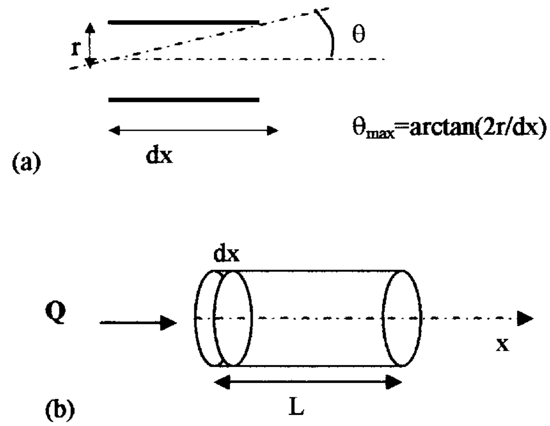

FIG. 5. Cylindrical tube scheme (a) on a surface element ds; (b) on all surface tube.

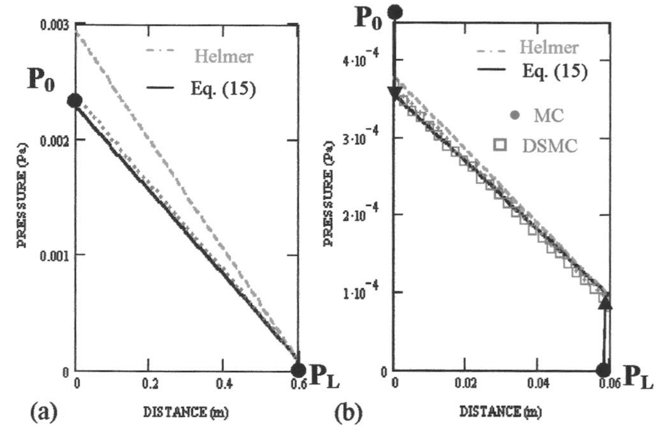

FIG. 6. Surface pressure distribution for cylindrical conductance for nitrogen at 20 °C, with a flux of 2.8×10^{-5} Pa m^3 s^{-1}, for total absorption (a) for $L/r=30$ with $r=2$ cm; (b) for $L/r=3$ with $r=2$ cm.

is explained by the fact that first one is calculated compared to the pressures upstream and downstream (entry and exit surfaces) while the second one is measured compared to the tube surface. Moreover, in our case, it is necessary to take into account the position of the gauge JAB1, between the two cones.

IV. CALCULATION OF THE SURFACE DISTRIBUTION OF PRESSURE

A. The case of the cylindrical tube

Initially, to calculate the collision number on a surface element, ds, one uses the Clausing $P_{\rm Cl}$ probability of the number of molecules crossing without collision.

$$P_{cl}=\int_0^{\theta_{\max}} 2\cos(\theta)\sin(\theta)S(\theta)d\theta, \tag{10}$$

with $S(\theta)$ the transmission probability of the molecules without collision for a given angle θ [Fig. 5(a)].

$$S(\theta)=\frac{2}{\pi}[\arccos(y)-y(1-y^2)^{1/2}]$$
$$\text{with } y=[L/(2r)]\tan(\theta), \tag{11}$$

thus for a length $L\to dx$ and a cosine distribution law of the molecules, the number of collisions for N emitted molecules is

$$N_{\rm choc}=N(1-P_{cl})=N\left(\frac{dx}{r}\right). \tag{12}$$

To determine the surface pressure at the beginning of a circular conductance, it is possible to use the Oatley method, by associating a tube of length dx and of transmission probability $W_1=N_{\rm choc}/2$, to a second length L of probability W_2 [see Fig. 5(b)].

At each passage, whatever the direction, one assumes that the number of hits on the surface $2\pi r dx$ is Ndx/r. It is obvious that this is only an approximation, not taking into account the "beam" effect, with variations of molecular spatial distributions for different crossings. In addition, in order to be able to simulate a flow, the rejected molecules are injected into the circuit, at the entrance of W_1. By assuming that W_1 is close to 1, we obtain

$$N_{\rm choc}=N\frac{dx}{r}\left(\frac{2-W_2}{W_2}\right). \tag{13}$$

Consequently, the surface pressure for $dx\to0$ can be expressed by

$$P_{S_0}=\frac{Q}{(1/4)V_m\pi r^2}\left(\frac{2-W_2}{2W_2}\right). \tag{14}$$

By deriving the $Q=\delta C dP$ equation (with δC the intrinsic conductance for a length dx) and while taking as initial condition P_{s_0} (obtained by the Oatley method), the surface pressure distribution, for a steady flow Q and for a perfect pump (sticking coefficient=1), we have

$$P_S(x)=\frac{-Q}{C_L L}x+P_{S_0}$$
$$=-\frac{Q}{\frac{1}{4}V_m\pi r^2 W_2}\left[\frac{1}{2}+(1-W_2)\left(\frac{1}{2}-\frac{x}{L}\right)\right]. \tag{15}$$

Helmer and Levi[9] obtained the same result by using a differential form of the Oatley equation and by taking $1/W_2=1+1/f$ with $f=2r/L$ as form factor. Figure 6 compares the pressure distribution calculated by Eq. (15), by the Helmer-Levi method, and also by the Monte Carlo method (MC or DSMC), for tubes of radius 0.02 m and length-radius ratios of 3 and 30. The flow Q is 2.8×10^{-5} Pa m^3 s^{-1} for nitrogen at 20 °C. The transmission probabilities W_2 are, respectively, 0.421 and 0.0797. For the short tube, we observe a good correlation between the $P_0[=Q/(C_eW_2)]$ of the upstream chamber and surface pressure P_{s_0}. According to Eq. (15), this difference is constant and independent of W_2, $\Delta P=P_0-P_{s_0}=Q/(2C_e)$ and it also applies to the difference in pressure downstream $P_{\rm SL}-P_L$ (in our case $P_L=0$). This variation of pressure between volume and surface seems to be a good

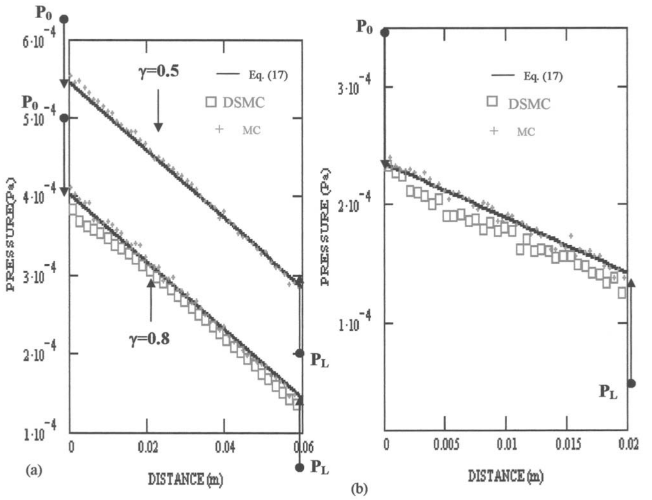

FIG. 7. Surface pressure distribution for cylindrical conductance for nitrogen at 20 °C, with a flux of 2.8×10^{-5} Pa m^3 s^{-1}, (a) for different sticking coefficients $\gamma=0.8$ and 0.5 and for $L/r=3$ with $r=2$ cm ($W_2=0.421$); (b) for a sticking coefficient $\gamma=0.8$ and for $L/r=1$ with $r=2$ cm ($W_2=0.672$).

approximation in spite of the fact that the change of the molecular spatial distribution is not taken into account. For the long tube, the ΔP becomes negligible with respect to P_0 ($\Delta P/P_0=W_2/2$ with small W_2), and the difference with the Helmer-Levi method arises from the fact that, according to a number of authors, the form factor f for a long tube tends towards $8r/(3L)$.

It is possible to extend the method of Oatley in order to take into account a pumping capacity $S_0=C_e\gamma$, with C_e the entrance conductance corresponding to the inlet pump and γ the sticking coefficient on this surface. The collision number on the element dx becomes

$$N_{\text{choc}}=N\frac{dx}{r}\left[\frac{2-W_2}{W_2}+\frac{2(1-\gamma)}{\gamma}\right]. \tag{16}$$

With the same reasoning as previously, the surface distribution of pressure is expressed by

$$P_S(x)=\frac{Q}{C_eW_2}\left[\frac{1}{2}+(1-W_2)\left(\frac{1}{2}-\frac{x}{L}\right)+\frac{W_2(1-\gamma)}{\gamma}\right]. \tag{17}$$

The upstream and downstream pressures are expressed classically by, respectively, $P_0=Q/(C_eW_2)[1+W_2(1-\gamma)/\gamma]$ and $P_L=(Q/C_e)[(1-\gamma)/\gamma]$ with $P_0-P_L=Q/(C_eW_2)$. Figure 7 represents the pressure distribution on the tube surface for the L/r ratio and for different sticking coefficients. Equation (17) gives, to a good approximation, the pressure measured by a gauge for a system of pumping connected by a tube to a large chamber. It should be noted that the difference between the surface pressures and the upstream and downstream pressures is constant as previously whatever the tube length and the pumping speed. It is equal to $\Delta P=P_0-P_{s_0}=P_{SL}-P_L=Q/(2C_e)$.

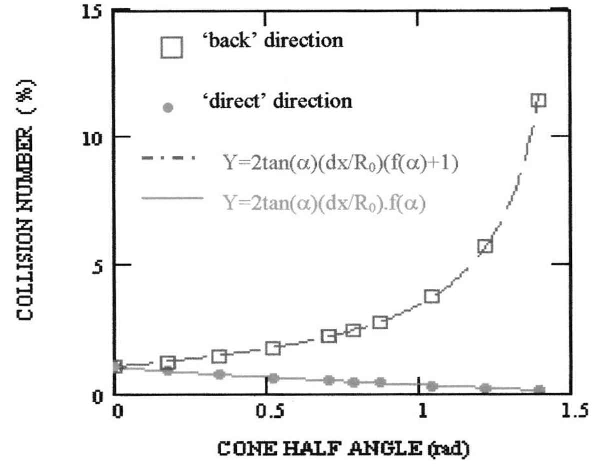

FIG. 8. Collision number on the cone surface for $L\rightarrow dx$ (10^{-5} m) and for radius $R_0=0.01$ m as a function of half-angle of cone.

B. The case of a conical tube

In the same way, to calculate the collision number on an surface element ds, one uses the probability of Clausing, P_{cone}, of the number of molecules crossing without collision.

$$P_{\text{cone}}=\int_0^{\theta_{\max}}2\cos(\theta)\sin(\theta)S_c(\theta)d\theta$$

$$\text{with } \theta_{\max}=\arctan[(R_k-R_0)/L], \tag{18}$$

with $S_c(\theta)$ the transmission probability of the molecules without collision for a given angle θ and the direct direction of the cone.

$$S_c(\theta)=\frac{1}{\pi}\left\{\arccos(y)-y(1-y^2)^{1/2}+\frac{R_K^2}{R_0^2}[\arccos(y_1)-y_1(1-y_1^2)^{1/2}]\right\},$$

with $y=[L/(2R_0)]\tan(\theta)+(R_0^2-R_k^2)/[2LR_0\tan(\theta)]$ and $y1=[L/(2R_k)]\tan(\theta)+(R_k^2-R_0^2)/[2LR_k\tan(\theta)]$. For θ ranging between 0 and α, $S_c(\theta)=1$. This equation can be solved numerically. For a length $L\rightarrow dx$, a cosine distribution law for the collision number for N emitted molecules is $N_{\text{choc}}=N(1-P_{\text{cone}})$. According to Fig. 8, we can express this collision number as a function of the cone half-angle, α.

$$N_{\text{choc}}=N2\frac{dx}{R_0}\tan(\alpha)f(\alpha)$$

$$\text{with } f(\alpha)=\frac{1}{2\tan(\alpha)}\left(1-\frac{8}{3}\frac{\alpha}{\pi}+\frac{4}{3}\frac{\alpha^2}{\pi}\right). \tag{19}$$

With the same reasoning, the collision number for a cone in the back direction is

$$N_{\text{choc}-r}=N2\frac{dx}{R_0}\tan(\alpha)[f(\alpha)+1]. \tag{20}$$

To determine the surface pressure, in the same way as that for a tube, it is possible to extend the Oatley method again,