Four nucleon systems: a zoom to the open problems in nuclear interaction

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Abstract. Faddeev-Yakubovski equations in configuration space are used to solve four nucleon problem for bound and scattering states. Different realistic interaction models are tested, elucidating open problems in nuclear interaction description. On one hand, by example of nonlocal Doleschall potential, we reveal possibility of reducing three-nucleon force. On the other hand we disclose discrepancies in describing n+^{3}H resonance, which seems to be hardly related with off-shell structure of nucleon-nucleon interaction.

Three and four nucleon systems are the cornerstones in understanding nuclear interaction. The new generation of nucleon-nucleon (*NN*) potentials provide almost perfect description of two-nucleon data, however most of them fail already to describe the binding energies of *3N* nuclei. It is considered that off-shell effects, not comprised in local *NN* interaction models, are the cause of the underbinding problem in the lightest nuclei. A standard practice to account for the missing off-shell physics in description of $N>2$ nuclei is implementation of *3N* forces. In such a manner it is not difficult to improve description of *3N* and *4N* binding energies. However such procedure, which relies only on good description of binding energies can be quite misleading.

Low energy few nucleon physics is dominated by large nucleon-nucleon scattering lengths and therefore, as can be understood from effective range physics [1], the binding energies of light nuclei depends only on very few parameters. Thus study of nuclear binding energies, even being of fundamental importance, can provide only very limited information on nuclear interaction structure. Only at larger energies physical observables should become sensible to particular form of the potential. However at larger energies relativistic effects become also stronger, while their impact is not very well understood. In this scope *4N* continuum, containing sensible structures as thresholds and resonances presents a perfect ground to understand nuclear interaction structure, see Fig. 1.

Recently several theoretical techniques have enabled to solve *4N* bound state problem accurately [2]. *4N* continuum is more challenging task and advances in it are more modest. Still a few different groups and using different methods have been able to obtain converged results for n+t elastic scattering with realistic interactions [3, 4, 5]. Employed formalisms enable to include Coulomb as well as to test different *NN* interactions, comprising nonlocal ones and models in conjunction with three nucleon force (*3NF*).

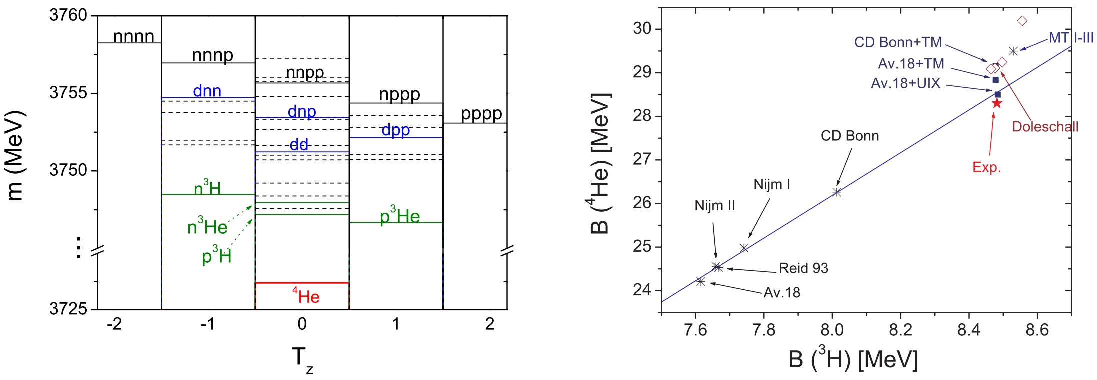


FIGURE 1. In left figure experimental spectra of *4N* bound and resonant states is presented. Figure on the right recapitulates various predictions for α-particle binding energies (Tjon-line). Some energies in this figure are taken from [6].

Theoretical model

To solve four-particle problem we use the Faddeev-Yakubovski (*FY*) equations in configuration space. In order to include *3NF FY* equations are rewritten in form, suggested by [7]. For four identical particles, these equations read:

$$(E-H_0-V_{12})K_{12,3}^4 = V_{12}(P^++P^-)\left[(1+Q)K_{12,3}^4+H_{12}^{34}\right]+V_{12,3}\Psi \tag{1}$$

$$(E-H_0-V_{12})H_{12}^{34} = V_{12}\tilde{P}\left[(1+Q)K_{12,3}^4+H_{12}^{34}\right] \tag{2}$$

with P^+, P^-, $\tilde{P}$ and Q being particle permutation operators:

$$P^+=(P^-)^-=P_{23}P_{12};\quad Q=\varepsilon P_{34};\quad \tilde{P}=P_{13}P_{24}=P_{24}P_{13}. \tag{3}$$

and

$$\Psi=\left[1+(1+P^++P^-)Q\right](1+P^++P^-)K_{12,3}^4+(1+P^++P^-)(1+\tilde{P})H_{12}^{34} \tag{4}$$

the total wave function.

Equation (2) becomes however non appropriate once long range interaction, in particular Coulomb, is present. In fact, *FY* components remain coupled even in far asymptotes, thus making implementation of correct boundary conditions hardly possible. The way to circumvent this problem is in detail described in [8].

Equations (**??**)in conjunction with the appropriate boundary conditions are solved by making partial wave decomposition of amplitudes $K_{12,3}^4$ and H_{12}^{34}:

$$K_i(\vec{x}_i,\vec{y}_i,\vec{z}_i) = \sum_{LST}\frac{\mathscr{K}_i^{LST}(x_i,y_i,z_i)}{x_iy_iz_i}\left[L(\hat{x}_i,\hat{y}_i,\hat{z}_i)\otimes S_i\otimes T_i\right] \tag{5}$$

$$H_i(\vec{x}_i,\vec{y}_i,\vec{z}_i) = \sum_{LST}\frac{\mathscr{H}_i^{LST}(x_i,y_i,z_i)}{x_iy_iz_i}\left[L(\hat{x}_i,\hat{y}_i,\hat{z}_i)\otimes S_i\otimes T_i\right] \tag{6}$$

The partial components $\mathscr{K}_i^{LST}$ and $\mathscr{H}_i^{LST}$ are expanded in the basis of three-dimensional splines. One thus converts integro-differential equations into a system of linear equations. More detailed discussion can be found in [8].

α-particle

The most general criticism of interaction models is based on their ability to describe nuclear binding energies. Local *NN* interaction models require attractive contribution of *3NF* to shield $\sim$0.8 *MeV* (or $\sim$10%) underbinding in triton and ^{3}He binding energies. Usually, once these *3N* binding energies are fixed, one obtains rather good description of the α-particle. This is due to well known correlation between α-particle and *3N* binding energies – Tjon line (see Fig. 1) – being a consequence of effective range theory [1]. However the use of *3NF*, to some extent, can be just a matter of taste. In fact, two different, but phase-equivalent, two-body interactions are related by an unitary, nonlocal, transformation [9]. One thus could expect that a substantial part of 3- and multi-nucleon forces could also be absorbed by nonlocal terms. A considerable simplification would result if the bulk of experimental data could be described by only using two-body nonlocal interaction.

The inclusion of nonlocal terms in CD-Bonn model considerably improves 3- and 4-*N* binding energies, nevertheless this improvement is still not sufficient to reproduce the experimental values. A very promising result, which takes profit from nonlocality in non relativistic nuclear models, has been obtained by Doleschall and collaborators [10]. They have managed to construct purely phenomenological nonlocal *NN* forces (called INOY), which are able to overcome the lack of binding energy in three-nucleon systems, without explicitly using *3NF* and still reproducing *2N* observables.

TABLE 1. Binding energy B (in MeV) and rms radius R (in fm) for ^{4}He ground state obtained with Doleschall, AV18 and AV18+UIX models.

Pot. Model	$<T>$	$-<V>$	B	R
INOY96	72.45	102.7	30.19	1.358
INOY03	69.54	98.79	29.24	1.373
INOY04	69.14	98.62	29.11	1.377
INOY04’	69.11	98.19	29.09	1.376
AV18	97.77	122.1	24.22	1.516
AV18+UIX	113.2	141.7	28.50 [6]	1.44 [19]
Exp			28.30	1.47

Using Faddeev-Yakubovski equations and by fully including Coulomb interaction we have tested ability of INOY interaction models to reproduce experimental α-particle binding energy. In Table 1 we summarize obtained results. One should mention that the convergence of numerical results guarantied at least three-digit accuracy. In contrary to *3N* system, experimental value of α-particle binding energy is overestimated by $\sim$800 keV. Nevertheless this discrepancy is considerably smaller than for conventional local interaction models and is on a par with the result obtained using CD-Bonn *NN* in conjunction with Tucson-Melbourne *3NF*.

Still it is risky to judge about the interaction models basing only on binding energy arguments, comparing other physical observables can give us hints about the origin of the existing drawbacks. In Table 1 one can clearly see that proton r.m.s. radius in α-particle predicted by INOY models is by $\sim$10% smaller than the experimental one. Note, these models provide experimental values of proton r.m.s in deuteron; in 3N compounds these radii become slightly smaller than the experimental ones [11]. Such tendency demonstrates that INOY interactions are too soft, resulting into a faster condensation of the nuclear matter. In order to improve these models one should try to increase the short range repulsion between the nucleons.

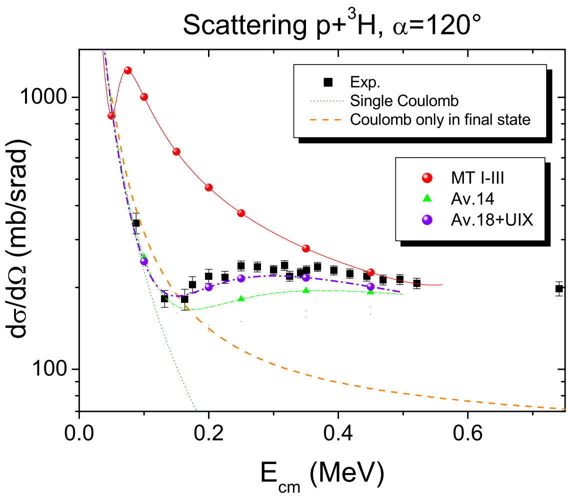
This study reveals enormous liberty one has in describing off-shell effects. Only careful analysis of continuum spectra, which enables one to test various space configurations, can impose stronger constrains to the off-shell structure of the nuclear interaction.

p+^{3}H scattering at very low energies

^{4}He continuum is the most complex *4N* system, its spectrum contains numerous resonances (see Fig. 1). In this study we have considered only very low energy region E_{cm}<500 keV region, below n+t threshold, fully described by proton scattering in S-waves. Nevertheless excitation function – $\frac{d\sigma}{d\Omega}(E)\big|_{\theta=120^\circ}$ – has complicated structure due to existence of the $\mathscr{J}^\pi = 0^+$ resonance in p+^{3}H threshold vicinity. This resonance, being the first excitation of α-particle, is located at $E_R \approx 0.4\ MeV$ above p+^{3}H threshold and with its width $\Gamma \approx 0.5\ MeV$ covers almost the entire region below n+^{3}He.

Separation of n+^{3}He and p+^{3}H channels requires proper treatment of Coulomb interaction, the task is furthermore burden since both thresholds are described by the same isospin quantum numbers. When ignoring Coulomb interaction, as was a case in the large number of nuclear scattering calculations, n+^{3}He and p+^{3}H thresholds coincide. In this case 0^+ resonant state moves below the joint threshold and becomes bound. Former fact is reflected in low energy scattering observables (see Fig. 2 dashed line): on one hand *N+NNN* scattering length in 0^+ state is found positive, on the other hand excitation function decreases smoothly with incident particles energy and does not show any resonant behavior. Only by properly taking Coulomb interaction into account, thus separating n+^{3}He and p+^{3}H thresholds, the ^{4}He excited state is placed in between and thus gives negative p+^{3}H singlet scattering length.

In order to reproduce the shape of experimental excitation function, *NN* interaction model is obliged with high accurately situate ^{4}He excited state. In fact, width of the resonance is strongly correlated with its relative position to p+^{3}H threshold. If this state is slightly overbound the resonance peak in excitation curve becomes too narrow and is situated at lower energies. This is a case for MT I-III model prediction, see Fig. 2. In case of underbinding one obtains excitation function, which is too flat and provides underestimated cross sections. This is a case of local *2NF* models, which predict too small (in absolute value) singlet scattering length and thus place the virtual state too far from threshold. Only once implementing UIX *3NF* in conjunction with Av18 *NN* model one obtains singlet scattering length as well as the excitation function in agreement with experimental data [12]. On contrary various model predictions do not differ much for



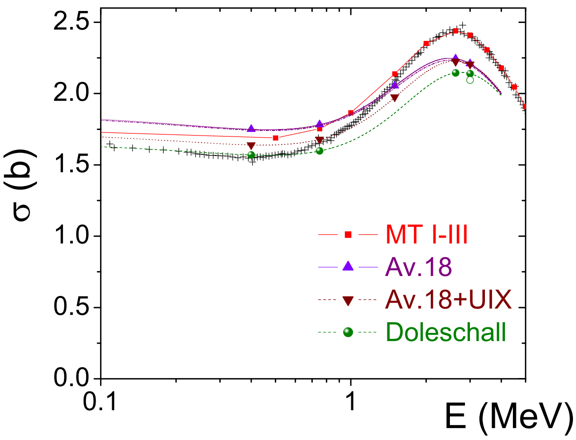


FIGURE 2. Various model calculations for p+^{3}H excitation function $\frac{d\sigma}{d\Omega}(E)\big|_{\theta=120^\circ}$ (figure in the left) and n+^{3}H cross sections (figure in the right). Calculations compared to experimental data from [12] and [13] respectively.

triplet scattering case, where resonance states are present only at considerably larger energies.

As have been shown, description of low energy p+^{3}H completely relies on success in reproducing position and width of the first excited state of α-particle. However position of this resonant state seems to be strongly correlated with position of predicted triton threshold and α-particle ground state. MT I-III model overbinds a ground state of α-particle, see Fig. 1, and thus its resonance is situated too close to p+^{3}H threshold. On contrary local realistic interaction models (without *3NF*) strongly underbind α-particle and thus predict resonant state at higher relative energies to triton threshold. Once triton and α-particle energies are reproduced (Av18+UIX case) one obtains also accurate description for ^{4}He excited state. These observations recalls to effective range theory [1]: in first order only one three-body scale is important in describing three and four body binding energies if two body physics is dominated by large scattering lengths.

TABLE 2. *4N* scattering lengths calculated using different interaction models.

	MT I-III		**Av14**		**Av18+UIX**		**INOY04**	
System	$J^\pi=0^+$	$J^\pi=1^+$	$J^\pi=0^+$	$J^\pi=1^+$	$J^\pi=0^+$	$J^\pi=1^+$	$J^\pi=0^+$	$J^\pi=1^+$
n-^{3}H	4.10	3.63	4.28	3.81	4.04	3.60	4.00	3.52
p-^{3}H	-63.1	5.50	-13.9	5.77	-16.5	5.39		

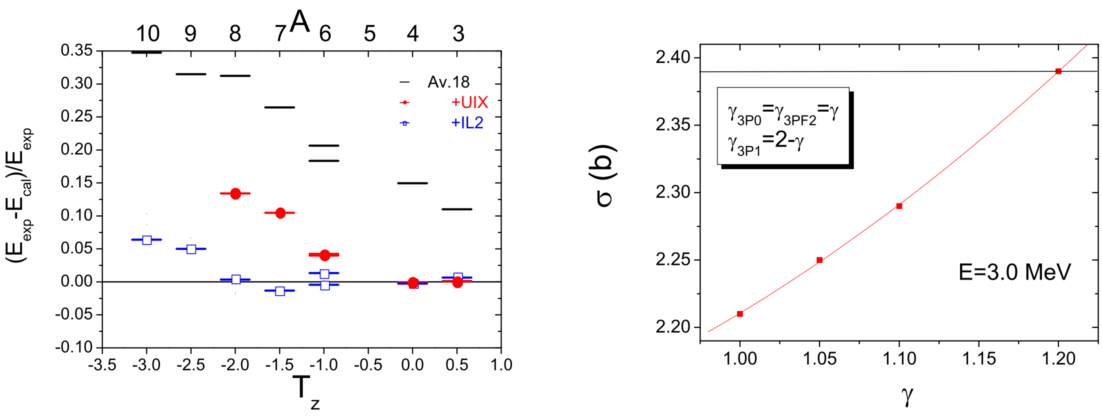


FIGURE 3. Comparison of calculated Ref.[19] and experimental binding energies for various He isotopes (figure in the left). Right figure shows n+^{3}H elastic cross section at $E_{cm} = 3$ MeV dependence on enhancement factor γ of n-n P-waves.

n+^{3}H elastic scattering

n+^{3}H elastic channel represents the simplest *4N* reaction. It is almost pure $\mathscr{T} = 1$ isospin state, free of Coulomb interaction in the final state as well as in the target nucleus. Nevertheless this system contains two, spin degenerated, narrow resonances at low energy ($E_{cm} \approx 3$ MeV), while ability of nuclear interaction models to describe scattering cross sections in this resonance region was put in doubt [15, 16].

Recently a few different groups have obtained the converged results for n+^{3}H elastic scattering when using realistic interaction models [3]. Our predictions are in full agreement with those of Pisa group, using Hyperspherical Harmonics method, determining n+t scattering observables with accuracy better than 1% [4].

As have been shown before [14, 15], pure *NN* local interaction models overestimate n+^{3}H zero energy cross sections, being consequence of overestimated size of triton (lower binding energy). Scattering cross sections in this very low energy region seems to be linearly correlated with triton binding energy. However even implementing UIX *3NF* for AV18 model, and thus reproducing triton binding energy, small discrepancies remain [11]. If zero energy cross section seems to fit experimental data, agreement for the coherent scattering lengths is less obvious. Nonlocal interaction model (INOY04) reproduces low n+^{3}H energy data better, this fact is best reflected near the elastic cross section minima around E_{cm} ~0.4 MeV, see Fig. 2.

Nevertheless all realistic interaction models, including INOY, underestimate by more than 10% elastic cross sections in the resonance region. Off-shell effects as 3NF or nonlocality of the force, which improve low energy behavior, do not give sizeable effect.

In order to understand the origin of this failure it is useful to study impact of different potential terms for scattering phaseshifts. This can be done using integral representation of the phaseshifts. As was shown in [11] for positive parity states – whether it is contribution in 3- and 4-*N* potential energies or in integral representation of scattering phaseshifts – *NN* S-waves plays a major role, whereas P-waves stay intact (less than 1%). On the other hand contribution of *NN* P-wave becomes almost as important as one of S-waves in negative parity states for n+^{3}H resonance region.

In fact low energy nuclear physics is dominated by *NN* S-waves, being a result of large *NN* scattering lengths and the fact that interaction in higher waves is weaker. On contrary rare physical observables, where *NN* P-waves are important, have tendency to disagree with the experimental data. One such example could be the *3N* analyzing powers [17, 18]. Higher *NN* waves should contribute in asymmetric nuclear systems; n+^{3}H system due to its large neutron excess is one such case. Other example can be given by plotting relative discrepancy in predicted binding energies by Argonne collaboration [19] of various He isotopes (see Fig. 3). The discrepancy increases linearly with neutron excess, even if *3NF* improves overall agreement it does not remove this tendency.

All these statements speaks against good description of *NN* interaction in P-waves. One should also recall that *NN* interaction is basically tuned on n-p and p-p data. Moreover, low energy p-p P-waves are overcasted by Coulomb repulsion, while n-n P-waves are not directly controlled by experiment at all. We have evaluated, see Fig. 3, how much n-n P-wave potential must be changed in order to describe resonance cross sections for Av18 model. It turns to be of order $\sim$20%, which is rather much to blame on charge dependence in P-waves as well as compared to $\sim$6-8% necessary to solve A_y puzzle in n-d scattering [17, 18]. However in n-d calculations n-n and n-p P-waves were modified simultaneously, whereas we have tuned only n-n P-waves. Still this study suggests that charge dependence effects can be sizeable in n-n P-waves and can provide a possible explanation for the disagreement observed in n-^{3}H resonance region.

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