Interplay between QCD and nuclear responses

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Abstract

We establish the interrelation between the QCD scalar response of the nuclear medium and its response to a scalar probe coupled to nucleons, such as the scalar meson responsible for the nuclear binding. The relation that we derive applies at the nucleonic as well as at the nuclear levels. Non trivial consequences follow. One concerns the scalar QCD susceptibility of the nucleon. The other opens the possibility of relating medium effects in the scalar meson exchange of nuclear physics to QCD lattice studies of the nucleon mass.

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1 Introduction

The spectrum of scalar-isoscalar excitations is quite different in the vacuum and in the nuclear medium. In the second case it includes low lying nuclear excitations and also two quasi-pion states *i.e.* pions dressed by particle hole excitations. All hese lie at lower energies than the vacuum scalar excitations which start at $2\,m_\pi$. We have shown in previous works [1, 2, 3, 4] that this produces a large increase of the magnitude of the scalar QCD susceptibility over its vacuum value. We have expressed the origin of this increase as arising from the mixing of the nuclear response to a scalar probe coupled to nucleonic scalar density fluctuations into the QCD scalar response.

It is natural to investigate also the reciprocal problem of the influence of the QCD scalar response to a probe which couples to the quark density fluctuations on the ordinary nuclear scalar response of nuclear physics, which is the object of the present work. We will study this influence not only for what concerns the nuclear excitations but also for a single nucleon for which only nucleonic excitations are involved. If this influence indeed exists, does it lead to non-trivial observable consequences? We will show that this is the case, with two main applications. One concerns the QCD scalar susceptibility of a single nucleon. The second is the possibility to infer medium effects in the propagation of the scalar meson which binds the nucleus from QCD results, such as the lattice ones on the evolution of the nucleon mass with the pion mass.

Our article is organized as follows. In section **2** we remind the mechanisms responsible for the mixing of the nuclear response into the QCD scalar susceptibility. We illustrate it in the framework of a nuclear chiral model with a scalar and vector meson exchange. We show that this mutual influence also exists at the nucleonic level. In section **3** we discuss the influence of the quark structure of the nucleon on the scalar response of nuclear physics in a general framework which is able to incorporate also confinement effects.

2 Mutual influence of the scalar QCD response and nuclear physics response

2.1 Study in a nuclear chiral model

We first remind how the usual nuclear physics response to a scalar field enters in the QCD susceptibility. For this, following ref. [2], we start from the expression of the modification of the quark condensate in the nuclear medium, $\Delta\langle\bar{q}q\rangle(\rho) = \langle\bar{q}q\rangle(\rho) - \langle\bar{q}q\rangle_{vac}$. We first use, as in [2], its expression for a collection of independent nucleons :

$$\Delta\langle\bar{q}q\rangle(\rho) = Q_S\,\rho_S. \tag{1}$$

where ρ_S is the scalar density of nucleons related to the chemical potential μ by :

$$\rho_S = 4\int \frac{d\vec{p}}{(2\pi)^3}\,\frac{M}{E_p}\,\Theta(\mu - E_p). \tag{2}$$

We have introduced the scalar charge of the nucleon, Q_S, volume integral of the nucleon scalar density of quarks. It is related to the nucleon sigma commutator σ_N and the current quark mass by :

$$Q_S == \frac{\sigma_N}{2m_q} = \int d\vec{r}\langle N|\bar{q}q(\vec{r}) - \langle\bar{q}q\rangle_{vac}|N\rangle. \tag{3}$$

The susceptibility of the nuclear medium χ_S^A is the derivative of the quark scalar density with respect to the quark mass. We define it in such a way that it represents a purely nuclear contribution with the vacuum susceptibility substracted off :

$$\chi_S^A = \left(\frac{\partial\Delta\langle\bar{q}q(\rho)}{\partial m_q}\right)_\mu = \left(\frac{\partial(Q_S\,\rho_S)}{\partial m_q}\right)_\mu. \tag{4}$$

It contains two terms. One arises from the derivative of Q_S, which by definition is the free nucleon QCD scalar susceptibility, $\chi_S^N = \partial Q_S/\partial m_q$. The second one involves the derivative of the nucleon density. This term itself is built of two pieces, one involves antinucleon excitations and is small [2]. The other one, which is larger, involves the nuclear response $\Pi_0 = -2M_N p_F/\pi^2$. In this case it is the free Fermi gas one since no interactions between nucleons have been introduced. The result of this derivation is contained in the following equation:

$$\chi_S^A = \rho_S\,\chi_N^S\,+\,2\,Q_S^2\,\Pi_0\,. \tag{5}$$

The nuclear susceptibility is thus the sum of a one body term and of a term due the nuclear excited states, the p-h excitations. This decomposition survives the introduction of the interaction, as will be shown next. In this case the free p-h polarization propagator is replaced by the full RPA one, while the free nucleon susceptibility can undergo medium modifications and become dependent on the density.
The previous result has been generalized in [3] to an assembly of nucleons interacting through a scalar and a vector meson exchange, working at the mean field level as in

relativistic mean field theories. In this work the condensate was obtained as the derivative of the grand potential with respect to the quark mass (Feynman-Hellman theorem) and the susceptibility as the derivative of the condensate, taken at constant chemical potential. The result is [3] :

$$\chi_S = \left(\frac{\partial\langle\bar{q}q\rangle}{\partial m_q}\right)_\mu \simeq -2\,\frac{\langle\bar{q}q\rangle^2_{vac}}{f_\pi^2}\,\left(\frac{\partial\bar{S}}{\partial c}\right)_\mu . \tag{6}$$

$\bar{S} \equiv f_\pi\, +\, \bar{s}$ is the expectation value of the chiral invariant scalar field and $c = f_\pi\, m_\pi^2$ is the symmetry breaking parameter of the model used in [3]. The quantity $\left(\partial\bar{S}/\partial c\right)_\mu$ is related to the in-medium sigma propagator :

$$\left(\frac{\partial\bar{S}}{\partial c}\right)_\mu = -D^*_\sigma = \frac{1}{m^{*2}_\sigma}\, -\, \frac{g_S^2}{m^{*2}_\sigma}\,\Pi_S(0)\,\frac{1}{m^{*2}_\sigma} \tag{7}$$

where $\Pi_S(0)$ is the full scalar polarization propagator, related to the bare one, Π_0 by :

$$\Pi_S(0) = \frac{M^*_N}{E^*_F}\,\Pi_0(0)\,\left[1 - \left(\frac{g_\omega^2}{m_\omega^2}\,\frac{E^*_F}{M^*_N}\, -\, \frac{g_S^{*2}}{m^{*2}_\sigma}\,\frac{M^*_N}{E^*_F}\right)\Pi_0(0)\right]^{-1} . \tag{8}$$

In the equations above, m^*_σ is the in-medium sigma mass, which is obtained from the second derivative of the energy density with respect to the order parameter :

$$m^{*2}_\sigma = \frac{\partial^2\varepsilon}{\partial\bar{s}^2} = V''(\bar{s})\, +\, \frac{\partial\,(g_S\,\rho_S)}{\partial\bar{s}} = m_\sigma^2\left(1\, +\, \frac{3\bar{s}}{f_\pi}\, +\, \frac{3}{2}\left(\frac{\bar{s}}{f_\pi}\right)^2\right) \tag{9}$$

where the potential V responsible for the spontaneous symmetry breaking is the standard quartic one of the linear sigma model. In the last expression we omit as in [3] the small antinucleon contribution. Moreover we do not consider, contrary to [3], the scalar response of the nucleon due to confinement, (*i.e.*, we ignore the medium renormalization of g_S). The mean scalar field $\bar{s}$ being negative, the term linear in $\bar{s}$ lowers the sigma mass by an appreciable amount ($\simeq$ 30 % at ρ_0). This is the chiral dropping associated with chiral restoration [8] and arising from the 3σ interaction as depicted in fig 1.

Since we are interested only in the medium effects the vacuum value of the quantity $\left(\partial\bar{S}/\partial c\right)_\mu = 1/{m_\sigma}^2$ has to be subtracted off in eq. 7 and the purely nuclear suceptibility, χ_S^A, writes :

$$\chi_S^A\; = 2\,\frac{\langle\bar{q}q\rangle^2_{vac}}{f_\pi^2}\left[\frac{3\,\bar{s}/f_\pi\, +\, \frac{3}{2}\,(\bar{s}/f_\pi)^2}{m^{*2}_\sigma}\, +\, \frac{g_S^2}{m^{*2}_\sigma}\,\Pi_S(0)\,\frac{1}{m^{*2}_\sigma}\right] . \tag{10}$$

We see that χ_S^A receives two types of contributions, the second being proportionnal to the full RPA scalar response Π_S. The corresponding proportionality factor r between this second contribution and Π_S writes, to leading order, *i.e.*, neglecting the medium modification of the sigma mass :

$$r = 2\,g_S^2\,\frac{\langle\bar{q}q\rangle^2_{vac}}{f_\pi^2\,m_\sigma^4} \simeq 2\,(Q_S^s)^2 \tag{11}$$

where we have introduced the nucleon scalar charge Q_S^s from the scalar field, defined below. In the sigma model the free nucleon sigma commutator is the sum of two contributions, one arising from the pion cloud, which depends on the mean value of the squared pion field, *i.e.*, on the scalar number of pions in the nucleonic cloud. The other one, Q_S^s, arises from the scalar meson [5, 6, 7] and is linear in the σ field :

$$Q_S^s = \frac{\sigma_N^s}{2m_q} = -\frac{\langle\bar{q}q\rangle_{vac}}{f_\pi}\int d^3x\langle N|\sigma(x)|N\rangle = -\frac{\langle\bar{q}q\rangle_{vac}}{f_\pi}\frac{g_S}{m_\sigma^2} \tag{12}$$

which establishes relation 11 if we ignore the in-medium modification of Q_S^s, *i.e.*, the difference beween m_σ^* and m_σ.

We now turn to the first part of χ_S^A which depends on the average scalar field $\bar{s}$. In the low density limit, $\bar{s}$ reduces to $\bar{s} = -g_S\,\rho_S/m_\sigma^2$, and we can ignore the term in $\bar{s}^2$ as well as the difference beween m_σ^* and m_σ. In this limit the first term in the expression (10) of χ_S^A is linear in the density. If we classify it in the decomposition of eq. 5 for χ_S^A, it obviously belongs to the individual nucleon contribution, $\rho_S\,\chi_S^N$, to the nuclear susceptibility. Writing the linear term explicitly in eq. 10 we deduce the free nucleon scalar susceptibility from the scalar field, $(\chi_S^N)^s$:

$$(\chi_S^N)^s \;=\; -2\,\frac{\langle\bar{q}q\rangle_{vac}^2}{f_\pi^3}\,\frac{3\,g_S}{m_\sigma^4}, \tag{13}$$

which is negative (*i.e*, similar to paramagnetism). It has been obtained here from the low density expression of χ_S^A. In fact it can also be obtained directly as the derivative with respect to the quark mass of Q_S^s, the part of the nucleon scalar charge originating in the scalar field written in eq. 12 :

$$(\chi_S^N)^s \;=\frac{\partial Q_S^s}{\partial m_q} = \frac{\partial}{\partial m_q}\left(-\,\frac{\langle\bar{q}q\rangle_{vac}}{f_\pi}\,\frac{g_S}{m_\sigma^2}\right). \tag{14}$$

Using the fact that, in the model, $\langle\bar{q}q\rangle_{vac}/f_\pi$ does not depend on m_q, only the derivative of the sigma mass with respect to m_q enters which, according to the Feynman-Hellmann theorem, is linked to the sigma commutator, σ_σ, of the σ. In the linear sigma model the derivative with respect to the quark mass is replaced by the derivative with respect to the symmetry breaking parameter, $c = f_\pi\,m_\pi^2$, keeping the other original parameters of the model, λ and v, constant. The result is :

$$\sigma_\sigma = m_q\,\frac{\partial m_\sigma}{\partial m_q} = \frac{3}{2}\,\frac{m_\pi^2}{m_\sigma}. \tag{15}$$

When inserted in eq. 14, it leads for $(\chi_S^N)^s$ to the expression of eq. 13.

We now turn to the scattering amplitude for the sigma meson on the nuclear system. In the same framework we will first show that the amplitude for the scattering of the scalar meson on the nucleon has the same relation to the nucleonic susceptibility as the case for the nuclear excitation part. Indeed in the expression (9) of m_σ^{*2} the term linear in density is obtained from the low density expression : $3\,\bar{s}\,m_\sigma^{*2} \simeq -(3g_S/f_\pi)\,\rho_S$. It represents an

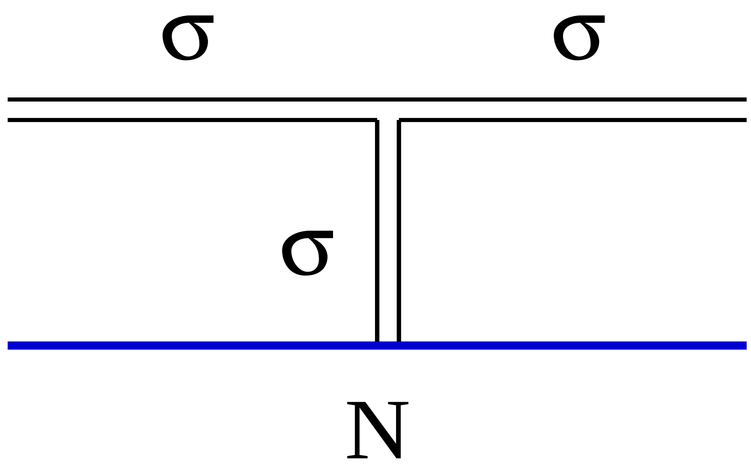

Figure 1: Contribution to the sigma-nucleon scattering amplitude responsible for the lowering of the sigma mass in the medium.

optical potential for the sigma propagation. The corresponding σN scattering amplitude, T_{σ_N}, which can also be evaluated directly from the graph of fig. 1, is equal to :

$$T_{\sigma N} = -3 g_S / f_\pi \tag{16}$$

We are now in a situation to relate the nucleon scalar susceptibility (eq. 13) to the sigma-nucleon amplitude of eq. 16, with the result :

$$(\chi_S^N)^s = \frac{2\,(Q_S^s)^2}{g_S^2}\, T_{\sigma N} \tag{17}$$

The proportionality factor, $2\,(Q_S^s)^2/g_S^2$, is the same as for the purely nuclear excitations. The quantity g_S which appears in this factor factor is due to the σNN coupling constant. Adding the two effects from the nucleonic and nuclear excitations the total QCD scalar susceptibility of the nuclear medium (vacuum value substracted) can therefore be related to the total response to the scalar field through :

$$\chi_S^A = \frac{2\,(Q_S^s)^2}{g_S^2} T^A \tag{18}$$

where the two members include both the individual nucleon contribution and the one arising from the nuclear excitations. We have in particular :

$$T_A = \rho_S\, T_{\sigma N} \,+\, g_S^2\, \Pi_{SS}. \tag{19}$$

The last term on the r.h.s. represents the (in-medium corrected) Born part of the σN amplitude while the first piece represents the non-Born part linked to nucleonic excitations.

Thus there exists a universal scaling factor between the responses of a nuclear or nucleonic system to probes which couple either to the nucleon scalar density fluctuations or to the quark ones. This relation has allowed us to infer the existence of a contribution to the QCD nucleon scalar susceptibility linked to the scalar meson. To the best of our knowledge this contribution to the nucleon susceptibility has not been discussed previously. It has a link, through the relation 17, to the optical potential for the σ propagation, which reduces the sigma mass in the medium.

2.2 Effect of the two pion propagator

In order to illustrate the coherence of this approach we will now extend the previous description to incorporate the effect of the two-pion propagator, G, which affects the nucleon susceptibility in the following way. The sigma propagator is renormalized by its coupling to two pion states, as discussed in [4]. At zero four-momentum we have :

$$-D_\sigma = \frac{1}{m_\sigma^2 \,+\, 3\,\lambda\,(m_\sigma^2 - m_\pi^2)\,\frac{G}{1-3\lambda G}} = \frac{1 - 3\,\lambda\,G}{m_\sigma^2 \,-\, 3\lambda\,m_\pi^2\,G} \simeq \frac{1}{m_\sigma^2} - \frac{3\,\lambda\,G}{m_\sigma^2} \tag{20}$$

where both D_σ and G are taken at zero four-momentum. In the last term we have restricted to the one pion loop level. We stress that this expression only holds for the sigma, chiral partner of the pion, which is not a chiral invariant field. It does not apply to the scalar field responsible for the nuclear binding which has to be a scalar invariant (that we have denoted s) and which is weakly coupled to two-pion states, while the σ is strongly coupled. Therefore this treatment is done for illustration purpose and not for an application to nuclear physics.

The medium correction to D_σ from the coupling of the σ to 2π states is :

$$\Delta D_\sigma \;=\; \frac{3\,\Delta G}{2\,f_\pi^2}, \tag{21}$$

where ΔG is the in-medium modification of the two-pion propagator. In ΔG, to lowest order, one and only one of the two pions of the two pion propagator has to be dressed by one p-h bubble. It is again possible to interpret the corresponding modification of the sigma propagator as representing a σN scattering amplitude, $T_{\sigma N}^{\pi}$, in which the sigma interacts with the nucleon pion cloud :

$$T_{\sigma N}^{\pi} = \frac{3\,m_\sigma^4}{2\,f_\pi^2}\,\frac{\Delta G}{\rho_S}. \tag{22}$$

This is to be compared to the nucleon scalar susceptibility from the pion cloud, which is [4] :

$$\chi_S^N = \frac{3\,\Delta G}{\rho_S}\,\frac{\langle\bar{q}q\rangle_{vac}^2}{f_\pi^4}. \tag{23}$$

The proportionality factor between the susceptibility (23) and the σN scattering amplitude (22) is the same as previously, $2\,Q_S^{s2}/g_S^2$. We find again that this relation holds not only at the level of p-h excitations but also for a single nucleon, at the level of the nucleonic excitations which in this specific case are of the pionic type.
In summary we have seen that the mixing between the quark density fluctuations and the

nucleon ones implies that the response of a probe which couples to the nucleonic density fluctuation is proportional to the QCD scalar response. This includes also the nucleonic contribution to these responses. As an exemple we have shown that the chiral dropping of the sigma mass in the medium has a counterpart in the form of a contribution of the scalar meson to the QCD scalar susceptibility of the nucleon.

3 Connection with lattice data

It is now interesting to connect our results to the available lattice simulations of the evolution of the nucleon mass with the pion mass, equivalently the quark mass. At present they do not cover the physical region but only the region beyond $m_\pi \simeq 400\,MeV$. The derivative $\partial M_N/\partial m_\pi^2 = \sigma_N/m_\pi^2$ provides the nucleon sigma commutator. In turn the derivative of σ_N leads to the susceptibility. Both quantities are strongly influenced by the pion cloud contribution which has a non-analytic behavior in the quark mass, preventing a polynomial expansion in this quantity. However the pionic self-energy contribution to the nucleon mass, Σ_π, has been separated out in [9] in a model dependent way with different cut-off forms for the pion loops (gaussian, dipole, monopole) with an adjustable parameter Λ. The remaining part is expanded in terms of m_π^2 as follows:

$$M_N(m_\pi^2) = a_0 + a_2\, m_\pi^2 + a_4\, m_\pi^4 + \Sigma_\pi(m_\pi). \tag{24}$$

The best fit value of the parameter a_4 which fixes the susceptibility shows little sensitivity to the shape of the form factor, with a value $a_4 \simeq -0.5\,GeV^{-3}$ while $a_2 \simeq 1.5\,GeV^{-1}$ (in a previous work [10] smaller values of a_2 and a_4 were given : $a_2 \simeq 1\,GeV^{-1}$ and $a_4 \simeq -0.23\,GeV^{-3}$). We can infer the non-pionic pieces of the sigma commutator and of the susceptibility from the expansion (24) :

$$\sigma_N^{non-pion} = m_\pi^2 \frac{\partial M}{\partial m_\pi^2} = a_2\, m_\pi^2 + 2\, a_4\, m_\pi^4 \simeq 29\,MeV\,. \tag{25}$$

It is largely dominated by the a_2 term. Notice that the value for $a_2 \simeq 1\,GeV^{-1}$ is $\sigma_N^{non-pion} = 20\,MeV$.
In turn the nucleon susceptibility is :

$$\chi_S^{N,non-pion} = 2\,\frac{\langle\bar{q}q\rangle_{vac}^2}{f_\pi^4}\frac{\partial}{\partial m_\pi^2}\left(\frac{\sigma_N^{non-pion}}{m_\pi^2}\right) = \frac{\langle\bar{q}q\rangle_{vac}^2}{f_\pi^4}\,4\,a_4 \simeq -5.4\,GeV^{-1} \tag{26}$$

The non-pionic susceptibility is found with a negative sign, as expected from the scalar meson term. In ref. [9] however, the negative sign is interpreted differently. It is attributed to deviations from the Gellman-Oakes-Renner (GOR) relation which links quark and pion masses. These deviations make the expansion of eq. 24 compatible with a positive value of the non-pionic susceptibility in spite of the negative sign of a_4. Here we take another attitude and stick to the usual GOR relation. The negative sign of a_4 then directly reflects that of the non-pionic suceptibility.

It is then interesting to test if the empirical values from the lattice are compatible with a pure scalar meson contribution. We thus tentatively make the following identifications :

$$Q_S^s \;=\; \frac{\langle\bar{q}q\rangle_{vac}}{f_\pi}\,\frac{g_S}{m_\sigma^2} \;=\; \frac{\sigma_N^{non-pion}}{(2\,m_q)} \simeq 2.4, \tag{27}$$

where the numerical value corresponds to $2\,m_q = 12\,MeV$ (taking $a_2 \simeq 1\,GeV^{-1}$ one would get $Q_S^s = 1.66$). It is interesting to translate this number into the value of the mean scalar field in the nuclear medium which, to leading order in density, is :

$$\bar{s} = \frac{g_s\,\rho_S}{m_\sigma^2} = \frac{Q_S^s\,f_\pi\,\rho_S}{\langle\bar{q}q\rangle_{vac}} = \frac{\sigma_N^{non-pion}}{(2\,m_q)}\,\frac{f_\pi\,\rho_S}{\langle\bar{q}q\rangle_{vac}} = \frac{a_2\,+\,a_4\,m_\pi^2}{f_\pi}\,\rho_S \tag{28}$$

At normal density the value $\bar{s}(\rho_0) \simeq 21\,MeV$, quite compatible with nuclear phenomenology. The second identification concerns the susceptibility. If the non pionic susceptibility would arise entirely from the scalar field, we should have :

$$\chi_S^{N,non-pion} \;=\; -\,\frac{2\,(Q_S^s)^2}{g_S^2}\,\frac{3\,g_S}{f_\pi} = \frac{\langle\bar{q}q\rangle_{vac}^2}{f_\pi^4}\,4\,a_4 \tag{29}$$

which would give, using the GOR relation :

$$-a_4 = \frac{3}{2}\,\frac{(\sigma_N^{non-pion})^2}{g_S\,f_\pi\,{m_\pi}^4} = 3.1\,GeV^{-3}, \tag{30}$$

much larger than the lattice value, $-a_4 = 0.5\,\mathrm{GeV}^{-3}$. Again, taking $a_2 \simeq 1\,GeV^{-1}$ one would get $-a_4 \simeq 1.2\,GeV^{-3}$, still larger than the empirical lattice value. Thus the linear σ model which fails to account for the saturation properties, due to the excess attraction produced by the chiral softening of the sigma mass, also leads to too large a susceptibility from the scalar meson. In fact in this work we show that the two problems are linked. Some mechanism suppresses the chiral softening of the σ mass as well as the large nucleonic susceptibility from the scalar meson, incompatible with lattice data. It is indeed likely that the scalar meson is not the only non-pionic contribution. In a previous work [3] we have invoked confinement and the quark meson coupling model (QMC) [11, 12] as a source of cancellation for the chiral softening of the sigma mass. It turns out that it has also a cancelling effect in the nucleon scalar susceptibility. Indeed, for 3 valence quarks confined in a bag of radius R, Guichon [13] derived $\chi_S^{N,bag} \simeq +0.25\,R \simeq 1\,GeV^{-1}$, for a value $R = 0.8\,fm$. Translated into the parameter a_4, one has $a_4^{bag} \simeq +0.1\,GeV^{-3}$. Contrary to the other components which are negative (of paramagnetic nature), it has a positive sign (of the diamagnetic type, linked to quark-antiquark excitations). The bag susceptibility indeed produces a mild cancellation effect.

It is then natural to try to extend the linear sigma model description so as to incorporate other effects than the scalar meson ones (or chiral symmetry breaking effects), for instance those arising from confined valence quarks.

4 Generalization and implications for nuclear physics

In view of the limitations of the linear sigma model discussed previously, a more general approach is desirable. The aim is to link the response of the nuclear medium to the scalar nuclear field and the QCD responses, in such a way that both quantitites include all the components of the individual nucleon contribution whatever their origin. Of course it is not possible to achieve this goal without some assumptions on the nature of the probe. We keep the basic assumption that the scalar field which couples to the nucleons, couples to the quarks of the nucleon condensate, as is the case in the linear sigma model. Thus its presence can induce a readjustement of the quark structure of the nucleon, that we evaluate in the way described below.

Consider a nuclear medium with a scalar nucleon density ρ_S. By definition the response of this medium to a scalar field which couples to the nucleon scalar density fluctuations (with a unit coupling constant) is the change in the nucleon scalar density for a small change of the nucleon mass. It is $\Pi_S = (\partial\rho_S/\partial M_N)_\mu$, the derivative being taken at constant chemical potential. With a coupling constant g_S this result should be multiplied by g_S^2. In the free Fermi gas case this derivative leads the the quantity $-2\,Mp_F/\pi^2$, the free Fermi gas response. For nucleons interacting via σ and ω exchange, the expression of the scalar nucleon density is :

$$\rho_S = \int \frac{4\,d^3p}{(2\pi)^3}\,\frac{M_N^*}{E_p^*}\,\Theta\left(\mu\,-\,E_p^*\,-\,\frac{g_\omega^2}{m_\omega^2}\,\rho\right). \tag{31}$$

where $M_N^* \;=\; M_N\,(1+\bar{s}/f_\pi)$ is the nucleon effective mass, linked to the mean scalar field $\bar{s}$ and $E_p^* \;=\; \sqrt{p^2\;+\;M_N^{*2}}$. The mean field $\bar{s}$ is obtained from the minimization equation of the energy density ϵ :

$$\frac{\partial\varepsilon}{\partial\bar{s}} = g_S\,\rho_S\,+\,V'(\bar{s}) = 0\,. \tag{32}$$

It is then possible to check that the derivative of the scalar nucleon density with respect to the nucleon mass leads to the full RPA scalar polarization propagator, Π_S, as defined in eq. 8. In this expression of the response as the derivative of the nucleon density the nucleon structure is not incorporated. It only includes the effect of the nuclear excitations and not that of the nucleonic ones. In order to include them we have to account for the internal nucleon structure, *i.e.*, the quark structure. It is the quark medium, and not only the nucleon one, which responds to the same excitation, *i.e.*, to the modification of the nucleon mass δM_N. Accordingly we make the following conjecture, writing the full response $\mathcal{R}_S^A$ as :

$$\mathcal{R}_S^A = \frac{1}{2\,Q_S}\left(\frac{\partial\rho_S^q}{\partial M_N}\right)_\mu \tag{33}$$

where ρ_S^q is the quark scalar density and the factor $1/2\,Q_S$ in front of the derivative is put for normalization purpose. Each nucleon containing a scalar number of quarks $2\,Q_S = \sigma_N/m_q$, the scalar density of quarks is $\rho_S^q = 2\,Q_S\,\rho_S$. The derivative involves two terms :

$$\mathcal{R}_S^A = \frac{1}{Q_S}\left(\frac{\partial}{\partial M_N}(Q_S\,\rho_S)\right)_\mu = \left(\frac{\partial\rho_S}{\partial M_N}\right)_\mu + \frac{\rho_S}{2\,Q_S^2}\,\frac{\partial Q_S}{\partial m_q}. \tag{34}$$

In the last term we have replaced the derivative with respect to the nucleon mass by the one with respect to the quark mass, with $\partial M_N/\partial m_q = 2\,Q_S$, which introduces the nucleon susceptibility χ_S^N. The overall result writes :

$$\mathcal{R}_S^A \;=\; \frac{\chi_S^N}{2\,Q_S^2}\,\rho_S \,+\, \Pi_S. \tag{35}$$

The interpretation of this equation is clear. This decomposition is obvious and analogous to the one of eq. 5. The term linear indensity represents the individual nucleon response from the nucleonic excitations. While the term in,Π_S embodies nuclear excitations. The new information is that the single nucleon response is proportional to the QCD one, χ_S^N, with the same proportionality factor $1/(2Q_S^2)$, as was found previously for the nuclear excitations. All in all, the eq. 35 writes :

$$\mathcal{R}_S^A \;=\; \frac{1}{2\,Q_S^2}\,\chi_S^A \tag{36}$$

where χ_S^A represents the total scalar QCD susceptibility of the nuclear medium (vacuum value substracted) and both members incorporate the individual nucleon contribution. This results holds for a unit coupling constant. For a coupling constant g_S (as is the case for the nuclear scalar field) the r.h.s should be multiplied by g_S^2. Accordingly the corresponding σN amplitude is :

$$T_{\sigma N} = \frac{\chi_S^N\,g_S^2}{2\,Q_S^2}. \tag{37}$$

We will now comment this result and we then will apply it to the problem of the propagation of the scalar field which mediates the nuclear attraction. Our relation 36 has a close resemblance to the previous one, 18, derived in the linear sigma model but here we do not inquire about the origin of the terms, χ_S^N and Q_S. With the values of the linear sigma model for these quantities we recover the previous result of this model. However at this stage we have to stress again the difference between the response to the chiral invariant s field responsible for the nuclear binding and the one to the σ (chiral partner of the pion) scalar field. At zero momentum the s field is weakly coupled to pions while the σ has a strong coupling to two-pion states. Thus the pionic pieces which appear in the susceptibility and thus show up in the response to the σ are eliminated from the response to the s field and only non-pionic quantities enter.

Our relation 37 is also very similar to the one of the quark-meson coupling model [11]. In QMC, the bag positive susceptibility manifests itself in the form of a repulsive interaction in the propagation of the scalar nuclear field. The corresponnding scattering amplitude is related to the bag suceptibility by a relation identical to our eq. 37, but only bag quantities appear and the scalar charge entering this relation is that of the bag, which is $Q_S^{bag} \simeq 0.7$. As QMC does not incorporate the chiral potential which implies the three-scalar coupling, only the repulsive three-body interaction from the bag structure enters. Our expression 37 thus covers the two extreme situations, when the nucleon mass originates totally from the condensate or when it is only due to confinement. It is legitimate to believe that it is able to describe a more general situation with a mixed origin.

We now turn to the quantitative applications. Since both the total (non-pionic) nucleonic susceptibility and scalar charge enter in the expression of the (chiral invariant) scalar-nucleon scattering amplitude, it is legitimate to use for these two quantities the phenomenological values obtained from the lattice data. We can therefore infer the medium effects in the propagation of the s field from the lattice results of eq 25 and 26 as :

$$-D_s^{-1} = m_\sigma^2 \,+\, \frac{g_S^2}{2\,Q_S^2}\,\chi_S^N\,\rho_S = m_\sigma^2 \,+\, g_S^2\,\frac{2\,a_4}{(a_2 \,+\, 2\,a_4\,m_\pi^2)^2}\,\rho_S \tag{38}$$

where in the first equation only non-pionic quantities enter, hence the introduction of the parameters a_2 and a_4 which have been defined in the lattice expansion. Numerically, at normal nuclear density, and for a value of the coupling constant $g_S = 10$, the second term on the rhs of the second equation takes the value $0.06\,GeV^2$ (a similar value is found for the other set of parameters a_2 and a_4). For a sigma mass of $m_\sigma = 0.75 GeV$, this represents only a 6% decrease of the mass, much less that the chiral dropping alone. Such a conclusion is in much better agreement with the nuclear phenomenology. A slight reduction of the sigma mass at normal density is indeed compatible with the saturation properties, binding energy and compressibility of nuclear matter [2].

5 Conclusion

In summary we have studied in this work the interplay between the nuclear responses to probes which couple either to nucleon or to quark scalar density fluctuations. We have found that the two responses are closely related, being proportional to each other. The scaling coefficient involves the nucleon scalar charge. Our result holds not only at the level of the nuclear excitations but also at the one of the nucleonic ones. It therefore applies to a single nucleon : the scalar response of a nucleon (for instance to the nuclear scalar field) is proportional to its QCD scalar susceptibility. Thus both responses incorporate the individual nucleon contributions.

One application of this relation concerns a free nucleon. In a chiral theory which implies the existence of a 3σ coupling, the corresponding σN amplitude (due to the scattering on the nucleon sigma field) has a counterpart in the QCD scalar susceptibility in the form of a negative (of the paramagnetic type) contribution. We have tested its existence in the lattice results on the nucleon mass evolution with the pion mass, as analyzed by [9]. The expansion of ref. [9] is indeed compatible with a negative component for the non-pionic susceptibility. However the magnitude does not fit, indicating the existence of other components, with a cancelling effect. In fact the same cancellation exists in the scalar response of the nucleon. The 3σ coupling is responsible for a chiral decrease of the σ mass, which produces too much attraction at large densities and is totally incompatible with the saturation properties of nuclear matter. In the optics of the present work, the two effects are related. The full σN scattering amplitude being proportional to the susceptibility, a cancelling effect in the amplitude is automatically reflected in the susceptibility. Confinement may be invoked as a natural mechanism for cancellation.

As for the mean scalar field, the one of interest is the one susceptible to induce an attraction between nucleons. The effect of the nucleonic pion cloud is cancelled in the NN interaction by other terms [4, 5]. This is why we do not keep in the nucleon scalar charge the pionic contribution. In the linear sigma model the remaining piece is entirely due to the scalar field. It can be obtained from the expansion parameter of the lattice results for the nucleon mass. This leads to a mean scalar field $\bar{s}(\rho_0) \simeq -21\,MeV$. With the introduction of confinement at least part of the scalar charge is concentrated in a small volume, leading to a short-range scalar field. Short-range correlations make it ineffective in the NN interaction, which reduces the remaining attraction. Making the crude assumption that the short-range part of Q_S is the bag one, as evaluated by P. Guichon $Q_S \simeq 0.7$ and that it is additive, the remaining part becomes $Q_S \simeq 2.4 - 0.7 = 1.7$ which brings the mean scalar field to the value $\bar{s}(\rho_0) \simeq -15\,MeV$. It is not directly linked to the expansion parameters and is model dependent but a conclusion can be drawn. A scalar field much stronger than $|\bar{s}(\rho_0)| \simeq 20\,MeV$ presumably implies a contribution from other sources, such as gluon exchange. This conclusion evidently relies on the present values of the expansion parameters, which may change with more data.
Our last result concerns the medium effects in the scalar exchange of the NN potential. We have shown that they can be obtained from the expansion parameters of the lattice results on the nucleon mass. They could be refined if lattice results will become available for lower values of the pion mass and open the possibility of performing a relativistic theory of nuclei compatible with the information of QCD results.

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References

[1] G. Chanfray, M. Ericson, EPJA16 (2003) 291.

[2] G. Chanfray, M. Ericson, P.A.M. Guichon, Phys. Rev C61 (2003) 035209.

[3] G. Chanfray, M. Ericson, EPJA25 (2005) 151.

[4] G. Chanfray, D. Davesne, M. Ericson and M. Martini, EPJA (2006).

[5] M. Birse, Phys. Rev. C49 (1994) 2212.

[6] J. Delorme, G. Chanfray, M. Ericson, Nucl. Phys. A603 (1996) 239.

[7] M. D. Scadron, F. Kleefeld, G. Rupp hep-ph/0601119.

[8] T. Hatsuda, T. Kunihiro and H. Shimizu, Phys. Rev. Lett. **82** (1999) 2840.

[9] A. W. Thomas, P. A. M. Guichon, D. B. Leinweber and R. D. Young, Progr. Theor. Phys. Suppl. 156 (2004) 124; nucl-th/0411014.

[10] A. W. Thomas, D. B. Leinweber and R. D. Young, Phys. Rev. Lett. 92 (2004) 242002.

[11] P.A.M. Guichon, Phys. Lett. B200 (1988) 235.

[12] P.A.M. Guichon, K. Saito, E. Rodionov and A. W. Thomas, Nucl. Phys. A601 (1996) 349.

[13] P.A.M. Guichon, private communication.