Low energy $n-3H$ scattering: A novel testground for nuclear interactions

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(Dated: December 22, 2004)

The low energy $n - {}^3\mathrm{H}$ elastic cross sections near the resonance peak are calculated by solving the 4-nucleon problem with realistic NN interactions. Three different methods – Alt, Grassberger and Shandas (AGS), Hyperspherical Harmonics and Faddeev-Yakubovsky – have been used and their respective results are compared. We conclude on a failure of the existing NN forces to reproduce the $n - {}^3\mathrm{H}$ total cross section.

PACS numbers: 21.45.+v, 11.80.Jy, 25.40.-h, 25.10.+s

I. INTRODUCTION

The four nucleon (4N) system represents a qualitative jump in complexity relative to the $A = 3$ case, as has been put forward in several papers [1, 2, 3, 4, 5, 6, 7, 8, 9]. It becomes already obvious when comparing the experimental $n-p$, $n-d$ and $n-{}^3\mathrm{H}$ cross sections displayed in Fig. (1); the smooth behaviour of $A = 2$ and $A = 3$ curves contrasts with the non trivial structure manifested in $A = 4$. This structure, in the energy region around E_{cm} ≈3 MeV, is commonly associated with negative parity, isospin $T = 1$ resonance states [10]. Actually, $A = 4$ is the smallest nuclear system which exhibit characteristic nuclear properties – such as saturation in the binding energy – and in which the simplest nuclear ($n + {}^3\mathrm{He} \to \mathrm{p} + {}^3\mathrm{H}$) and fusion ($d + d \to n + {}^3\mathrm{He}$) reactions take place. Moreover, many reactions involving four nucleons, like $p + {}^3\mathrm{He} \to {}^4\mathrm{He} + \nu_\mathrm{e} + \mathrm{e}^+$ (the *hep* process), are of extreme astrophysical interest, as they play important roles in solar models and big-bang nucleosynthesis; the *hep* process, for instance, is the source of the highest energy neutrinos from the Sun.

The study of the 4N system is particularly interesting as a "theoretical laboratory" to test new models of the nuclear force. Unlike the $A = 3$ systems, $A = 4$ shows a delicate and rich structure of excited states in the continuum [10] whose position and width depends critically on the underlying nucleon-nucleon (NN) interaction [9]. In fact, the effect of (i) the NN P-wave and of (ii) the three-nucleon (3N) force are believed to be larger than in the $A = 2$ or 3 systems. Moreover, it is the simplest system where the 3N interaction in the channels of total isospin $T = 3/2$ can be studied. Therefore it is of the utmost importance to have reliable few-body techniques powerful enough to deal with this system. Ever increasing computer power, development of novel numerical methods, and significant refinements of well-established techniques have come together to show that the solution of the four-nucleon bound state problem with realistic Hamiltonians is reliable.

In Ref. [11] the binding energies and other properties of the α-particle were studied using the AV8′ [12] NN interaction; several different techniques [13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25] produce results in very close agreement with each other (at the level of less than 1%). More recently the Faddeev–Yakubovsky (FY) equations [25, 26] were solved for different realistic NN + 3N force models. These calculations are fully converged in terms of NN partial waves and the results agree with identical works using the Green's function Monte Carlo (GFMC) [21] or Hypherspherical Harmonics (HH) methods [18].

In this paper, we consider the problem of solving the $4N$ scattering problem with realistic Hamiltonians. As discussed before, the $A = 4$ system is very rich and several elastic and rearrangement channels are indeed possible already at low energy, depending on the total charge ($n - {}^3\mathrm{H}$, $p - {}^3\mathrm{H} \leftrightarrow \mathrm{n} - {}^3\mathrm{He} \leftrightarrow \mathrm{d} - \mathrm{d}$, $p - {}^3\mathrm{He}$). Among them, the $n - {}^3\mathrm{H}$ is the simplest one: free from Coulomb interaction and, to a very good approximation, a pure isospin $T = 1$ state. However, it presents a rich dynamics since a resonance structure is visible at center of mass energy $E \approx 3$ MeV, as already shown in Fig. (1).

Up to now only two of the techniques used in bound state calculations (FY and HH) as well as methods based on

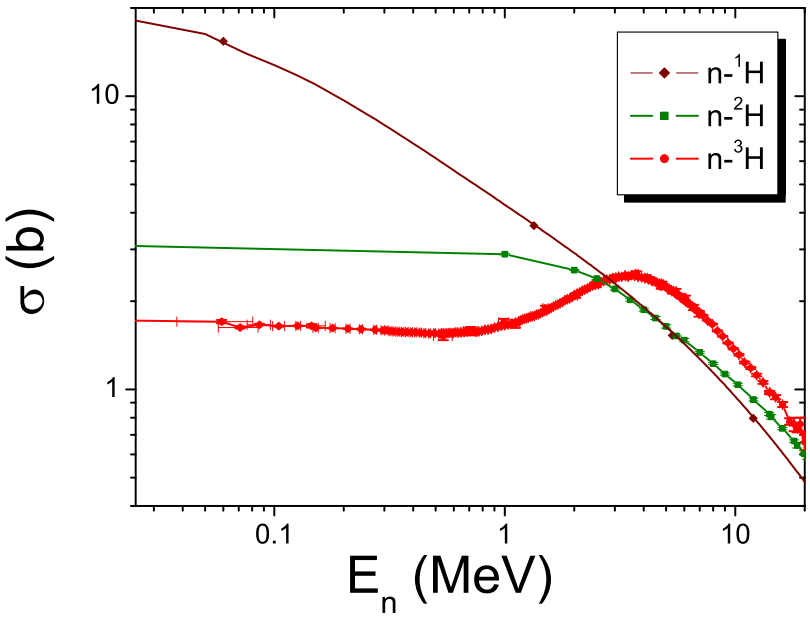

FIG. 1: Comparison between n-p, n-d and $n-{}^3$H cross sections for neutron energies in laboratory frame.

the resonating group model (RGM) [27] and on the solution of Alt, Grassberger and Sandhas (AGS) equations [6, 28] have been employed to attack this problem with realistic Hamiltonians (for a review of earlier results, see Ref. [10]). Some previous FY calculations in configuration space suggested that realistic NN interactions could fail in reproducing the experimental total and differential cross sections [1, 4, 6]. These calculations were however limited to a relatively small number of partial waves in the expansion of FY amplitudes, thereby such lack of convergence could be advocated to explain the failure. Using AGS equations, the author of Ref. [6] was able to substantially increase the number of partial wave amplitudes and obtain a fairly good agreement for the total cross section at the resonance peak. However, these calculations were based on a rank-one separable expansion of the 2-body t-matrix, which also raises some doubts on the reliability of the results. The calculations performed by expanding the scattering wave functions on the correlated Hyperspherical Harmonic (CHH) basis were able to obtain a reasonable estimate of the various phase-shift and mixing-angle parameters [5, 8]. However, the problem of convergence could not be solved completely due to numerical difficulties when adding more CHH components. For this reason, a new expansion on the (uncorrelated) HH basis is considered here.

The results obtained using these techniques were so far at rather large variance between each other. Clearly, this situation should be clarified before questioning the ability of present NN + 3N force models to describe the experimental data beyond the binding energy of ^{4}He, for which they have proven to be rather successful [21]. This is the purpose of the present paper in which we compare low energy $n-{}^3$H scattering results obtained by three different groups, using independent methods to solve the four-body equations. They concern AGS calculations [6] based on rank-one expansion of the NN t-matrix, variational solutions of the Schrödinger equation using HH method [18] and the solutions of FY equations [29, 30] with a substantial improvement in the number of partial waves [31]; AV18 [32] and NIJM-II [33] NN interaction models are used in present calculations.

We have considered the center of mass energies $E = 0.40, 0.75, 1.50, 2.625, 3.0$ MeV. They constitute a good set of data for interpolating other values of the phase shifts and allow to determine the low energy parameters (scattering length and effective range). In addition, values $E = 0.75, 1.50, 2.625$ MeV correspond to the measured differential cross sections.

To describe scattering solutions, we will expand the different J^π $n-{}^3$H states in terms of the asymptotic hamiltonian channels $|L, S; J^\pi>$, where L denotes the $n-{}^3$H relative orbital angular momentum and $\vec{S} = \vec{s}_n + \vec{s}_t$ the total spin. The following J^π states will be considered:

$$\begin{aligned}
|0^+\rangle &= \quad |0,0;0^+\rangle \\
|1^+\rangle &= c_{1^+} \; |0,1;1^+\rangle + d_{1^+}|2,1;1^+\rangle \\
|0^-\rangle &= \quad |1,0;0^-\rangle \\
|1^-\rangle &= c_{1^-} \; |1,1;1^-\rangle + d_{1^-}|1,0;1^-\rangle
\end{aligned}$$

$$|2^-\rangle = \ c_{2^-} \ \ |1,1;2^-\rangle + d_{2^-}|3,1;2^-\rangle| \tag{1}$$

Coefficients c and d are fixed by the dynamics and make the solutions eigenstates of the S-matrix.

As it will be demonstrated later, these states are the only relevant ones for the low energy scattering considered here. Moreover, the coupling $L \leftrightarrow L+2$ is very weak and is neglected for $J^\pi = 2^-$ state.

II. METHODS

We briefly describe the methods used for solving the 4N problem. Two of them (FY and HH) work in configuration space while AGS in momentum space.

A. AGS equations

The starting point involves the AGS equations [28] for the transition operators involving all (2N)+(2N) and N+(3N) channels. For local NN potentials such equations are three-vector variable integral equations which after partial wave decomposition reduce to a set of coupled equations in three continuous scalar variables. Since scattering calculations require a great number of channels for convergence, we follow an approach based on the separable representation of subsystem amplitudes in order to reduce the equations to two or one continuous variable. The integral equations we use are the same as in Ref. [34] and result from the modified AGS equations [35] after one has: (a) represented the original NN t-matrix by an operator of rank one; (b) represented the resulting 3N t-matrix by a finite rank operator and taken as many terms as needed for convergence. Since in the modified AGS equations the 2N+2N subamplitudes are expressed in terms of a convolution integral involving two non-interacting pair-propagators, as first proposed in Ref. [36], the sole approximation in this approach involves a rank one representation of the 2N t-matrix which may be obtained from the well-known method of Ernst, Shakin, and Thaler (EST) [37]. The multi-term representation of the 3N t-matrix is done using the Energy Dependent Pole Expansion (EDPE) method developed in Ref. [38]. This latter representation for the 3N t-matrix is well under control since one may check the convergence rate of 4N observables for increasing rank in the 3N t-matrix.

This method was first used in Ref. [34] to calculate the binding energy of ^{4}He and later confirmed to be accurate by the exact work of Kamada and Glöckle [13]. More recently [4] the results of our calculations for $n-{}^3$H elastic scattering were shown to agree with the exact results of the Grenoble group [3], for both Malfliet-Tjon (MT) and AV14 potentials taken in 2N partial waves with total angular momentum j $\leq 1^+$ (^{1}S$_0$,^{3}S$_1 - ^3$D$_1$). In the present calculations we extend the work in Ref. [6] by increasing the number of 2N partial waves to $j \leq 2$. Therefore we add ^{3}F$_2$, ^{1}D$_2$ and ^{3}D$_2$ to the partial waves ^{1}S$_0$,^{3}S$_1 - ^3$D$_1$, ^{1}P$_1$, ^{3}P$_0$, ^{3}P$_1$ and ^{3}P$_2$ already included in Ref. [6]. Moreover we limit the particle-pair orbital angular momentum $\ell_y \leq 2$ and include all J_3 and $\ell_z \leq 2$ for a given total four-body angular momentum J; J_3 is total angular momentum of any given three-body subsystem and ℓ_z is the orbital angular momentum of that subsystem relative to the fourth particle. The results we find here are consistent with those obtained in Ref. [6] and converged relative to the number of terms in the finite rank expansion of the underlying 3N t-matrix. The number of terms in the expansion range from four to six depending on J_3.

B. Faddeev-Yakubovsky method

In the case of four identical fermions, interacting via a pair-wise potential V, the FY equations result into a set of two integrodifferential equations, coupling two FY components, namely K and H:

$$(E - H_0 - V)\, K \ = \ V(P^+ + P^-)\,[(1+Q)K + H] \tag{2}$$

$$(E - H_0 - V)\, H \ = \ V\tilde{P}\,[(1+Q)K + H] \tag{3}$$

with P^+, P^-, $\tilde{P}$ and Q being particle permutation operators:

$$P^+ = (P^-)^- = P_{23}P_{12};\ Q = \varepsilon P_{34};\ \tilde{P} = P_{13}P_{24} = P_{24}P_{13}, \tag{4}$$

and ε is a Pauli factor for exchange of two identical particles, which in case of fermions is -1. The wavefunction is given by:

$$\Psi = \left[1 + (1 + P^+ + P^-)Q\right](1 + P^+ + P^-)K + (1 + P^+ + P^-)(1 + \tilde{P})H \tag{5}$$

Each FY component F=(K,H) is considered as a function, described in its proper set of Jacobi coordinates $\vec{x},\vec{y},\vec{z}$, defined respectively by

$$\begin{array}{rclcrcl} \vec{x}_K &=& \vec{r}_2-\vec{r}_1 & & \vec{x}_H &=& \vec{r}_2-\vec{r}_1 \\ \vec{y}_K &=& \sqrt{\frac{4}{3}}\left(\vec{r}_3-\frac{\vec{r}_1+\vec{r}_2}{2}\right) & & \vec{y}_H &=& \vec{r}_4-\vec{r}_3 \\ \vec{z}_K &=& \sqrt{\frac{3}{2}}\left(\vec{r}_4-\frac{\vec{r}_1+\vec{r}_2+\vec{r}_3}{3}\right) & & \vec{z}_H &=& \sqrt{2}\left(\frac{\vec{r}_3+\vec{r}_4}{2}-\frac{\vec{r}_1+\vec{r}_2}{2}\right) \end{array} \tag{6}$$

and expanded in angular variables for each coordinate according to

$$\langle \vec{x}\vec{y}\vec{z}|F\rangle = \sum_\alpha \frac{F_\alpha(xyz)}{xyz}\, Y_\alpha(\hat{x},\hat{y},\hat{z}). \tag{7}$$

The quantities F_α are called regularized FY amplitudes and Y_α are tripolar harmonics, containing spin, isospin and angular momentum variables. The label α holds for the set of 10 intermediate quantum numbers describing a $J^\pi, T=1$ state . They are defined in a j-j coupling scheme as

$$\begin{aligned} K &\equiv \left\{[(t_1t_2)_{\tau_x}t_3]_{T_3}\, t_4\right\}_{T=1} \otimes \left\{\left[(l_x(s_1s_2)_{\sigma_x})_{j_x}\,(l_y s_3)_{j_y}\right]_{J_3}(l_z s_4)_{j_z}\right\}_J \\ H &\equiv \left[(t_1t_2)_{\tau_x}(t_3t_4)_{\tau_y}\right]_{T=1} \otimes \left\{\left[(l_x(s_1s_2)_{\sigma_x})_{j_x}\left(l_y(s_3s_4)_{\sigma_y}\right)_{j_y}\right]_{j_{xy}} l_z\right\}_J \end{aligned}$$

where s_i and t_i are the spin and the isospin of the individual particles and T, J the isospin and total angular momentum of the four-body system. Each of the $N_c = N_K + N_H$ amplitudes in the expansion (7) is labelled by 12 quantum numbers, which are further conditioned by the antisymmetry properties $(-)^{\sigma_x+\tau_x+l_x} = \varepsilon$ for K and $(-)^{\sigma_x+\tau_x+l_x} = (-)^{\sigma_y+\tau_y+l_y} = \varepsilon$ for H.

The boundary conditions for the 1+3 scattering problem are implemented by imposing at large enough value of z the Dirichlet-type condition

$$\begin{aligned} K(x,y,z) &= t(x,y) \\ H(x,y,z) &= 0 \end{aligned}$$

$t(x,y)$ being the triton Faddeev component with quantum numbers $\left[(l_x(s_1s_2)_{\sigma_x})_{j_x}\,(l_y s_3)_{j_y}\right]_{J_3}$. They ensure a solution which, e.g. for a relative $n-{}^3$H S-wave, behaves asymptotically like

$$K(x,y,z) \;\sim\; t(x,y)\sin(qz+\delta)$$

where δ is the $n-{}^3$H phase shift and q, the conjugate momentum of the z-Jacobi coordinate in K-amplitudes, is related to the center of mass $n-{}^3$H kinetic energy $E_{\rm cm}$ and the physical momentum k by

$$E_{\rm cm} = \frac{3}{4}E_{\rm lab} = \frac{\hbar^2}{m}q^2 = \frac{2}{3}\frac{\hbar^2}{m}k^2\ . \tag{8}$$

The convergence is reached in respect to expansion (7).

FY calculations have been performed in the $j-j$ coupling scheme. The following truncations in the partial wave expansion of amplitudes were used: *(i)* V_{NN} waves limited to $l_x \le 3$, always including tensor-coupled partners, i.e. involving the set ${}^1S_0$,${}^3SD_1$, ${}^1P_1$,${}^3P_0$,${}^3PF_2$,${}^3P_1$, ${}^1D_2$,${}^3DG_3$,${}^3D_2$, ${}^1F_3$,${}^3FH_4$,${}^3F_3$ and *(ii)* $l_x+l_y+l_z \le 10$.

The convergence was studied as a function of j_{yz}=max(j_y,j_z) for K-like components and j_{yz}=max(j_y,l_z) for H-like, starting with $j_{yz}=1$. In the second column of the Table I we present the values of the relevant phase shifts for AV18 model at the peak energy E_{cm}=3.0 MeV. The couplings $L \leftrightarrow L+2$ in the asymptote of the 1^+ ($L=0 \leftrightarrow L=2$) and 2^- ($L=1 \leftrightarrow L=3$) states are found to be very small (see next section) and have been omitted here. For the 1^- state, we give the eigenphaseshifts - $\delta_1(1^-), \delta_2(1^-)$ – and the mixing parameter ϵ. One can remark that results displayed on Table I converge pretty well as a function of j_{yz}. For the most of the phaseshifts three-digit accuracy is reached already with $j_{yz}=2$. The most unstable were found $J^\pi = 2^-$ phaseshift and $J^\pi = 1^-$ mixing parameter, which were requiring inclusion of $j_{yz}=3$ amplitudes to converge at a one percent level.

C. Hyperspherical harmonic method

In the HH method, the wave function is written as the sum of an asymptotic part, which form is known (except for the parameter of the S-matrix), and an internal part, which is expanded in HH functions. Explicitly, the wave function $\Psi_{1+3}^{LSJJ_z}$ describing a $n-{}^3$H scattering state with incoming orbital angular momentum L and channel spin S ($S=0,1$) coupled to total angular JJ_z, is expressed as

$$\Psi_{1+3}^{LSJJ_z} = \Psi_C^{JJ_z\pi} + \Psi_A^{LSJJ_z} \ , \qquad \pi = (-)^L \ , \tag{9}$$

where $\Psi_C^{JJ_x\pi}$ vanishes in the limit of large intercluster separations, and hence describes the system in the region where the particles are close to each other and their mutual interactions are strong. On the other hand, $\Psi_A^{LSJJ_z}$ describes the relative motion of the two clusters in the asymptotic regions, where the $n-{}^3$H interaction is negligible and can be decomposed as a linear combination of the following functions

$$\Omega^{\pm}_{LSJJ_z} = \frac{1}{\sqrt{4}} \sum_{i=1}^{4} \Big[[s_i \otimes \phi_3(jlp)]_S \otimes Y_L(\hat{\mathbf{r}}_i) \Big]_{JJ_z} \Big(-f_R(r_i) y_L(kr_i) \pm \mathrm{i} j_L(kr_i) \Big) \ , \tag{10}$$

where r_i is the distance between the neutron (particle i) and ^{3}H (particles jlp), k is the magnitude of the relative momentum between the two clusters defined in Eq. (8), ϕ_3 is the ^{3}H wave function, which has been obtained by solving the corresponding three-body problem using the pair-correlated HH expansion [39] and j_L and y_L are the spherical Bessel functions of the first and second kind, respectively. The function $f_R(r_i)$ has been introduced to regularize $y_L(kr_i)$ at small r_i, and $f_R(r_i) \to 1$ as r_i is large, thus not affecting the asymptotic behavior of $\Psi_{1+3}^{LSJJ_z}$. Note that for large values of kr_i,

$$-f_R(r_i) y_L(kr_i) \pm \mathrm{i} j_L(kr_i) \to \frac{\cos(kr_i - L\pi/2) \pm \mathrm{i}\sin(kr_i - L\pi/2)}{kr_i} = \frac{\exp\Big[\pm \mathrm{i}(kr_i - L\pi/2)\Big]}{kr_i} \ . \tag{11}$$

Therefore, $\Omega^{+}_{LSJJ_z}$ ($\Omega^{-}_{LSJJ_z}$) describes in the asymptotic regions an outgoing (ingoing) $n-{}^3$H relative motion. Finally,

$$\Psi_A^{LSJJ_z} = \sum_{L'S'} \Big[\delta_{LL'}\delta_{SS'} \Omega^{-}_{LSJJ_z} - \mathcal{S}^J_{LS,L'S'}(k) \Omega^{+}_{LSJJ_z} \Big] \ , \tag{12}$$

where the parameters $\mathcal{S}^J_{LS,L'S'}(k)$ are the S-matrix elements which determine phase-shifts and (for coupled channels) mixing angles at the energy $(2/3)k^2/m$. Of course, the sum over L' and S' is over all values compatible with a given J and parity.

The "core"wave function $\Psi_C^{JJ_z\pi}$ has been here expanded in the HH basis as for the bound-state wave function [18],

$$\Psi_C^{JJ_z\pi} = \sum_{\alpha=1}^{N_c} \sum_{n_2,n_3=0}^{n_2+n_3\le N(\alpha)} u_{\alpha n_2 n_3}(\rho) \mathcal{Y}_{\alpha n_2 n_3}(\vec{x}_K, \vec{y}_K, \vec{z}_K) \ , \tag{13}$$

where ρ is the hyperradius, $\rho = \sqrt{x_K^2 + y_K^2 + z_K^2}$. The known functions $\mathcal{Y}_{\alpha n_2 n_3}(\vec{x}_K, \vec{y}_K, \vec{z}_K)$ are given as the anti-symmetrized product of spin-isospin states and HH functions [40], the latter being given by the product of tripolar spherical harmonics and two Jacobi polynomials of indexes n_2 and n_3 given, respectively, in terms of "hyperangles" defined by

$$\cos\varphi_2 = \frac{y_K}{\sqrt{z_K^2 + y_K^2}} \ , \qquad \cos\varphi_3 = \frac{x_K}{\rho} \ . \tag{14}$$

The channels index α denotes collectively the orbital angular momenta, spin and isospins quantum numbers. For a given channel α, the HH states with indexes n_2 and n_3 chosen in the range $0 \le n_2 + n_3 \le N(\alpha)$ are included in the expansion. The values of the number of channels N_c and the various $N(\alpha)$ are increased until the desired degree of convergence in the quantity of interest is obtained (for the choice of which channels to include see the discussion below).

The functions $u_{\alpha n_2 n_3}(\rho)$ occurring in the expansion of $\Psi_C^{JJ_z\pi}$ and the matrix elements $\mathcal{S}^J_{LS,L'S'}(k)$ are determined by making the functional

$$[\overline{\mathcal{S}}^J_{LS,L'S'}(k)] = \mathcal{S}^J_{LS,L'S'}(k) - \frac{m}{\sqrt{6}\mathrm{i}} \langle \Psi_{1+3}^{L'S'JJ_z} | H - E_3 - T_{\rm cm} | \Psi_{1+3}^{LSJJ_z} \rangle \tag{15}$$

stationary with respect to variations in the $\mathcal{S}^J_{LS,L'S'}$ and $u_{\alpha n_2 n_3}$ (Kohn variational principle). Here E_3 is the ^{3}H ground-state energy. By applying this principle, a set of second order differential equations for the functions $u_{\alpha n_2 n_3}(\rho)$ are obtained. By replacing the derivatives with finite differences, a linear system is obtained,which have been solved using the Lanczos algorithm. The procedure is very similar to that outlined in the appendix of Ref. [41] and it will not be repeated here. A large number of equation can be quite easily be solved.

Let us now discuss briefly the choice of the HH states to be included in the expansion. As for the α-particle bound state [18], the "brute force" inclusion of HH states is not possible and one has to select the appropriate subset of functions. We have found convenient [18] to separate the HH functions in classes depending on the value of $\mathcal{L} = \ell_x + \ell_y + \ell_z$: channels with low values as possible of $\mathcal{L}$ have to be included in the expansion first. For each group of channels of a given $\mathcal{L}$, the corresponding values of $N(\alpha)$ are increased until convergence is reached. Then, channels with a larger value of $\mathcal{L}$ are included in the expansion and so on. The advantage of this procedure is that the classes of channels with large values of $\mathcal{L}$ have small contributions. Consequently, the corresponding values of $N(\alpha)$ can be also taken smaller and smaller.

An example of convergence is presented in Table I, in the last three columns. There, the phase shifts at $E_{cm} = 3$ MeV obtained by including in the HH expansion the channels with increasing values of $\mathcal{L}$ have been reported. For S-wave phase shifts, the inclusion of the $\mathcal{L} = 0$ channels is sufficient to have already rather good estimates. The contribution of the $\mathcal{L} \geq 4$ channels is found to be negligible. For the P-wave phase shifts, the channels with $\mathcal{L} = 1$ give the dominant contribution at low energies ($E_{c.m.} \leq 1.5$ MeV), but in the peak region they are clearly insufficient to obtain good estimates. Including the channels with $\mathcal{L} = 3$, the HH predictions already differ less of 4% with respect to the final FY results. There exist a very large number of $\mathcal{L} = 5$ channels and we have limited ourselves to the inclusion of those (usually) most important, namely, those having $\ell_x = \ell_y = 2$ and $\ell_z = 1$, plus permutations (this is sufficient to double the number of channels). However, the changes of the P-wave phase shifts are rather tiny. Therefore, we expect that the contribution of the disregarded channels $\mathcal{L} = 5$ (and those with $\mathcal{L} > 5$) to be practically negligible. This issue is currently being investigated [42].

It must be noticed that in the present HH calculation only states constructed in terms of the Jacobi vectors of the set K have been considered (hereafter referred to set-K HH functions). Obviously, the HH functions of a given set of Jacobi vectors form a complete basis by themselves. However, the HH functions constructed with the set H of Jacobi vectors (set-H HH functions) should be more suitable for taking into account contributions from the $\{2+2\}$ cluster structures. It is rather obvious that the inclusion of HH functions of both sets should speed up the convergence in constructing the state of the system. In the present calculation, however, we have included only the set-K HH functions since we were able to include a number of states sufficient to reach, in the studied cases, a satisfactory degree of convergence.

TABLE I: Convergence of $n-^3$H phase shifts and mixing parameter for AV18 potential at E$_{cm}$=3 MeV. The coupling $L \to L+2$ in the asymptotics for the $J^\pi = 1^+$ and 2^- states have been neglected. See the text for explanations.

	FY				HH		
J^π	$j_{yz} \leq 1$	$j_{yz} \leq 2$	$j_{yz} \leq 3$	$j_{yz} \leq 4$	$\mathcal{L} = 0$	$\mathcal{L} = 2$	$\mathcal{L} = 4$
0^+	-70.11	-69.98	-70.00	-70.00	-70.1	-69.8	-69.8
1^+_1	-63.61	-61.94	-61.97		-62.6	-62.2	-62.2
J^π	$j_{yz} \leq 1$	$j_{yz} \leq 2$	$j_{yz} \leq 3$	$j_{yz} \leq 4$	$\mathcal{L} = 1$	$\mathcal{L} = 3$	$\mathcal{L} = 5$
0^-	26.28	23.86	23.60	23.58	16.9	23.0	23.4
1^-_1	25.32	22.40	22.27		20.7	22.0	22.2
1^-_2	18.00	40.70	40.88		31.5	41.5	42.0
ϵ	9.04	-43.52	-44.39		-40.9	-44.9	-45.3
2^-	-	43.82	45.41	45.44	27.9	44.0	44.5

III. RESULTS

The $n-{}^3$H scattering lengths have already been presented in Ref. [5, 6, 43]. For completeness, in Table II we summarize the latest obtained values with the improved precision.

In Table III we present phase shifts, mixing parameters and the corresponding total cross sections for the most relevant partial waves at some selected energies. The coupling $L \leftrightarrow L+2$ between the different asymptotics in the $J^\pi = 1^+, 2^-$ states turned out to be very small. In AGS and FY calculations they still have been taken into account for $J^\pi = 1^+$ but neglected for $J^\pi = 2^-$ state. In fact, even at $E_{cm} = 3$ MeV, the largest energy we have considered, $L = 2$ phaseshift in $J^\pi = 1^+$ is only $\sim 1^\circ$ and contributes by only 0.01% in total cross section. Contribution of $L = 3$ phaseshifts, which were neglected in $J^\pi = 2^-$ calculations, is at least by one more order of magnitude smaller.

TABLE II: Singlet $a_1(J^\pi{=}0^+)$ and triplet $a_3(J^\pi{=}1^+)$ scattering lengths calculated with Nijm-II and AV18 NN interactions.

Pot.	a_1 (fm)	a_3 (fm)	$\sigma(0)$ (b)	Method
AV18	4.09	3.70	1.82	AGS
	4.25	3.74	1.88	FY
	4.28	3.73	1.89	HH
Nijm-II	4.09	3.70	1.82	AGS
	4.25	3.74	1.88	FY

One can see that positive parity states are under full control by the different methods. Agreement between HH and FY results is perfect for positive parity phaseshifts (in the most of cases all three significant digits coincide, whereas the largest discrepancy does not exceed 0.3%). AGS results also agree by a few percent.

The situation is more delicate for negative parity states, where resonances are present and various techniques suffer from slower convergence. Nevertheless FY and HH results stay in close agreement to better than few percent for phaseshifts, though it could be perfect if one or more convergence step is effected (see Table I). Still these small, convergence related, correlations do not shelter the excellent agreement in total cross sections. On the other hand disagreement with AGS results becomes important, reaching 30% for the phaseshifts in $J^\pi = 0^-$ state. All the negative parity phases are strongly overestimated by this method, thus resulting a larger total cross sections in the resonance region.

We believe that the underlining reason for the disagreement with AGS results is due only to the rank-one separable expansion for 2-body t-matrix used in this calculations. This lowest order expansion may be enough to describe systems dominated by spherical symmetric wave functions (i.e. positive parity states) but fails for more complicated structures, not being able to account for strong compensations present in NN P-waves [43], which turn out to be sizeable in negative parity states [44].

TABLE III: AV18 $n-{}^3$H S- and P-wave phase-shifts (degrees) as a function of the c.m. kinetic energy E_{cm} (MeV). Numbers in brackets are the mixing parameters in degrees for corresponding scattering states.

E_{cm}	0^+	1_1^+	1_2^+		0^-	1_1^-	1_2^-		2^-	σ (b)	
0.40	-27.8	-25.0	-0.018		2.24	2.87	3.89		3.56	1.73	AGS
	-28.8	-25.1	-0.0165	(0.199)	1.81	2.66	3.56	(-39.6)	3.38	1.75	FY
	-28.8	-25.1			1.78	2.70	3.76	(-44.2)	3.32	1.76	HH
0.75	-37.3	-33.6	-0.072		5.59	6.69	9.95		9.15	1.79	AGS
	-38.7	-33.8	-0.059	(0.358)	4.41	6.11	8.85	(-41.3)	8.57	1.78	FY
	-38.7	-33.8			4.36	6.08	9.32	(-45.4)	8.53	1.79	HH
1.50	-50.7	-46.1	-0.274		14.5	15.1	25.9		24.3	2.22	AGS
	-52.8	-46.2	-0.330	(0.670)	11.5	13.4	21.9	(-42.9)	22.5	2.06	FY
	-52.6	-46.3			10.8	13.3	23.0	(-45.6)	22.1	2.06	HH
2.625	-63.7	-58.3	-0.649		27.5	24.7	44.8		44.2	2.51	AGS
	-66.5	-58.5	-0.850	(1.08)	20.9	20.7	37.3	(-43.5)	41.0	2.24	FY
	-66.3	-58.7			20.6	20.5	38.6	(-45.5)	40.1	2.24	HH
3.0	-67.4	-61.9	-0.764		30.9	26.8	48.5		48.5	2.48	AGS
	-70.0	-62.0	-1.180	(1.21)	23.9	22.4	40.7	(-43.5)	45.4	2.21	FY
	-69.8	-62.2			23.4	22.2	42.0	(-45.3)	44.5	2.21	HH

The total cross sections are plotted in Fig. 2 and compared to the experimental values taken from Ref. [45]. The results of HH and FY (dashed line) are in agreement for three significant digits and thus are not distinguishable by eye.

The AGS cross sections (solid line) are slightly smaller at very low energies, whereas they provide considerably larger cross sections in the resonance region. The disagreement of theoretical results with experimental data at zero energy can be attributed to the ^{3}H underbinding by local NN interaction models. It can be improved by including a three nucleon force (3NF), i.e. the Urbana version IX (UIX), and thus reproducing the 3N binding energies [3, 5, 30, 31]. In fact, HH and FY results for AV18+UIX model agree on $\sigma(0) = 1.73$ b total cross section value, which is consistent with experimental one $\sigma(0) = 1.70 \pm 0.03$ b of Ref. [45]. Three nucleon forces, and UIX in particular, have however very small effect in the resonance region as shown in Fig 2; it has even the tendency to diminish the cross section due to triton rescaling [3, 30, 31]. The very weak sensitivity of total cross sections to "standard" 3NF, may indicate that this disagreement could have its origin in the NN forces themselves and their P-waves in particular [44].

Let us remember that in $N - d$ scattering there exist other large discrepancies between theory and experiment, the most known example is the so called A_y puzzle [46, 47, 48]. To explain these discrepancies, speculations about either the deficiency of the NN potentials in ^3P_j waves [49], or the presence of exotic 3N force terms not contemplated so far [50, 51], have been advanced. Whether such discrepancies in 3N and 4N systems originate from the same deficiency in the nuclear Hamiltonian is still under debate.

Some doubts could be casted on the reliability of the experimental data in the resonance region, due to existence of a single accurate data set [45]. However, due to the near agreement with calculations at low energies, its hardly believable that experimental points could contain a normalization error near the resonance peak. A normalization error can also be excluded by analyzing differential cross sections [43]: they indicate that theoretical results agree with experimental ones at $\theta_{cm} = 90$ deg, where positive parity phaseshifts dominate, while diverge at forward and backward directions, where negative parity phaseshifts become important. Therefore the experimental data of ref. [45] seems to be accurate and coherent. In addition, at near-resonance energies, this data seems to be also in agreement with older and less accurate measurements [52]. Nevertheless, in view of the very few experimental studies of n-^{3}H it would be very interesting to have an independent confirmation of the data.

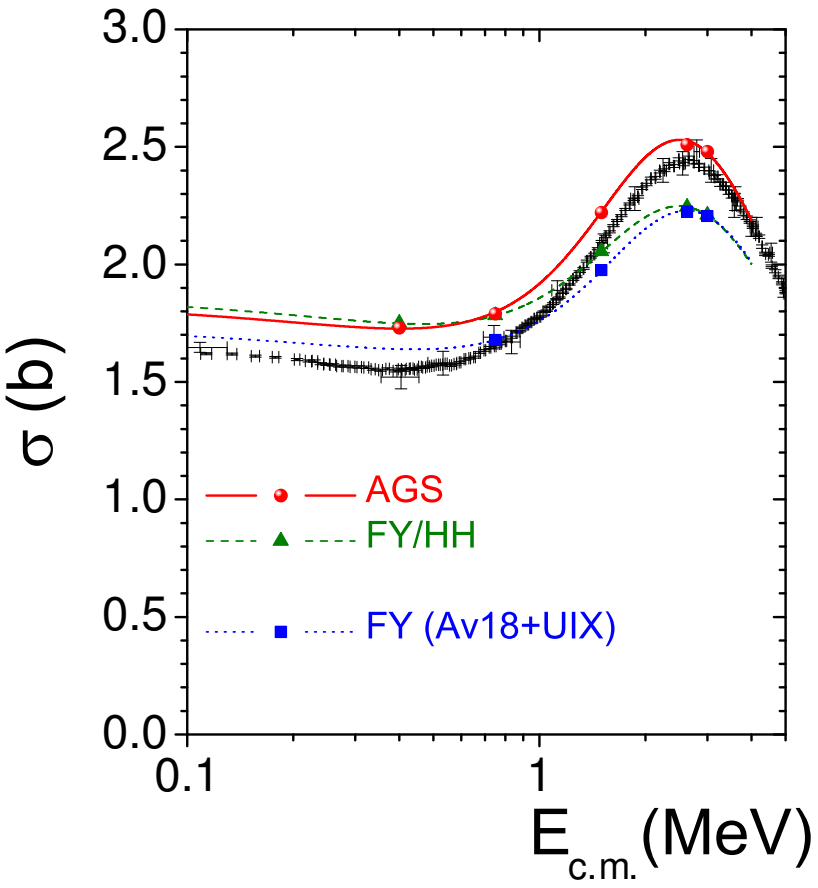

FIG. 2: Comparison between experimental and theoretical n-^{3}H total cross section calculated using different methods and with AV18 potential. Dotted curve presents AV18+UIX model results of Ref. [44].

A similar analysis has been performed with FY and AGS methods using Nijm-II model. The obtained results are summarized in Table IV. Note that a particular choice of the local NN-interaction model has no qualitative impact. Thus Nijm-II predictions are very close to the AV18 ones and lead to the same discrepancies.

TABLE IV: Nijm-II $n-{}^3$H S- and P-wave phase-shifts (degrees) as a function of the center of mass kinetic energy E_{cm} (MeV). Numbers in brackets are the mixing parameters in degrees for corresponding scattering states.

E_{cm}	0^+	1_1^+	1_2^+		0^-	1_1^-	1_2^-		2^-	σ (b)	
0.40	-27.81	-25.03	-0.0183		2.28	2.84	3.89		3.47	1.73	AGS
	-28.66	-25.03	-0.0167	(0.196)	1.751	2.633	3.537	(-39.44)	3.349	1.74	FY
0.75	-37.27	-33.68	-0.072		5.67	6.63	9.94		8.92	1.79	AGS
	-38.57	-33.73	-0.696	(0.354)	4.252	6.040	8.763	(-41.03)	8.505	1.77	FY
1.50	-50.68	-46.11	-0.271		14.66	14.98	25.83		23.87	2.21	AGS
	-52.48	-46.22	-0.334	(0.661)	10.74	13.20	21.72	(-42.69)	22.37	2.04	FY
2.625	-63.75	-58.38	-0.639		27.77	24.51	44.66		43.69	2.51	AGS
	-66.49	-58.51	-0.813	(1.067)	20.22	20.44	36.96	(-43.31)	40.82	2.23	FY
3.0	-67.18	-61.81	-0.790		32.00	26.77	48.13		48.04	2.47	AGS
	-70.04	-62.67	-1.10	(1.195)	23.09	22.13	40.35	(-43.29)	45.25	2.20	FY

IV. CONCLUSION

In this work we have presented the low energy $n-{}^3$H elastic cross sections calculated by solving the 4-nucleon problem with realistic NN interactions and by using three different approaches.

The results of Hyperspherical Harmonics and Faddeev-Yakubovsky methods are converged to 1% accuracy and are in close agreement with each other. AV18 and Nijm-II nucleon-nucleon potentials underestimate the experimental cross section by 10% near the resonance peak. This disagreement is not corrected by UIX three-nucleon forces, whose effect in this energy region is small and tends to further decrease the theoretical value upto 15% [30, 31, 44].

The results of AGS equations are closer to the experimental data. However they have been obtained using a rank-one approximation to the two-body t-matrix. This approximation, which was found to provide excellent results for nuclear systems dominated by S-waves (3N problem or positive parity 4N states), seems to provide an excess of attraction in the negative parity states of $n-{}^3$H system, where P-waves turn out to be important. We believe that accurate treatment of these resonant states requires the inclusion of higher rank terms.

The results presented here concern local NN potentials. However similar conclusions were found in previous works [44] using non local ones (Bonn [53] and Doleschall [54]). This indicates a serious difficulty of the existing NN force models in describing the simplest nuclear resonance, i.e. the $n-{}^3$H system. This difficulty can hardly be solved by the inclusion of a standard type of 3NF, used to reproduce the few-nucleon binding energies. Their origin could rather lie either in the NN forces themselves, or in the presence of 3NF of unknown type.

Acknowledgment: The numerical calculations have been performed with the CCRT (CEA) and IDRIS (CNRS). We thanks the staff members of both computer centers for their constant help. The work of (ACF) was supported in part by grant POCTI/FNU/37280/2001.

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