 1 shows clearly that the reduction of the number of fitted parameters or/and the strengthening of the Ω_M prior, reduces the contours.

The second line of Table 2 presents errors with the paradigm of a perfectly flat universe $(\Omega_T = 1)$ and the 5-fit reduces to a 4-fit $(M_s, \Omega_M, w_0, w_1)$. It appears that for the SNAP statistics, one gets better estimates of w_0 and Ω_M but the error on w_1 is always large. For SNLS, even the error on w_0 is large.

Adding some prior knowledge of Ω_M improves greatly the situation: the same 4-fit $(M_s, \Omega_M, w_0, w_1)$ for SNAP and SNLS with a weak prior on Ω_M $(0.2 < \Omega_M < 0.4)$ yields

Table 2: Statistical errors obtained with a Fisher matrix analysis on the cosmological parameters for various fitting procedures with the SNLS and SNAP data repectively. The fiducial model is a cosmological constant with $M_s^F = -3.6$, $\Omega_M^F = 0.3$, $\Omega_X^F = 0.7$, $w_0^F = -1$, $w_1^F = 0$. The weak (strong) Ω_M prior corresponds to the constraint $\Omega_M = 0.3 \pm 0.1$ ($\Omega_M = 0.3 \pm 0.01$). The labels 5-fit, 4-fit and 3-fit corresponds to the fitting procedures 5-fit$(M_s, \Omega_M, \Omega_X, w_0, w_1)$, 4-fit$(M_s, \Omega_M, w_0, w_1)$ and 3-fit(M_s, Ω_M, w_0), respectively. $\sigma(\Omega_X)$ has been omitted from the table since $\Omega_X = 1 - \Omega_M$ is no longer a free parameter except for the 5-fit where $\sigma(\Omega_X)$ is very large. $\sigma(M_S)$ has been omitted from the table since its value, roughly 1% , changes weakly for the various fitting procedures thanks to the inclusion of the Nearby SN sample.

Fit	Ω_M prior	assumptions	$\sigma(\Omega_M)$ SNAP	$\sigma(w_0)$ SNAP	$\sigma(w_1)$ SNAP	$\sigma(\Omega_M)$ SNLS	$\sigma(w_0)$ SNLS	$\sigma(w_1)$ SNLS
5-fit	no	no	1.51	4.15	11.07	19.10	63.37	>100
4-fit	no	$\Omega_T = 1$	0.14	0.11	1.31	1.08	1.21	7.94
4-fit	weak	$\Omega_T = 1$	0.082	0.076	0.77	0.10	0.161	0.89
4-fit	strong	$\Omega_T = 1$	0.01	0.055	0.174	0.01	0.117	0.511
3-fit	no	$\Omega_T = 1$,$w_1 = 0$	0.016	0.063	/	0.069	0.19	/
3-fit	weak	$\Omega_T = 1$,$w_1 = 0$	0.016	0.063	/	0.057	0.158	/
3-fit	strong	$\Omega_T = 1$,$w_1 = 0$	0.008	0.040	/	0.01	0.047	/

a good estimation of w_0 but still a very large error for w_1 (see line 3 of Table 2). Only a strong prior on Ω_M ($\Omega_M = 0.3 \pm 0.01$) really improves the situation for w_1 as can be seen from line 4 of Table 2. These conclusions are in agreement with other published results [15, 16, 17, 18, 19, 20, 21].

One can conclude that the determination of a possible redshift dependence of the EoS remains the most difficult task. All authors agree on the following statement: to get a good precision on the parameter governing the redshift evolution of the EoS, a fit with a strong prior on Ω_M (”strong” means at the percent level) is needed within the $\Omega_T = 1$ paradigm.

2.3.2 The bias problem

Several comments on the previous approach are in order :

- The setting of strong priors inside the fit should be taken with caution.
  First of all, the cosmological parameter Ω_M is far from being measured at the percent level [31]. So, this kind of prior cannot be applied blindly, even if there is hope that this parameter might be well measured by the time SNAP provides data.

- Even in a future context, some doubts can be raised about the relevance of very precise priors [15, 16] and their use has to be set with some cautions : all the Ω's should be obtained under the same hypothesis (astrophysical, experimental, DE properties) and correlations among fitted parameters have to be taken properly into account for the combined statistical analysis to be relevant.

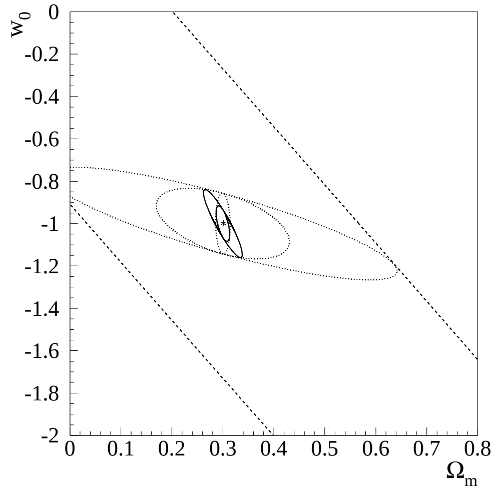

Figure 1: Fisher contours in the plane (Ω_M, w_0) at 68,3 % CL for a cosmological constant as the fiducial model. The large dashed contour corresponds to the 5-fit$(M_s, \Omega_M, \Omega_X, w_0, w_1)$. The three dotted contours correspond to the 4-fit$(M_s, \Omega_M, w_0, w_1)$ with no prior (largest), with a weak Ω_M prior (i.e. $\Omega_M = 0.3 \pm 0.1$) and with a strong Ω_M prior (i.e. $\Omega_M = 0.3 \pm 0.01$, smallest ellipse). The solid contours correspond to the 3-fit(M_s, Ω_M, w_0) with a weak Ω_M prior (largest) and with a strong Ω_M prior (smallest). For this fitting procedure, there is almost no difference between the cases "no" and "weak" prior for Ω_M.

In this paper, we attempt to extract the informations from SN data, therefore avoiding the (consistency) problems encountered when constraining Ω_M with external data.

The results from Table 2 and Fig. 1 lead us to leave aside the determination of the redshift dependence of the EoS and to consider w_0 as an "effective" constant w^{eff} EoS parameter. Then we concentrate on the best strategy to extract w_0 from present or future SN data with minimal assumptions on priors : we use the paradigm $\Omega_T = 1$ either with no priors or with weak priors on Ω_M. We focus on a 3-fit (M_s, Ω_M, w_0) or on a 4-fit $(M_s, \Omega_M, w_0, w_1)$ where we know already that the precision on w_1 will be low.

A 3-fit should always provide better constraints than a 4-fit. Table 2 shows, however, that the reduction of the errors is more important for Ω_M than for w_0 and since the w_0 error is not strongly improved, the use of the 4-fit seems the best strategy for extracting w_0 [21, 14]. We want to point out that this conclusion is only valid for the Λ fiducial model. In some other fiducial models, the 3-fit can increase considerably the constraints on w_0 with respect to the 4-fit: Figure 2 gives the expected contours in the plane (Ω_M,w_0) for the two fiducial models ($w_0^F = -2$,$w_1^F = 0$) and ($w_0^F = -1$,$w_1^F = 0.2$). The best constraints on Ω_M and w_0 are given by the 3-fit.

On the other hand, doing a complete minimization procedure shows that the central values given by the 3-fit can be far away from the fiducial values. This fact has been pointed out by several authors [15, 16, 20, 22]. This happens when the fitting hypothesis are not verified by the fiducial model, namely a constant EoS in the 3-fit and a varying one in the fiducial model. In this case *bias are introduced on the estimate of the cosmological parameters.* This bias problem is due to the degeneracy of the fitting function among

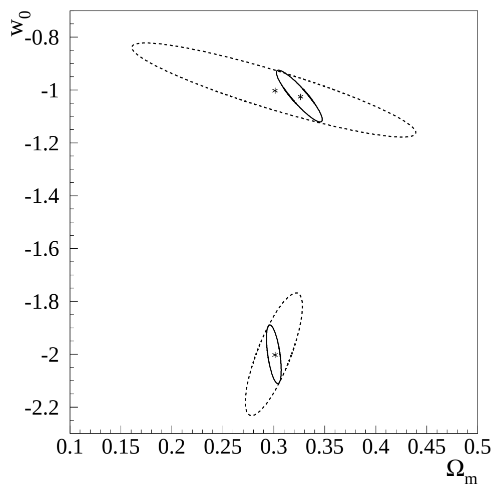

Figure 2: Contours in the plane (Ω_M, w_0) at 68,3 % CL for the fiducial models ($w_0^F = -2$, $w_1^F = 0$) and ($w_0^F = -1$, $w_1^F = 0.2$). The large dashed (small solid) contours correspond to the 4-fit (3-fit). A weak Ω_M prior has been used.

the various cosmological parameters [15, 16]. A wrong hypothesis on one parameter is compensated in a non trivial way by the value of the others. We know also from the various Fisher analysis presented in the litterature [18, 19, 21, 14] that the *errors* depend on the central values of the cosmological parameters. Since these last ones are biased, their errors are also biased [21, 14].
Then the question of choosing between the 3-fit and the 4-fit is not so obvious and appears to be fiducial model dependent.

In the next section we present in detail a study of this bias by exploring the range of fiducial models which is affected, in order to conclude on the validity of a 3-fit.
Let us point out that other bias would be introduced by a departure from the $\Omega_T = 1$ paradigm in the fiducial model or if the central value of the Ω_M prior used in the fitting procedure is not the same as the fiducial one. These bias will be studied in a future paper [27].

3 How to extract w_0 : a quantitative analysis of the validity of a 3-fit

To be able to choose the best strategy for extracting w_0, we want to test the relevance of the fitting procedure. We have performed for that purpose a complete study of the bias introduced by the different fitting procedures. After a description of the problem through a short illustration, we present the results of a full scan of the (w_0^F, w_1^F) plane for SNAP, SNLS and a sample corresponding to present data. The scanning is done by fixing the fitting procedure and varying the various parameters of the fiducial model ; we perform a 3-fit and compare it to the result of the 4-fit where w_1 is left free.

3.1 Illustration of the bias effect

Let us start with some illustrations of the bias introduced by the 3-fit procedure. We choose three different models and look at the central values of the parameters obtained from the fit:

• The one of Maor et al. [16] ($w_0^F = -0.7$, $w_1^F = 0.8$). The 3-fit applied to this model clearly provides some erroneous results : $\Omega_M = 0.62 \pm 0.013$ and $w_0 = -1.548 \pm 0.194$. Note that our central values are slightly different from the ones of [16], this is due to small differences between the chosen SN samples.

Figure 3 shows the two elliptical contours from the 3-fit and the 4-fit : the 4-fit ellipse is centered on the fiducial values as expected whereas the 3-fit ellipse is very far away. This is due to the large chosen w_1^F value and this behavior has been already pointed out by Maor et al. (see Fig. 5 of ref. [16]).

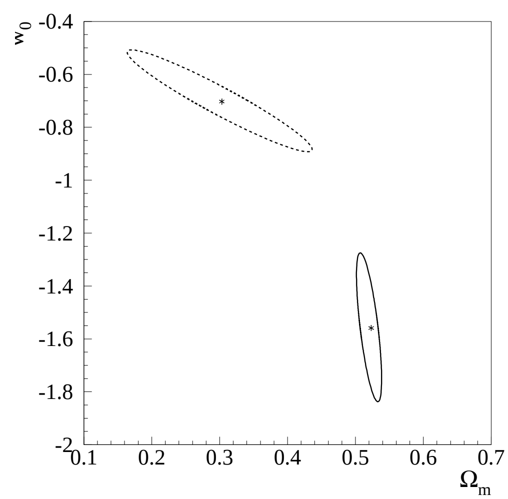

Figure 3: Contours in the plane (Ω_M, w_0) at 68,3 % CL for the fiducial models ($w_0^F = -0.7$, $w_1^F = 0.8$)(Maor et al. model[16]). The dashed (solid) contour corresponds to the 4-fit (3-fit) contour. A weak Ω_M prior has been used. We can observe the large differences in the central values coming from the two 3-fit and 4-fit fitting procedures.

• If we take the model ($w_0^F = -1$, $w_1^F = 0.2$), which has a "small" w_1^F value, it leads to a smaller bias : we get $\Omega_M = 0.324 \pm 0.016$ and $w_0 = -1.021 \pm 0.067$. We see that there is a bias of the order of 1.5σ for Ω_M, and a small bias within the statistical error for w_0. The central values (crosses) displayed in Figure 2 of the ellipses for the 3-fit and the 4-fit are different but the 3-fit ellipse is included in the 4-fit one, indicating a small bias on one parameter (w_0) and a not too big bias on the second (Ω_M).

• Finally, with the model ($w_0^F = -2$, $w_1^F = 0$) which has no z dependence of the EoS, there is no bias and both ellipses are centered on the same point (see Figure 2).

Note that there is no significant bias for the normalization parameter M_S in the three cases.

3.2 Validity zones for the 3-fit

To quantify the bias, we scan the full plane $\mathcal{P}$ (w_0^F, w_1^F) using the procedure 3-fit, assuming a flat universe. We first define different zones in this plane where the effects of the bias can be quantitatively estimated.

The plane $\mathcal{P}$ is divided in three distinct zones :

- The Non Converging Zone (NCZ) where the fit has some detectable problems: the fit is bad either because $\chi^2 > 3n$ (n being the number of degrees of freedom), or because the error on one of the fitted parameter is above 1. We also reject fits yielding some Ω_M values far away from current expectations, namely we require $0.1 - \sigma(\Omega_M) < \Omega_M < 1. + \sigma(\Omega_M)$.
  This zone is shaded in our plots. None of the fiducial models located in this NCZ can be fitted with the procedure 3-fit. Therefore, with real data, the fit procedure will be excluded.

- The Biased Zone (BZ) is the part of $\mathcal{P}$ where the fit converges perfectly, but where we know from the simulation that we are far from the fiducial values: We quantify this bias as the difference between the central value of a fitted parameter $\mathcal{O}$ and the fiducial value $\mathcal{O}^{\mathcal{F}}$. Namely the "bias" is defined as

  $$B(\mathcal{O}) \;=\; |\mathcal{O}^F - \mathcal{O}| \tag{8}$$

  In general, $B \neq 0$ and we define the biased zone as the part of $\mathcal{P}$ where $B(\mathcal{O}) > \sigma(\mathcal{O})$ where $\sigma(\mathcal{O})$ is the statistical error on $\mathcal{O}$.

  The important point is that this zone is undetectable with real data. It is the region we want to identify and to minimize through an appropriate choice of the fitting procedure.

- The Validity Zone (VZ) is the remaining part of $\mathcal{P}$ where the fit converges perfectly and where $B(\mathcal{O}) < \sigma(\mathcal{O})$. If we require this condition for the full set of fitted parameters together, we can define a "full" validity zone (full VZ).

  Consequently, the contours separating the BZ and the VZ in the plane $\mathcal{P}$ correspond to the intrinsic limitation of the fitting procedure which is considered.

We now apply this strategy of analysis to the SNAP and SNLS simulated data. The impact of the bias on the present SN statistical sample is also presented.

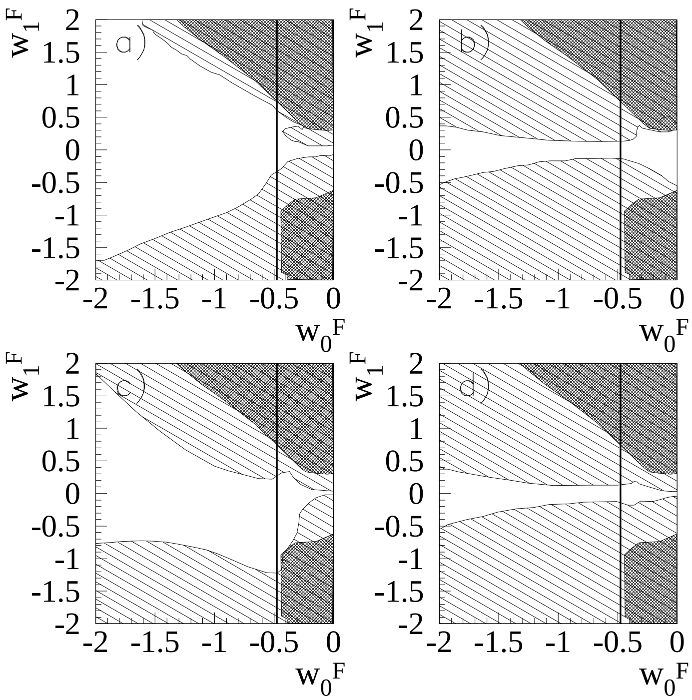

Figure 4: Validity Zones(white), Biased Zones (hatched) and Non Converging Zones (black) for a SNAP like data set for M_S only (a), Ω_M (b), w_0 (c) and all the parameters together (d). The procedure is a 3-fit with a weak Ω_M prior. The vertical line at $w_0^F = -0.48$ separates decelerating models from accelerating ones.

3.3 Fitting procedures for SNAP

3.3.1 Determination of the 3-fit validity zones

We apply this method to determine the validity zones using the 3-fit on the SNAP simulated data. A weak prior on Ω_M, $\Omega_M = 0.3 \pm 0.1$, has been used.
Figure 4a,b,c,d give the different validity zones for M_s (Figure 4a), Ω_M (Figure 4b), w_0 (Figure 4c) and for the three parameters taken together (Figure 4d).

In all the figures, the line $w_1^F = 0$ corresponds to the actual unbiased fiducial models. One can see that the Biased Zone is always limited by the NCZ. The line $w_0^F = -1/(3\Omega_X^F) \simeq -0.48$ separates the models which correspond to an accelerating universe today ($w_0^F < -0.48$) from the decelerating ones ($w_0^F > -0.48$).

Looking at the validity zone for each parameter individually :

- M_s (Figure 4a) is essentially unbiased for models which correspond to acceleration ($w_0^F < -0.48$) except for large negative values of w_1^F. We notice that M_s is always measured with an error better than the percent thanks to the "Nearby" sample. Therefore it is easy to get a bias greater than this error. However this bias has no real consequence on the determination of the other parameters.

- Ω_M (Figure 4b), conversely, is unbiased only for a small band around the line $w_1^F = 0$. It means that for most of DE models with varying EoS, the fitted Ω_M is strongly biased in this fitting procedure.

- w_0 (Figure 4c) is valid in a large part of the reduced plane $\mathcal{P}_{\mathcal{R}}$, which corresponds to the present acceleration region ($w_0^F < -0.48$). Otherwise we fall in the Biased Zone which, for instance, contains the previously quoted model of Maor et al.[16]. Outside $\mathcal{P}_{\mathcal{R}}$, for large $\mid w_1^F \mid$ values, we are in the Biased Zone. Consequently, one has to take some cautions when using the 3-fit for extracting w_0.

The full Validity Zone is shown in Fig. 4d ans is driven mainly by the bias on the Ω_M parameter. One can conclude that the 3-fit is valid on all parameters together for fiducial models which have a small dependence in redshift: when $w_0^F > -1$, the validity is for $|w_1^F| < 0.2$, otherwise it increases to $|w_1^F| < 0.4$.

One has to emphasize that the experimental error will also contain systematic errors. These latter have been estimated for SNAP [14]. We have simulated this case and the result is given on Fig. 5, where we have included a constant uncorrelated systematic error on each magnitude measurement. The consequence of increasing the error is an increase of the Validity Zones. Compared to Fig. 4c, the VZ for w_0 is substantially enlarged. Note that the NCZ is reduced in the same time. The VZ for M_s follows the same behaviour, the one of Ω_M being much less affected.

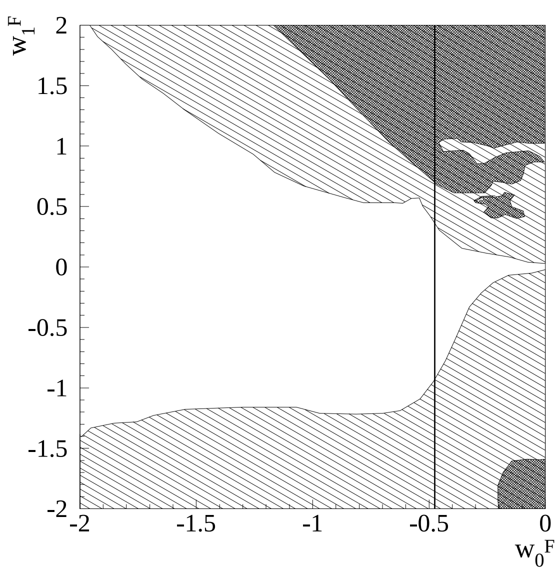

Figure 5: Same as Fig.4c for w_0 in the case where we add an uncorrelated and constant in redshift systematic error of 0.02 on the magnitudes.

We have also looked at the impact of the Ω_M prior on our results:

• When there is no prior, the results are very comparable (but slightly worse) to the weak prior case described above.

• When a strong prior is used (e.g., $\sigma(\Omega_M) = 0.01$), the error on the parameters are reduced but the VZ also. Fig. 6a gives the VZ for w_0 with this prior. Due to the strong prior on Ω_M, all the bias effect is reported on the w_0 parameter which is strongly biased as shown on Figure 6a. Looking more closely at the reduced $\mathcal{P}_{\mathcal{R}}$, one sees that the VZ is limited to the line $w_1^F = 0$. Therefore we lose the fact that w_0 is in general well fitted by this fitting procedure.

The conclusion is that the fitting procedure 3-fit used in the case of a high SN statistics experiment like SNAP, can be useful to constrain w_0 for a large part of the plane $\mathcal{P}$, which corresponds to accelerating models, only when no prior or a weak prior on Ω_M is used. The other cosmological parameters (Ω_M especially) are strongly biased, and strengthening the Ω_M prior worsens the situation. Concerning DE models which lead to a present deceleration, it appears clearly from the preceding figures, that the cosmological parameters are biased even if the time dependence of the EoS is weak.

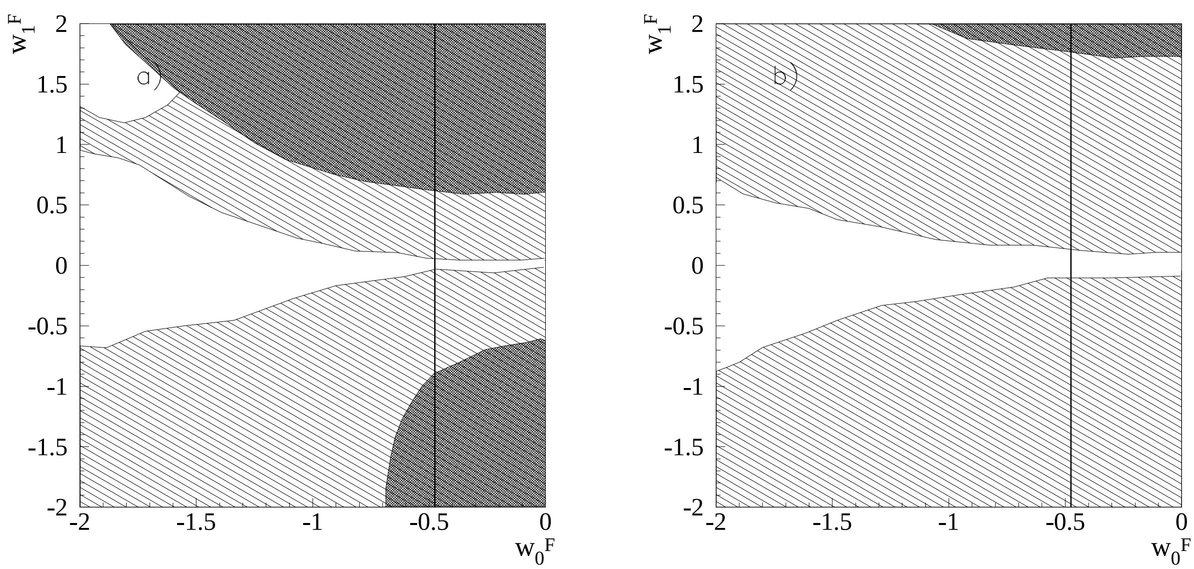

Figure 6: Same as Fig.4c for w_0 in the case of a strong Ω_M prior a) : SNAP, b) : SNLS.

3.3.2 Comparison between the 3-fit and the 4-fit

We have shown previously that the 3-fit procedure may be useful to extract w_0 even in the presence of the bias due to the hypothesis $w_1 = 0$. The simplest 4-fit procedure (with no constraint on w_1) allows the extraction of w_0 without any bias since w_1 is included in the fit, but with an increase of the statistical errors on each parameter. We adress now a comparison of the errors obtained on w_0 with these two different fitting procedures.

To combine the bias error and the statistical error, we choose a fit quality estimator of the procedure as $E(w_0) = \sqrt{\sigma^2(w_0) + B^2(w_0)}$ which reduces to $\sigma(w_0)$ for the 4-fit. Figure 7 compares the two procedures by displaying the difference of errors $E(w_0, 3-\text{fit}) - \sigma(w_0, 4-\text{fit})$ with a weak prior on Ω_M. The 3-fit is preferred, with this estimator, in a large part of the plane $\mathcal{P}$ which corresponds to models with present acceleration.

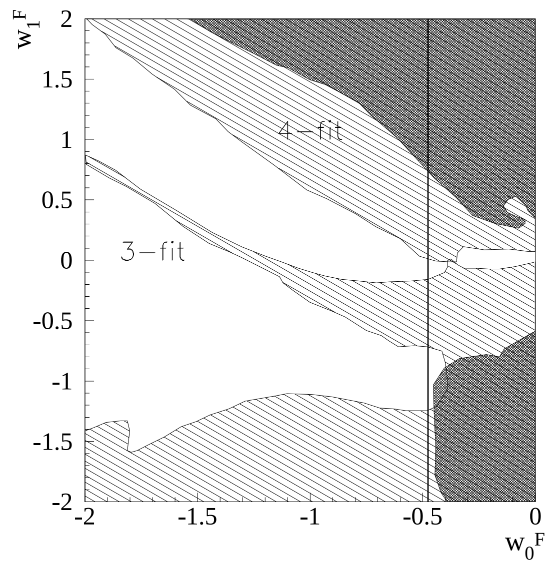

Figure 7: $E(w_0, 3-\text{fit}) - \sigma(w_0, 4-\text{fit})$ for the SNAP data and a weak Ω_M prior. The white part corresponds to a positive difference, the hatched part to a negative difference, the black region corresponds to the Non Converging zones.

The two black areas correspond to the NCZ of both fitting procedures. Therefore, one cannot conclude for a preference for the 4-fit in this zone. The two hatched zones, corresponding to large values of $|w_1^F|$, indicate a preference for the 4-fit. This is due to the bias on w_0 inherent to the 3-fit. There is also a narrow hatched band where the 4-fit is still preferred although the bias is small. This appears to be a zone where the correlation between w_0 and w_1 is small and then, the errors from the 4-fit and the 3-fit are similar, the 4-fit being slightly better (less than 2%).

In the white regions, the two parameters w_0 and w_1 are strongly correlated and then the 3-fit has clearly a smaller error.

So, we can conclude that with a weak, thus conservative, prior on Ω_M the 3-fit is in general better than the 4-fit to extract w_0 for accelerating models.

3.4 Analysis for SNLS

3.4.1 Determination of the 3-fit validity zones

A similar analysis has been performed for a simulated sample corresponding to the SNLS-like statistics.

The various Validity Zones obtained for SNLS with a weak Ω_M prior are shown in Figures 8a,b,c,d

We deduce from this figure that :

- M_S is almost unbiased for accelerating models and also for decelerating ones provided the redshift dependence is small ($|w_1^F| < 0.6$) (Fig. 8a).

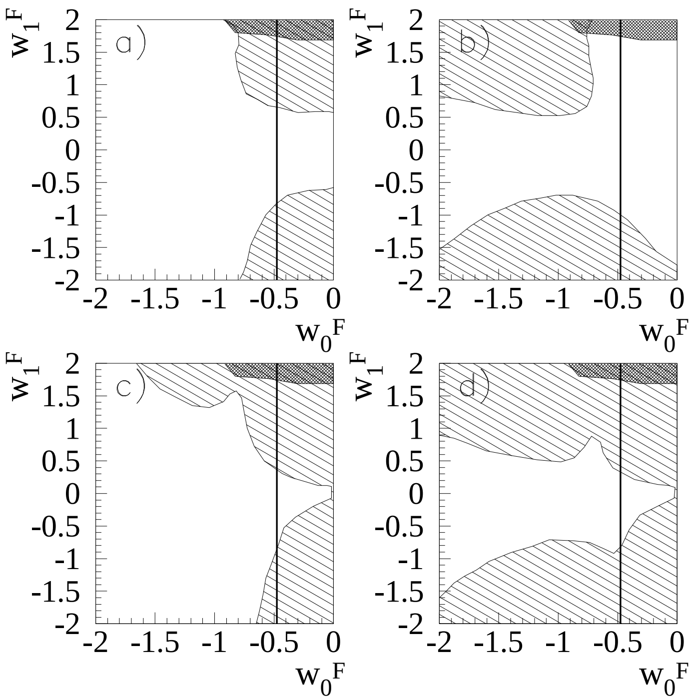

Figure 8: Validity Zones (white), Biased Zones (hatched) and Non Converging Zones (black) for a SNLS like data set for M_S only (a), Ω_M (b) and w_0 (c) and all the parameters together (d). The procedure is a 3-fit with a weak Ω_M prior.

- Ω_M is not biased in most of the reduded plane $\mathcal{P}_{\mathcal{R}}$ and it is biased outside this plane if $|w_1^F| > 0.6$ (Fig. 8b).
- w_0 is not biased for an important part of the plane corresponding to acceleration ($w_0 < -0.48$) (Fig. 8c).
- The 3-fit is a good fitting procedure for reconstructing *all* cosmological parameters simultaneously for accelerating models ($w_0 < -0.48$) when the redshift dependence is small $|w_1^F| < 0.6$ (see Fig. 8d). Otherwise the result is biased.
- The use of a strong prior on Ω_M worsens the situation : w_0 is biased for most of the plane $\mathcal{P}$ as shown on Fig. 6b.

Comparing these results with the SNAP case shows that the conclusions are quite similar but the validity zones for SNLS are much larger than for SNAP. Due to the lower statistics our criterion $B(\mathcal{O}) < \sigma(\mathcal{O})$ is less constraining. Note that the NCZ zones are strongly reduced as expected with the lower statistics.

3.4.2 Comparison between the 3-fit and the 4-fit

We perform the same comparison of the errors on w_0 obtained from the two procedures 3-fit and 4-fit, still with a weak prior on Ω_M.

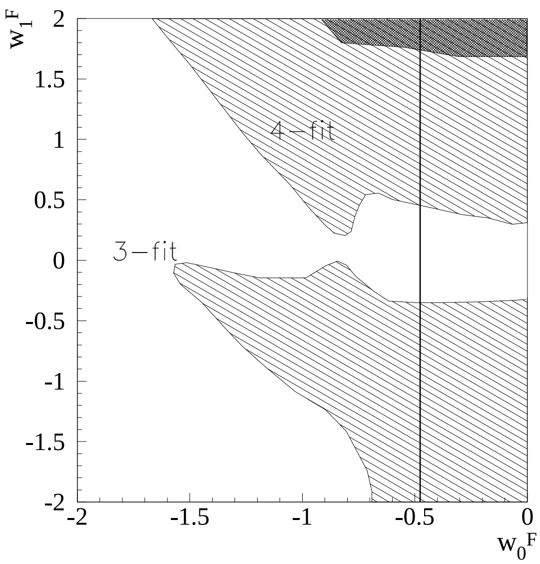

Figure 9: $E(w_0, 3-\text{fit}) - \sigma(w_0, 4-\text{fit})$ on w_0 for a SNLS like data set and a weak Ω_M prior. The white part corresponds to a positive difference, the dashed part to a negative difference, the black region corresponds to the Non Converging Zones.

Figure 9 shows the difference of errors between the 3-fit and the 4-fit. The 3-fit is preferred when the bias is small or when the two parameters w_0 and w_1 are strongly correlated. When $w_1^F < 0$ the hatched region in the accelerating zone has similar origin and properties as the narrow hatched zone of Fig. 7, namely that both fitting procedures provide in fact very comparable errors.

3.5 Analysis for a statistical sample corresponding to the SCP data

We can define the validity zones for the statistics of the present SN sample from the SCP collaboration [4] introduced in Section 2.1.

Figure 10a gives the full Validity Zone obtained without any prior on Ω_M. The large size of this zone shows that the results are not biased (at 1σ) for most of the plane $\mathcal{P}$. In fact only Ω_M is affected by a bias.

A weak Ω_M prior of $\sigma(\Omega_M) = 0.1$ has been used to draw Fig. 10b, giving the Validity Zone for w_0 (identical to the full VZ). The VZ is slightly reduced compared to the no prior case but it remains quite important within the plane $\mathcal{P}$. More precisely, it appears that the upper part of the biased zones are comparable for both no and weak prior cases. Only the lower part of the biased zone of Fig. 10b is specific to the weak prior case. Note that changing the error value of the prior has little consequences on the shape of the VZ.

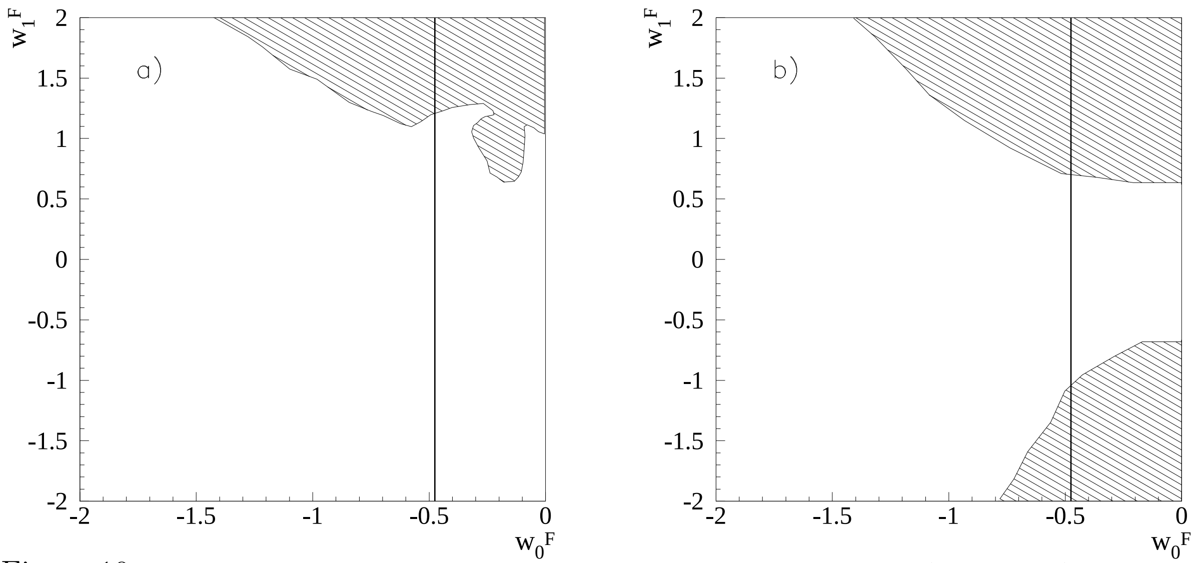

Figure 10: Full Validity Zones and Biased Zones for a SCP like data set with a) no prior b) a weak Ω_M prior. The NCZ are not plotted due to the large errors on Ω_M and w_0.

This fitting procedure is widely used to extract informations from present SN data [1, 2, 3, 4, 32]. The results published in these references are valid for models with constant EoS. From our findings, one can extend the validity of these results and conclude that models with a redshift dependent EoS which verify $\mid w_1^F \mid < 0.7$ may be fitted by this 3-fit procedure with a bias which stays below the statistical error.
So, we conclude that we can trust (at 1σ) the results based on present SN data obtained with a 3-fit where we neglect the redshift dependence of the EoS, using a Ω_M prior or not. Only (fiducial) models with a strong recent variations of the EoS may lead to biased results.

The comparison of the potentialities of the 3-fit and the 4-fit to extract w_0 does not seem very relevant. However, we have checked this point, and it appears that the 3-fit provides some constraints on w_0 which are roughly 30% better than with the 4-fit for most of $\mathcal{P}$.

Finally, we have remarked that the central values of w_0 and Ω_M are shifted as follows: if w_1^F is positive we have $w_0 > w_0^F$ and $\Omega_M > \Omega_M^F$, and the converse if $w_1^F < 0$. We do not recover this simple behaviour with the SNAP and SNLS samples.

4 Summary and conclusions

In this paper, we have studied the best strategy to extract the cosmological parameters and in particular the dark energy component from the future supernovae data within the $\Omega_T = 1$ paradigm. We have compared results from the present statistics with the ones we can expect from future large ground-based surveys and from a future space experiment like SNAP.
In order to be as model independent as possible we have looked at a large range of fiducial

models by varying the parameters of the equation of state of the dark energy w_0^F and w_1^F where $w^F(z) = w_0^F + w_1^F z$ [17].

With present statistics or the one expected from a survey like SNLS, precision on w_1 is very weak and the physical question concerns the extraction of w_0 and Ω_M. For a SNAP like experiment a good precision on w_1 is only possible with a strong prior on Ω_M.

To avoid the use of strong priors on Ω_M, we have focused on the extraction of w_0 only and neglected the w_1 contribution by fixing it in the fitting procedure. We have to face in this case the problem of introducing a bias if the fiducial value of w_1^F is far away from the fixed one. We compare the bias to the statistical error which is expected for each survey.

To study the validity of such a strategy, we have presented an extensive study of fiducial models in a flat universe in which we vary w_0^F and w_1^F in a complete range $-2 < w_0^F < 0$ and $-2 < w_1^F < 2$ corresponding to a large variety of DE theoretical models. On the other hand M_s^F and Ω_M^F are fixed. In the fit we have used in general a weak prior on $\Omega_M(\sigma(\Omega_M) < 0.1)$ and we have looked at the bias introduced by fixing $w_1 = 0$. We call this procedure the 3-fit (M_s, Ω_M, w_0) procedure.

For SNAP (and SNLS) like statistics, we obtain the following conclusions :

- w_0 is not biased for an important part of the scanned plane (w_0^F, w_1^F) which corresponds to models describing a Universe which is today in acceleration.
- for M_S, we get the same conclusion but we emphasize that this parameter is so much constrained in the fitting procedure that a small bias has no visible effect.
- Ω_M is strongly biased if the redshift dependence is large ($|w_1^F| > 0.2(0.6)$).
- If a strong prior on Ω_M is used, w_0 is biased for most of the plane. Namely, the 3-fit procedure is relevant to extract w_0 if only a weak Ω_M prior is used.

We have compared the accuracy of the results on the w_0 parameter to the ones obtained if w_1 is left free as in a 4-fit $(M_s, \Omega_M, w_0, w_1)$ procedure.

We found that, in spite of the bias, the 3-fit procedure, where w_1 is fixed, can be used for SNAP and SNLS to test accelerating models and give better or equivalent results than the simplest 4-fit where w_1 is completely free. This is no longer true if $| w_1^F |$ is large. For decelerating models the 4-fit is mandatory. Note that, concerning the choice of the number of parameters to be fitted, the bayesian method presented in [33] seems promising.

These conclusions have been set using only the value $\Omega_M^F = 0.3$ for the fiducial models. If the true value is different the areas of the different Validity Zones increase if the contribution of DE is smaller, namely if Ω_M^F is larger. Conversely, if $\Omega_M^F < 0.3$, the validity of the 3-fit is reduced.

Using the presently available statistics of the SCP collaboration [4], it appears that the procedure where w_1 is fixed can be trusted for almost all the fiducial models considered here. Only fiducial models with strong recent variations of the EoS ($| w_1^F |> 0.7$) may lead to biased results.

Let us point out that other bias would be introduced, by a departure from the $\Omega_T = 1$ paradigm in the fiducial model, or if the central value of the Ω_M prior itself (used in the fitting procedure) is not the same as the fiducial value. Indeed a small bias in the prior could induce some strong bias for the other cosmological parameters. These bias will be studied in a future paper [27].

It has been advocated recently [34] that it should be easier to constrain the DE density $\rho_X(z)$ and its time derivative $\rho'_X(z)$, instead of constraining the equation of state $w(z)$.

These authors have simulated a large variety of fiducial models in the range $-1.2 \leq w_0^F \leq -0.5$ and $-1.5 \leq w_1^F \leq 0.5$, which is almost completely contained in our Validity Zone for SNAP (see Fig. 5). They interpret physically parts of this plane in term of increasing, decreasing or non-monotonic DE densities $\rho_X(z)$.

Their approach is interesting to answer if the DE energy density is a constant or not. Nevertheless, focusing on the equation of state itself remains mandatory if, for instance, one wants to validate precisely the acceleration since, in this case, a precise knowledge of w_0 (and also on Ω_M and Ω_X) is necessary. Finally, we would like to point out that trying to answer any particular question on the nature of DE requires an investigation on the choice of the most relevant fitting procedures.

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