Effects of spin-orbit interaction on nuclear response and neutrino mean free path

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(Dated: April 10, 2006)

The effects of the spin-orbit component of the particle-hole interaction on nuclear response functions and neutrino mean free path are examined. A complete treatment of the full Skyrme interaction in the case of symmetric nuclear matter and pure neutron matter is given. Numerical results for neutron matter are discussed. It is shown that the effects of the spin-orbit interaction remain small, even at momentum transfer larger than the Fermi momentum. The neutrino mean free paths are marginally affected.

I. INTRODUCTION

The mean field theory of nuclear systems is a well developed tool for the microscopic study of finite nuclei as well as infinite matter. Many effective two-body interactions have been determined with the goal of achieving a self-consistent mean field description of nuclear ground states and excited states through the Hartree-Fock (HF) and random phase approximation (RPA) approaches. The Skyrme-type interactions are among the most frequently used effective forces, and this is due to the fact that they are relatively simple and yet, they can give a fairly accurate description of finite nuclei data[1, 2].

Homogeneous infinite matter is just an idealized object but it can be a very useful testing ground for various theories because practical computations are easier to carry out than in finite nuclear systems. It is also a good representation of the internal regions of stellar objects such as neutron stars. The matter equation of state and the response of matter to various external probes are important physical properties. For instance, neutrino mean free paths can be deduced from nuclear response functions[3, 4]. Therefore, HF and RPA studies of infinite matter have always been carried out along with finite nuclei studies. The formalism for computing RPA response functions in infinite Fermion systems with Skyrme-type interactions has been shown in Ref.[5] where applications were made for symmetric nuclear matter while calculations for neutron matter have been performed in Ref.[6]. The two-body spin-orbit component has been ignored in the previous studies of the nuclear response function. Whereas it is true that the spin-orbit term does not contribute to quantities as the equation of state of saturated spin systems, one should consider also situations where an external operator can induce spin oscillations which can manifest in the response function. The aim of this work is to give a complete treatment of the full Skyrme interaction in the case of symmetric nuclear matter and pure neutron matter. The spin-orbit component of the particle-hole (p-h) interaction couples the spin channels in the response function. However, it will be shown that the effect remains small, even at momentum transfer larger than the Fermi momentum, while the neutrino mean free paths are marginally affected.

The outline of the paper is as follows. In Sec. II we present the general method for calculating RPA response functions with the full Skyrme interaction, generalizing the method of Garcia-Recio et al.[5] to the case of the spin-orbit interaction. In Sec. III the results obtained for response functions in symmetric nuclear matter and pure neutron matter, as well as neutrino mean free paths are discussed. Concluding remarks are given in Sec. IV.

II. FORMALISM

A. Definitions

A general two-body interaction in momentum representation depends at most on 4 momenta. Because of momentum conservation there are actually 3 independent momenta. For the p-h case we choose these independent variables to be

the initial (final) momentum $\mathbf{k_1}$ ($\mathbf{k_2}$) of the hole and the external momentum transfer $\mathbf{q}$. This is illustrated by Fig. 1. We will denote by $\alpha = (S, M; I, Q)$ the spin and isospin p-h channels with $S=0$ (1) for the non spin-flip (spin-flip) channel, $I=0$ (1) the isoscalar (isovector) channel, M and Q being the third components of S and I.
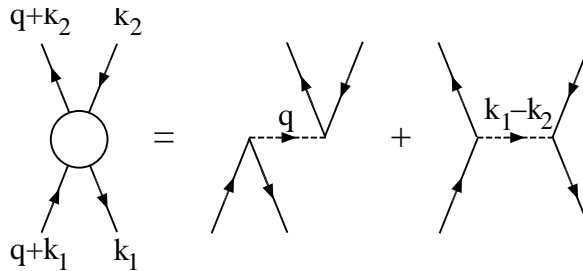

FIG. 1: Direct and exchange parts of the ph interaction.

Let us consider an infinite nuclear medium at zero temperature and unpolarized both in spin and isospin spaces. At mean field level this system is described as an ensemble of independent nucleons moving in a self-consistent mean field generated by the starting effective interaction treated in the Hartree-Fock (HF) approximation. The momentum dependent HF mean field, or self-energy determines the single-particle spectrum $\epsilon(\mathrm{k})$ and the Fermi level $\epsilon(\mathrm{k}_F)$.

To calculate the response of the medium to an external field it is convenient to introduce the Green's function, or retarded p-h propagator $G^{(\alpha)}(\mathbf{q}, \omega, \mathbf{k_1})$. From now on we choose the z axis along the direction of $\mathbf{q}$. In the HF approximation, the p-h Green's function is the free retarded p-h propagator [7]:

$$G_{HF}(\mathrm{q}, \omega, \mathbf{k_1}) = \frac{f(\mathrm{k_1}) - f(|\mathbf{k_1} + \mathbf{q}|)}{\omega + \epsilon(\mathrm{k_1}) - \epsilon(|\mathbf{q} + \mathbf{k_1}|) + i\eta} \ , \tag{1}$$

where the Fermi-Dirac distribution f is defined for a given temperature T and chemical potential μ as $f(\mathrm{k}) = [1 + e^{(e(\mathrm{k})-\mu)/T}]^{-1}$. The HF Green's function G_{HF} does not depend on the spin-isospin channel α. To go beyond the HF mean field approximation one takes into account the long-range type of correlations by resumming a class of p-h diagrams and one obtains the well-known random phase approximation [7]. The interaction appearing in the RPA is the p-h residual interaction whose matrix element including exchange can be written as:

$$V_{ph}^{(\alpha,\alpha')}(q, \mathbf{k_1}, \mathbf{k_2}) \equiv \langle \mathbf{q} + \mathbf{k_1}, \mathbf{k_1^{-1}}, (\alpha)|V|\mathbf{q} + \mathbf{k_2}, \mathbf{k_2^{-1}}(\alpha') \rangle \ . \tag{2}$$

The RPA correlated Green's function $G_{RPA}^{(\alpha)}(\mathrm{q}, \omega, \mathbf{k_1})$ satisfies the Bethe-Salpeter equation:

$$G_{RPA}^{(\alpha)}(\mathrm{q}, \omega, \mathbf{k_1}) = G_{HF}(\mathrm{q}, \omega, \mathbf{k_1}) + G_{HF}(\mathrm{q}, \omega, \mathbf{k_1}) \sum_{(\alpha')} \int \frac{\mathrm{d}^3 k_2}{(2\pi)^3} V_{ph}^{(\alpha,\alpha')}(\mathrm{q}, \mathbf{k_1}, \mathbf{k_2}) G_{RPA}^{(\alpha')}(\mathrm{q}, \omega, \mathbf{k_2}) \ . \tag{3}$$

Finally, the response function $\chi^{(\alpha)}(\mathrm{q}, \omega)$ in the infinite medium is related to the p-h Green's function by:

$$\chi_{RPA}^{(\alpha)}(\mathrm{q}, \omega) = g \int \frac{\mathrm{d}^3 k_1}{(2\pi)^3} G_{RPA}^{(\alpha)}(\mathrm{q}, \omega, \mathbf{k_1}) \ , \tag{4}$$

where the spin-isospin degeneracy factor g is 4 for symmetric nuclear matter and 2 for pure neutron matter. The Lindhard function χ_{HF} is obtained when the free p-h propagator G_{HF} is used in Eq. (4).

In the following parts of this work we will often deal with integrals similar to those appearing in Eqs. (3-4) and we will adopt the notation $\langle V_{ph}^{(\alpha\alpha')} G_{RPA}^{(\alpha')} \rangle$, $\langle G_{RPA}^{(\alpha)} \rangle$ for such quantities.

B. The p-h interaction

The central component of the p-h interaction can be written in the general form:

$$V_{ph}^{(\alpha,\alpha')}(q, \mathbf{k_1}, \mathbf{k_2}) = \delta(\alpha, \alpha') \left\{ W_1^{(\alpha)} + W_2^{(\alpha)}[\mathrm{k}_1^2 + \mathrm{k}_2^2] - 2W_2^{(\alpha)} \frac{4\pi}{3} \mathrm{k}_1 \mathrm{k}_2 \sum_{\mu} Y_{1\mu}^*(\hat{k_1}) Y_{1\mu}(\hat{k_2}) \right\} \ , \tag{5}$$

where the $W_i^{(\alpha)}$ are combinations of the Skyrme parameters (t_i, x_i) and of the transferred momentum q. Their detailed expressions are given in Refs.[5] and [6] for the symmetric nuclear matter and pure neutron cases, respectively. One can note that there is no coupling between the different spin and isospin channels. The general case of matter with an arbitrary neutron-to-proton ratio has been studied in Ref.[8].

The Skyrme interactions also contain a zero-range spin-orbit term [1]. It has the form $iW_{so}(\sigma_1 + \sigma_2) \cdot [\mathbf{k}' \times \delta(r_{12})\mathbf{k}]$, where σ_i is the spin operator of particle i, and $\mathbf{k}$ and $\mathbf{k}'$ are the relative momentum operator of the particles acting to the right and left, respectively. To calculate the contribution of this term to the p-h interaction one has to evaluate the matrix element (2) of this spin-orbit interaction. As this term is density-independent there is no rearrangement contribution and the result is just adding the following term

$$
-\delta(I,I')\, w(I) \sqrt{\frac{4\pi}{3}}\, q\, W_{so} \Bigg\{ \quad \delta(S,1)\delta(S',0) M_S \left[k_1 Y_{1-M_S}(\hat{k_1}) - k_2 Y_{1-M_S}(\hat{k_2}) \right]
$$

$$
+\delta(S,0)\delta(S',1) M'_S \left[k_1 Y_{1M'_S}(\hat{k_1}) - k_2 Y_{1M'_S}(\hat{k_2}) \right] \Bigg\} , \tag{6}
$$

to Eq. (5). We have defined $w(I) = 2 + (-1)^I$ in the case of symmetric nuclear matter, and $w(I) = 2$ for pure neutron matter. The effect of the spin-orbit component is to couple both $S = 0$ and 1 channels.

C.   Response function

To obtain the RPA response function of Eq. (4) one has to calculate the correlated Green's function $G_{RPA}^{(\alpha)}$. The technical details are given in the Appendix. The response function can then be written in the form:

$$
\frac{\chi_{HF}}{\chi_{RPA}^{(\alpha)}} = 1 - \widetilde{W}_1^{(\alpha)}\chi_0 - 2W_2^{(\alpha)} \left[\frac{q^2}{4} - \left(\frac{\omega m^*}{q}\right)^2 \frac{1}{1 - \frac{m^* k_F^3}{3\pi^2}W_2^{(\alpha)}} \right] \chi_0
$$

$$
+2W_2^{(\alpha)}\left(\frac{q^2}{2}\chi_0 - k_F^2\chi_2\right) + [W_2^{(\alpha)}k_F^2]^2 \left[\chi_2^2 - \chi_0\chi_4 + \left(\frac{\omega m^*}{k_F^2}\right)^2 \chi_0^2 - \frac{m^*}{6\pi^2 k_F}q^2\chi_0 \right] . \tag{7}
$$

In this expression k_F is the Fermi momentum while m^* denotes the nucleon effective mass. The functions χ_0, χ_2 and χ_4 are generalized free response functions, defined as [5]

$$
\chi_{2i} = \frac{1}{2}\langle \left[\left(\frac{k^2}{k_F^2}\right)^i + \left(\frac{|\mathbf{k} + \mathbf{q}|^2}{k_F^2}\right)^i \right] G_{HF} \rangle , \tag{8}
$$

with $\chi_0 = \langle G_{HF} \rangle$, and $\chi_{HF} = g\chi_0$.

The coupling between the two spin channels appears implicitly in the function $\widetilde{W}_1^{(\alpha)}$ which can be expressed in terms of the quantities β_i introduced in Ref.[5] (their definitions are recalled in the appendix). One obtains:

$$
\widetilde{W}_1^{(\alpha)} = W_1^{(\alpha)} + C^{(\alpha)}w^2(I)W_{so}^2\, q^4\, \frac{\beta_2 - \beta_3}{1 + W_2^{(\alpha')}q^2(\beta_2 - \beta_3)}, \tag{9}
$$

where α' is defined with respect to α as $S' = 1 - S$, $I' = I$ (the third components M' and Q' are irrelevant here since $W_2^{(\alpha')}$ does not depend on them), and

$$
\begin{aligned}
C^{(\alpha)} &= 1 \quad \text{if} \quad S = 0 \\
&= \frac{1}{2}M^2 \quad \text{if} \quad S = 1
\end{aligned} \tag{10}
$$

If we replace $\widetilde{W}_1^{(\alpha)}$ in Eq.(7) by $W_1^{(\alpha)}$ we obtain the results of Ref.[5], as it should be. It is worth noticing that the spin-orbit interaction induces a complex coupling between the $S=0$ and $S=1$ channels. This coupling is seen in Eqs.(A1-A3) of the appendix. The β_i's are complex and therefore, $\widetilde{W}_1^{(\alpha)}$ is a complex function of q and ω.

Finally, the quantity of interest is the dynamical structure function $S^{(\alpha)}(q,\omega)$. At zero temperature, it is just proportional to the imaginary part of the response function at positive energies. At finite temperature T, one can also relate it to the response function by the detailed balance relation and obtain[9]:

$$
S^{(\alpha)}(q,\omega,T) = -\frac{1}{\pi}\frac{\text{Im}\chi^{(\alpha)}(q,\omega,T)}{1 - e^{-\omega/T}} , \tag{11}
$$

where the energy ω can be either positive or negative.

III. RESULTS

We apply the above formalism to the case of neutron matter. The calculations are performed with the Skyrme interaction SLy230b which is designed for reproducing the neutron matter equation of state[10]. We can make a global assessment of our numerical accuracy by calculating the energy-weighted integrals of our dynamical structure functions, and compare them with the energy-weighted sum rule obtained by the double commutator method [11]. Indeed, the sum rule value must not depend on the spin-orbit interaction.

A. The spin-orbit induced interaction

The function $\widetilde{W}_1^{(\alpha)}$ describes the coupling between spin channels induced by the spin-orbit interaction. While the interaction parameter $W_1^{(\alpha)}$ is real and independent of ω and of the temperature, the new function $\widetilde{W}_1^{(\alpha)}$ is complex and it depends on both ω and T through the β_2, β_3 terms.
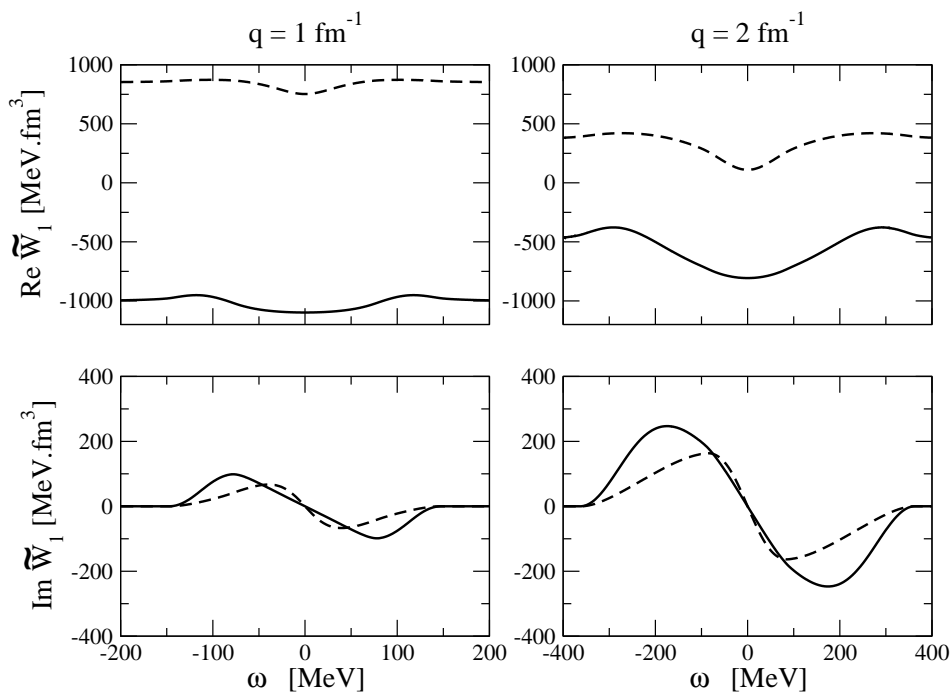

FIG. 2: The interaction parameter $\widetilde{W}_1^{(\alpha)}(q, \omega)$ as a function of ω, for two different values of the momentum transfer. Solid and dashed lines correspond to $S = 0$ and $S = 1$ channels, respectively. The results are for neutron matter at $T = 0$ and $\rho = \rho_0$.

In Fig. 2 are plotted the real and imaginary parts of $\widetilde{W}_1^{(\alpha)}(q, \omega, T)$, as a function of ω for two different values of q. The density has been fixed to the value ρ_0 of saturation density of symmetric nuclear matter, and the temperature is $T = 0$. It is interesting to notice that the ω-dependence of the difference $\beta_2 - \beta_3$ has the same symmetry properties as the response function, namely, the real (imaginary) part is symmetric (antisymmetric) with respect to $\omega=0$ for a fixed value of q. This property is fulfilled in spite of the fact that each of the functions β_2 and β_3 does not satisfy it separately. Such symmetry properties are seen on Fig. 2.

When $\omega \to \pm\infty$, both β_i functions of Eq.(9) go to zero and $\widetilde{W}_1^{(\alpha)}$ tends to $W_1^{(\alpha)}$. Therefore, the amplitude of the oscillations of the curves in Fig. 2 show the deviations of $\widetilde{W}_1^{(\alpha)}$ with respect to $W_1^{(\alpha)}$. As for the real parts, these deviations are most visible around $\omega=0$. For the imaginary parts, they are always zero at $\omega=0$ and the deviations become significant at larger values of $|\omega|$. In any case, one can see that the change of the p-h interaction due to the spin-orbit force seems to be relatively small, except at higher values of the transferred momentum q. This increase with q reflects the q^4 power explicitly appearing in Eq.(9).

B. Dynamical structure functions

We now turn to the effect of the spin-orbit interaction on the dynamical structure function $S^{(\alpha)}(q,\omega)$. In Figs. 3-4 are plotted the values of $S^{(\alpha)}(q,\omega)$ as functions of ω, calculated at zero temperature and at $T=20$ MeV. The calculations are done at the fixed values of momentum transfer q and neutron matter densities indicated in the figures. We show separately the $S=0$ and $S=1$ cases, calculated with and without the spin-orbit force. A general observation is that the effect of spin-orbit interaction increases as q increases. This can be easily understood according to the previous analysis of the interaction parameter $\widetilde{W}_1^{(\alpha)}(q,\omega)$.

We first discuss the $T=0$ results of Fig. 3. For the smaller value of q the effect increases slightly with increasing density, this effect being more visible in the $S=0$ channel. For the larger value of q the increasing effect with increasing density becomes more dramatic in the $S=1$ channel. The reason for this behaviour is due to the fact that, at $\rho=\rho_0$ and $q=2$ fm^{-1} we are approaching the point of instability in the $S=1$ channel. This instability is characteristic of Skyrme-type interactions [12]. Indeed, the response function depends not only on $\widetilde{W}_1^{(\alpha)}$ but also on $W_2^{(\alpha)}$ as shown in Eq.(7). Now, the interaction SLy230b gives $W_2^{S=0}=163.55$ MeV.fm^5, $W_2^{S=1}=-163.55$ MeV.fm^5. From Fig. 2 one also sees that the spin-orbit force contributes some important negative amount in the $S=1$ channel, thus enhancing the tendency to instability. One can conclude that the effect of the spin-orbit force is generally small but it can become dramatic when one approaches the instability region of the $S=1$ channel.
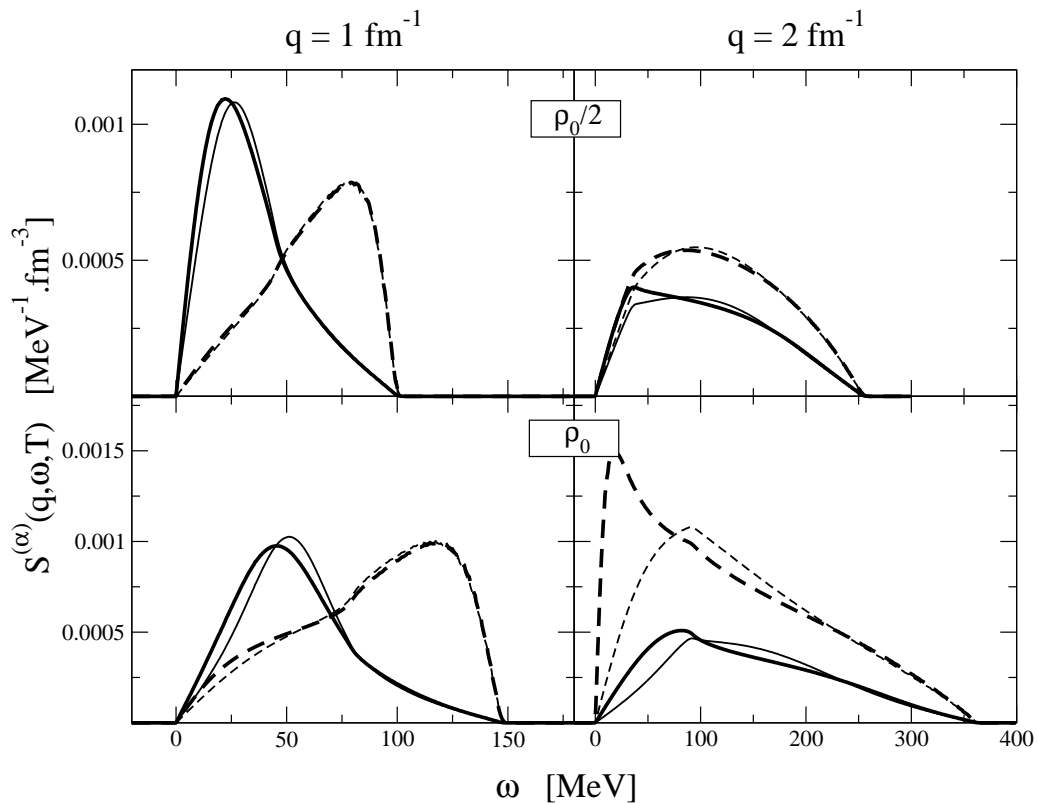

FIG. 3: The dynamical structure function $S^{(\alpha)}(q,\omega)$ at $T=0$ as a function of ω, for two values of the momentum transfer q and at densities $\rho=\rho_0/2$ and ρ_0 . Solid and dashed lines correspond to $S=0$ and $S=1$ channels, respectively. The thin lines represent the structure function without spin-orbit interaction.

The above conclusions remain valid at finite temperature but they are further amplified, as one can see from Fig. 4 in the case of $T=20$ MeV. As expected, noticeable modifications of the dynamical structure function are found for ω around zero and q larger than k$_F$. One must keep in mind that the effective interaction parameter $\widetilde{W}_1^{(\alpha)}(q,\omega)$ containing the contribution of the spin-orbit force is temperature dependent through the β_2, β_3 functions. In the $T=20$ MeV case the vicinity of the instability point is clearly seen for $\rho=\rho_0$ and $q=2$ fm^{-1}.
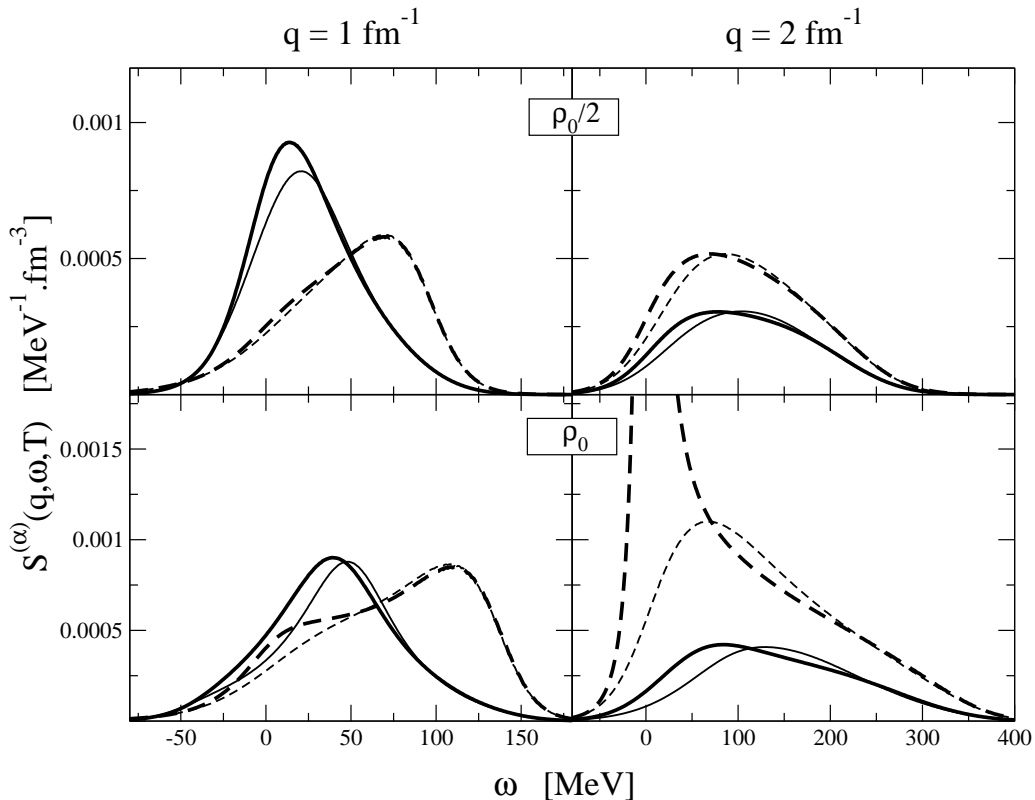
FIG. 4: Same as Fig.3, for $T = 20$ MeV.

C. The neutrino mean free paths

We examine now the effect of the spin-orbit interaction on neutrino mean free paths in neutron matter under various conditions of density and temperature. The scattering of neutrinos on neutrons is mediated by the neutral current of the electro-weak interaction. In the non-relativistic limit and in the case of non-degenerate neutrinos, the mean free path λ of a neutrino with initial momentum $\mathbf{k}_1$ is given by [3, 4]

$$1/\lambda(\mathbf{k}_1, T) = \frac{G_F^2}{32\pi^3} \int d\mathbf{k}_3 \left(c_V^2 (1 + \cos\theta) \, \mathcal{S}^{(0)}(q, T) + c_A^2 (3 - \cos\theta) \, \mathcal{S}^{(1)}(q, T) \right) , \qquad (12)$$

where T is the temperature, G_F is the Fermi constant, c_V (c_A) the vector (axial) coupling constant. The final neutrino momentum is $\mathbf{k}_3$, the four-vector $q = k_1 - k_3$ stands for the transferred energy-momentum, and $\cos\theta = \hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_3$. The dynamical structure factors $\mathcal{S}^{(S)}(q, T)$ describe the response of neutron matter to excitations induced by neutrinos, and they contain the relevant information on the medium. The vector (axial) part of the neutral current gives rise to density (spin-density) fluctuations, corresponding to the $S = 0$ ($S = 1$) spin channel.

We have calculated the neutrino mean free path at different densities (ρ_0 and $2\rho_0$) and temperatures (10, 20 and 30 MeV). The energy of the incoming neutrino is chosen to be $E_\nu = 3T$. Results are shown in Table I. The first line (λ_{HF}) shows the results for the neutrino mean free path calculated at the self-consistent mean field approximation, i.e., without RPA correlation effects. The spin-orbit interaction has no effect in this case, since we are in a homogeneous medium and the spin-orbit force does not contribute to the HF properties. Next, we show the results of the complete calculation with or without spin-orbit interaction. For $T = 10$ MeV, the spin-orbit interaction modify the mean free path by only 1%, 3-4% for $T = 20$ MeV and 5-10% for $T = 30$ MeV. We thus conclude that the effects of the spin-orbit interaction on the neutrino mean free paths are at the level of a few percent.

IV. CONCLUDING REMARKS

We have investigated the effects of the spin-orbit component of the p-h interaction V_{ph} on the RPA nuclear response functions and their possible consequences on the neutrino mean free paths. This study is carried out in the framework

	ρ/ρ_0	1			2		
	T (MeV)	10	20	30	10	20	30
λ_{HF} [m]	-	45.1	5.83	1.84	67.0	8.19	2.33
$\lambda_{RPA}/\lambda_{HF}$	with spin-orbit	1.03	0.93	0.79	0.54	0.44	0.27
	without spin-orbit	1.04	0.96	0.83	0.55	0.48	0.38

TABLE I: Neutrino mean free paths (in meters) in neutron matter, calculated in HF and RPA schemes. The calculations correspond to the different densities and temperatures as indicated. The neutrino energies are $E_\nu = 3T$.

of a Skyrme-type, zero-range effective interaction. While the central component of V_{ph} keeps the $S=0$ and $S=1$ spin channels separated, the spin-orbit component couples these channels together. However, within the specific form of Skyrme-type interactions this coupling appears only implicitly through a modified interaction parameter $\widetilde{W}_1^{(\alpha)}(q,\omega,T)$, and the calculation of the response function is formally identical to the case without spin-orbit interaction.

The modified interaction parameter $\widetilde{W}_1^{(\alpha)}$ is shown to be complex and it depends on the energy-momentum transfer (ω,q) and temperature T. Its behaviour at large ω shows that the effect of the spin-orbit force tends to zero for increasing ω. The overall effects on the response functions remain small in neutron matter at densities up to ρ_0. However, in the specific example of the SLy230b force that we have considered, a pole in the response function and hence an instability occur in the $S=1$ channel at $\rho \simeq \rho_0$ and $q \simeq 2$ fm^{-1}. In this case, even a small modification brought about by the spin-orbit force produces a large change of the $S=1$ response function near the pole.

As for the T-dependence of the spin-orbit effects, all the remarks made above remain true with increasing T, the only difference being that the effects are amplified at higher temperature. Finally, the neutrino mean free paths in neutron matter are very moderately affected by the spin-orbit component of the p-h interaction.

The numerical applications have been presented here for the case of neutron matter, but similar results are obtained in symmetric and asymmetric nuclear matter. We also note that the zero-range nature of the spin-orbit force studied here is reflected in the q^4 dependence of the modified p-h interaction, and therefore the fact that the spin-orbit effects increase with increasing q would be altered for a finite range interaction.

Acknowledgments

APPENDIX A: THE COUPLED INTEGRAL EQUATIONS FOR THE $G_{RPA}^{(\alpha)}$ GREEN'S FUNCTIONS

In this Appendix we shall show the way to transform the Bethe-Salpeter equation (3) into a set of coupled algebraic equations for three integrated quantities depending on $G_{RPA}^{(\alpha)}$. To alleviate the notation, we will omit the (q,ω)-dependence, and specify only the spin variables, as there is no isospin coupling. The expressions are formally valid for both symmetric nuclear matter and neutron matter. The only differences will be in the value of the spin-isospin degeneracy factor g of Eq. (4), and the factor $w(I)$ of Eq. (6).

One must note that the multipole expansion of G_{HF} (see Eq. 1) only involves terms of the type $Y_{L,0}(\hat{k}_1)$. Therefore integrals of the type $\langle f(k)Y_{L,M}G_{HF}\rangle$ or $\langle f(k)Y_{L,M}Y_{L',M'}G_{HF}\rangle$ vanish unless $M \neq 0$ or $M + M' \neq 0$, respectively. This will simplify the response function equations. Other integrals involving G_{HF} are also needed and they can be expressed in terms of the quantities β_i introduced in [5]:

$$\langle G_{HF}\rangle = \beta_0 \ , \qquad\qquad \langle k^2 G_{HF}\rangle = q^2\beta_2 \ , \qquad\qquad \langle k^4 G_{HF}\rangle = q^4\beta_5 \ ,$$

$$\langle kY_{1,0}G_{HF}\rangle = q\sqrt{\tfrac{3}{4\pi}}\beta_1 \ , \qquad \langle k^3 Y_{1,0}G_{HF}\rangle = q^3\sqrt{\tfrac{3}{4\pi}}\beta_4 \ ,$$

$$\langle k^2|Y_{1,0}|^2 G_{HF}\rangle = q^2\tfrac{3}{4\pi}\beta_3 \ , \qquad \langle k^2|Y_{1,1}|^2 G_{HF}\rangle = q^2\tfrac{3}{8\pi}\left(\beta_2 - \beta_3\right) \ .$$

Let first consider the $S = 0$ channel. With the p-h interaction given by Eqs. (5-6), the Bethe-Salpeter equation is

written as

$$
\begin{aligned}
G_{RPA}^{(0)}(\mathbf{k}_1) =\ & G_{HF}(\mathbf{k}_1) + W_1^{(0)} G_{HF}(\mathbf{k}_1)\langle G_{RPA}^{(0)}\rangle + W_2^{(0)} \mathrm{k}_1^2 G_{HF}(\mathbf{k}_1)\langle G_{RPA}^{(0)}\rangle \\
& + W_2^{(0)} G_{HF}(\mathbf{k}_1)\langle \mathrm{k}^2 G_{RPA}^{(0)}\rangle - 2W_2^{(0)}\frac{4\pi}{3}\sum_\mu \mathrm{k}_1 Y_{1\mu}^*(\hat{1}) G_{HF}(\mathbf{k}_1)\langle \mathrm{k}Y_{1\mu} G_{RPA}^{(0)}\rangle \\
& - 2\sqrt{\frac{4\pi}{3}} W_{so}\,\mathrm{q}\sum_{M_S'} M_S'\,\mathrm{k}_1 Y_{1M_S'}(\hat{1}) G_{HF}(\mathbf{k}_1)\langle G_{RPA}^{(1,M_S')}\rangle \\
& + 2\sqrt{\frac{4\pi}{3}} W_{so}\,\mathrm{q}\sum_{M_S'} M_S'\, G_{HF}(\mathbf{k}_1)\langle \mathrm{k}Y_{1M_S'} G_{RPA}^{(1,M_S')}\rangle\ .
\end{aligned}
\tag{A1}
$$

Integrating over $\mathbf{k}_1$ we get

$$
\begin{aligned}
\langle G_{RPA}^{(0)}\rangle =\ & \beta_0 + W_1^{(0)}\beta_0\langle G_{RPA}^{(0)}\rangle + W_2^{(0)}\mathrm{q}^2\beta_2\langle G_{RPA}^{(0)}\rangle + W_2^{(0)}\beta_0\langle \mathrm{k}^2 G_{RPA}^{(0)}\rangle - 2W_2^{(0)}\sqrt{\frac{4\pi}{3}}\mathrm{q}\beta_1\langle \mathrm{k}Y_{10} G_{RPA}^{(0)}\rangle \\
& + 2\sqrt{\frac{4\pi}{3}} W_{so}\,\mathrm{q}\,\beta_0\sum_{M_S'} M_S'\,\langle \mathrm{k}Y_{1M_S'} G_{RPA}^{(1,M_S')}\rangle\ .
\end{aligned}
\tag{A2}
$$

One can see that the quantity $\langle G_{RPA}^{(0)}\rangle$ we are interested in is coupled to $\langle \mathrm{k}^2 G_{RPA}^{(0)}\rangle$, $\langle \mathrm{k}Y_{10} G_{RPA}^{(0)}\rangle$, and $\langle \mathrm{k}Y_{1M_S'} G_{RPA}^{(1,M_S')}\rangle$. Two new equations are obtained multiplying Eq. (A1) with k_1^2 and $\mathrm{k}_1 Y_{10}(\hat{k}_1)$, and integrating over $\mathbf{k}_1$. These factors are such that there is no contribution from the term $\langle G_{RPA}^{(1,M_S')}\rangle$ entering Eq. (A1). The coupling with the $S=1$ channel is thus contained in the last term entering Eq. (A2). From the Bethe-Salpeter equation for the $S=1$ channel the following expression is obtained

$$
\sum_{M_S'}\langle M_S'\mathrm{k}Y_{1,M_S'} G_{RPA}^{(1,M_S)}\rangle = 2\sqrt{\frac{3}{4\pi}} W_{so}\mathrm{q}^3\frac{\beta_2-\beta_3}{1+W_2^{(1)}\mathrm{q}^2(\beta_2-\beta_3)}\langle G_{RPA}^{(0)}\rangle\ .
\tag{A3}
$$

This means that the effect of the spin-coupling can be simply absorbed in an effective $\widetilde{W}_1^{(0)}$ coefficient. The equations for $S=1$ channel are obtained proceeding along similar lines. The explicit expression of $\widetilde{W}_1^{(\alpha)}$ is given in Eq.(9).

Finally, the system of algebraic equations can be written in a compact form for both channels as

$$
\left(1 - \widetilde{W}_1^{(\alpha)}\beta_0 - W_2^{(\alpha)}\mathrm{q}^2\beta_2\right)\langle G_{RPA}^{(\alpha)}\rangle - W_2^{(\alpha)}\beta_0\langle \mathrm{k}^2 G_{RPA}^{(\alpha)}\rangle + 2W_2^{(\alpha)}\mathrm{q}\beta_1\sqrt{\frac{4\pi}{3}}\langle \mathrm{k}Y_{10}(\hat{k}) G_{RPA}^{(\alpha)}\rangle = \beta_0
$$

$$
-\left(\widetilde{W}_1^{(\alpha)}\mathrm{q}^2\beta_2 + W_2^{(\alpha)}\mathrm{q}^4\beta_5\right)\langle G_{RPA}^{(\alpha)}\rangle + \left(1 - W_2^{(\alpha)}\mathrm{q}^2\beta_2\right)\langle \mathrm{k}^2 G_{RPA}^{(\alpha)}\rangle + 2W_2^{(\alpha)}\mathrm{q}^3\beta_4\sqrt{\frac{4\pi}{3}}\langle \mathrm{k}Y_{10}(\hat{k}) G_{RPA}^{(\alpha)}\rangle = \mathrm{q}^2\beta_2
$$

$$
-\left(\widetilde{W}_1^{(\alpha)}\mathrm{q}\beta_1 + W_2^{(\alpha)}\mathrm{q}^3\beta_4\right)\langle G_{RPA}^{(\alpha)}\rangle - W_2^{(\alpha)}\mathrm{q}\beta_1\langle \mathrm{k}^2 G_{RPA}^{(\alpha)}\rangle + \left(1 + 2W_2^{(\alpha)}\mathrm{q}^2\beta_3\right)\sqrt{\frac{4\pi}{3}}\langle \mathrm{k}Y_{10}(\hat{k}) G_{RPA}^{(\alpha)}\rangle = \mathrm{q}\beta_1
$$

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