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# Precision measurement of the neutron spin asymmetries and spin-dependent structure functions in the valence quark region 

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We report on measurements of the neutron spin asymmetries $A_{1,2}^{n}$ and polarized structure functions $g_{1,2}^{n}$ at three kinematics in the deep inelastic region, with $x=0.33,0.47$, and 0.60 and $Q^{2}=2.7,3.5$, and $4.8(\mathrm{GeV} / c)^{2}$, respectively. These measurements were performed using a 5.7 GeV longitudinally polarized electron beam and a polarized ${ }^{3} \mathrm{He}$ target. The results for $A_{1}^{n}$ and $g_{1}^{n}$ at $x=0.33$ are consistent with previous world data and, at the two higher- $x$ points, have improved the precision of the world data by about an order of magnitude. The new $A_{1}^{n}$ data show a zero crossing around $x=0.47$ and the value at $x=0.60$ is significantly positive. These results agree with a next-to-leading-order QCD analysis of previous world data. The trend of data at high $x$ agrees with constituent quark model predictions but disagrees with that from leading-order perturbative QCD (PQCD) assuming hadron helicity conservation. Results for $A_{2}^{n}$ and $g_{2}^{n}$ have a precision comparable to the best world data in this kinematic region. Combined with previous world data, the moment $d_{2}^{n}$ was evaluated and the new result has improved the precision of this quantity by about a factor of 2 . When combined with the world proton data, polarized quark distribution functions were extracted from the new $g_{1}^{n} / F_{1}^{n}$ values based on the quarkparton model. While results for $\Delta u / u$ agree well with predictions from various models, results for $\Delta d / d$ disagree with the leading-order PQCD prediction when hadron helicity conservation is imposed.

## I. INTRODUCTION

Interest in the spin structure of the nucleon became prominent in the 1980s when experiments at CERN [1] and SLAC [2] on the integral of the proton polarized structure function $g_{1}^{p}$ showed that the total spin carried by quarks was very small, $\approx(12 \pm 17) \%$ [1]. This was in contrast to the simple relativistic valence quark model prediction [3] in which the spin of the valence quarks carries approximately $75 \%$ of the proton spin and the remaining $25 \%$ comes from their orbital angular momentum. Because the quark model is very successful in describing static properties of hadrons, the fact that the quark spins account for only a small part of the nucleon spin was a big surprise and generated very productive experimental and theoretical activities to the present. Current understanding [4] of the nucleon spin is that the total spin is distributed among valence quarks, $q \bar{q}$ sea quarks, their orbital angular momenta, and gluons. This is called the nucleon spin sum rule:

$$
\begin{equation*}
S_{z}^{N}=S_{z}^{q}+L_{z}^{q}+J_{z}^{g}=\frac{1}{2} \tag{1}
\end{equation*}
$$

where $S_{z}^{N}$ is the nucleon spin, $S_{z}^{q}$ and $L_{z}^{q}$ represent, respectively, the quark spin and orbital angular momentum (OAM), and $J_{z}^{g}$ is the total angular momentum of the gluons. Only about (20-30)\% of the nucleon spin is carried by the spin of the quarks. To further study the nucleon spin, one thus needs to know more precisely how it decomposes into the three components and to measure their dependence on $x$. Here $x$ is the Bjorken scaling variable, which in the quark-parton model [5] can be interpreted as the fraction of the nucleon momentum carried by the quark. For a fixed target experiment one has $x=Q^{2} /(2 M \nu)$, with $M$ the nucleon mass, $Q^{2}$ the four-momentum transfer squared, and $\nu$ the energy transfer from the incident electron to the target. However, due to experimental limitations, precision data have been collected so far only in the low and moderate $x$ regions. In these regions, one is sensitive to contributions from a large amount of $q \bar{q}$ sea and gluons and the nucleon is hard to model. Moreover, at large distances corresponding to the size of a nucleon, the theory of the strong interaction-quantum chromodynamics (QCD)—is highly nonperturbative, which makes the investigation of the roles of quark orbital angular momentum and gluons in the nucleon spin structure difficult.

Our focus here is the first precise neutron spin structure data in the large $x$ region $x \geqq 0.4$. For these kinematics, the valence quarks dominate and the ratios of structure functions can be estimated based on our knowledge of the interactions between quarks. More specifically, the virtual photon asymmetry $A_{1}$, defined as

$$
A_{1}\left(x, Q^{2}\right) \equiv \frac{\sigma_{1 / 2}-\sigma_{3 / 2}}{\sigma_{1 / 2}+\sigma_{3 / 2}}
$$

(the definitions of $\sigma_{1 / 2,3 / 2}$ are given in Appendix A), which at large $Q^{2}$ is approximately the ratio of the polarized and the unpolarized structure functions $g_{1} / F_{1}$, is expected to approach unity as $x \rightarrow 1$ in perturbative QCD (PQCD). This is a dramatic prediction, not only because this is the only kinematic region where one can give an absolute prediction for
the structure functions based on PQCD, but also because all previous data on the neutron asymmetry $A_{1}^{n}$ in the region $x$ $\gtrsim 0.4$ have large uncertainties and are consistent with $A_{1}^{n}$ $\leqslant 0$. Furthermore, because both $q \bar{q}$ sea and gluon contributions are small in this region, it is a relatively clean region to test the valence quark model and to study the role of valence quarks and their OAM contribution to the nucleon spin.

Deep inelastic scattering (DIS) has served as one of the major experimental tools to study the quark and gluon structure of the nucleon. The formalism of unpolarized and polarized DIS is summarized in Appendix A. Within the quarkparton model (QPM), the nucleon is viewed as a collection of noninteracting, pointlike constituents, one of which carries a fraction $x$ of the nucleon's longitudinal momentum and absorbs the virtual photon [5]. The nucleon cross section is then the incoherent sum of the cross sections for elastic scattering from individual charged pointlike partons. Therefore the unpolarized and the polarized structure functions $F_{1}$ and $g_{1}$ can be related to the spin-averaged and spin-dependent quark distributions as [6]

$$
\begin{equation*}
F_{1}\left(x, Q^{2}\right)=\frac{1}{2} \sum_{i} e_{i}^{2} q_{i}\left(x, Q^{2}\right) \tag{2}
\end{equation*}
$$

and

$$
\begin{equation*}
g_{1}\left(x, Q^{2}\right)=\frac{1}{2} \sum_{i} e_{i}^{2} \Delta q_{i}\left(x, Q^{2}\right) \tag{3}
\end{equation*}
$$

where $q_{i}\left(x, Q^{2}\right)=q_{i}^{\uparrow}\left(x, Q^{2}\right)+q_{i}^{\downarrow}\left(x, Q^{2}\right)$ is the unpolarized parton distribution function (PDF) of the $i$ th quark, defined as the probability that the $i$ th quark inside a nucleon carries a fraction $x$ of the nucleon's momentum, when probed with a resolution determined by $Q^{2}$. The polarized PDF is defined as $\Delta q_{i}\left(x, Q^{2}\right)=q_{i}^{\uparrow}\left(x, Q^{2}\right)-q_{i}^{\downarrow}\left(x, Q^{2}\right)$, where $q_{i}^{\uparrow}\left(x, Q^{2}\right)\left[q_{i}^{\downarrow}\left(x, Q^{2}\right)\right]$ is the probability to find the spin of the $i$ th quark aligned parallel (antiparallel) to the nucleon spin.

The polarized structure function $g_{2}\left(x, Q^{2}\right)$ does not have a simple interpretation within the QPM [6]. However, it can be separated into leading-twist and higher-twist terms using the operator expansion method [7]:

$$
\begin{equation*}
g_{2}\left(x, Q^{2}\right)=g_{2}^{\mathrm{WW}}\left(x, Q^{2}\right)+\bar{g}_{2}\left(x, Q^{2}\right) \tag{4}
\end{equation*}
$$

Here $g_{2}^{\mathrm{WW}}\left(x, Q^{2}\right)$ is the leading-twist (twist-2) contribution and can be calculated using the twist-2 component of $g_{1}\left(x, Q^{2}\right)$ and the Wandzura-Wilczek relation [8] as

$$
\begin{equation*}
g_{2}^{\mathrm{WW}}\left(x, Q^{2}\right)=-g_{1}\left(x, Q^{2}\right)+\int_{x}^{1} \frac{g_{1}\left(y, Q^{2}\right)}{y} d y \tag{5}
\end{equation*}
$$

The higher-twist contribution to $g_{2}$ is given by $\bar{g}_{2}$. When neglecting quark mass effects, the higher-twist term represents interactions beyond the QPM, e.g., quark-gluon and quark-quark correlations [9]. The moment of $\bar{g}_{2}$ can be related to the matrix element $d_{2}$ [10]:

$$
\begin{equation*}
d_{2}=\int_{0}^{1} d x x^{2}\left[3 g_{2}\left(x, Q^{2}\right)+2 g_{1}\left(x, Q^{2}\right)\right]=3 \int_{0}^{1} d x x^{2} \bar{g}_{2}\left(x, Q^{2}\right) . \tag{6}
\end{equation*}
$$

Hence $d_{2}$ measures the deviations of $g_{2}$ from $g_{2}^{W W}$. The value of $d_{2}$ can be obtained from measurements of $g_{1}$ and $g_{2}$ and can be compared with predictions from lattice QCD [11], bag models [12], QCD sum rules [13], and chiral soliton models [14].

In this paper we first describe available predictions for $A_{1}^{n}$ at large $x$. The experimental apparatus and the data analysis procedure will be described in Secs. III-V. In Sec. VI we present results for the asymmetries and polarized structure functions for both ${ }^{3} \mathrm{He}$ and the neutron, a new experimental fit for $g_{1}^{n} / F_{1}^{n}$, and a result for the matrix element $d_{2}^{n}$. Combined with the world proton and deuteron data, polarized quark distribution functions were extracted from our $g_{1}^{n} / F_{1}^{n}$ results. We conclude the paper by summarizing the results for $A_{1}^{n}$ and $\Delta d / d$ and speculating on the importance of the role of quark OAM on the nucleon spin in the kinematic region explored. Some of the results presented here were published previously [15]; the present publication gives full details on the experiment and all of the neutron spin structure results for completeness.

## II. PREDICTIONS FOR $\boldsymbol{A}_{1}^{n}$ AT LARGE $\boldsymbol{x}$

In Secs. II A-II F we present predictions of $A_{1}^{n}$ at large $x$. Data on $A_{1}^{n}$ from previous experiments did not have the precision to distinguish among different predictions, as will be shown in Sec. II G.

## A. $\operatorname{SU}(6)$ symmetric nonrelativistic constituent quark model

In the simplest nonrelativistic constituent quark model (CQM) [16], the nucleon is made of three constituent quarks and the nucleon spin is fully carried by the quark spin. Assuming $\mathrm{SU}(6)$ symmetry, the wave function of a neutron polarized in the $+z$ direction has the form [17]

$$
\begin{align*}
|n \uparrow\rangle= & \frac{1}{\sqrt{2}}\left|d^{\uparrow}(d u)_{000}\right\rangle+\frac{1}{\sqrt{18}}\left|d^{\uparrow}(d u)_{110}\right\rangle-\frac{1}{3}\left|d^{\downarrow}(d u)_{111}\right\rangle \\
& -\frac{1}{3}\left|u^{\uparrow}(d d)_{110}\right\rangle+\frac{\sqrt{2}}{3}\left|u^{\downarrow}(d d)_{111}\right\rangle, \tag{7}
\end{align*}
$$

where the three subscripts are the total isospin, total spin $S$, and spin projection $S_{z}$ along the $+z$ direction for the "diquark" state. For the case of a proton one needs to exchange the $u$ and $d$ quarks in Eq. (7). In the limit where $\operatorname{SU}(6)$ symmetry is exact, both diquark spin states with $S=1$ and $S=0$ contribute equally to the observables of interest, leading to the predictions

$$
\begin{gather*}
A_{1}^{p}=5 / 9 \text { and } A_{1}^{n}=0 ;  \tag{8}\\
\Delta u / u \rightarrow 2 / 3 \text { and } \Delta d / d \rightarrow-1 / 3 . \tag{9}
\end{gather*}
$$

We define $u(x) \equiv u^{p}(x), d(x) \equiv d^{p}(x)$, and $s(x) \equiv s^{p}(x)$ as parton distribution functions for the proton. For a neutron
one has $u^{n}(x)=d^{p}(x)=d(x), d^{n}(x)=u^{p}(x)=u(x)$ based on isospin symmetry. The strange quark distribution for the neutron is assumed to be the same as that of the proton, $s^{n}(x)$ $=s^{p}(x)=s(x)$. In the following, all PDF's are for the proton, unless specified by a superscript $n$.

In the case of DIS, exact $\mathrm{SU}(6)$ symmetry implies the same shape for the valence quark distributions, i.e., $u(x)$ $=2 d(x)$. Using Eqs. (2) and (A4), and assuming that $R\left(x, Q^{2}\right)$ is the same for the neutron and the proton, one can write the ratio of neutron and proton $F_{2}$ structure functions as

$$
\begin{equation*}
R^{n p} \equiv \frac{F_{2}^{n}}{F_{2}^{p}}=\frac{u(x)+4 d(x)}{4 u(x)+d(x)} \tag{10}
\end{equation*}
$$

Applying $u(x)=2 d(x)$ gives

$$
\begin{equation*}
R^{n p}=2 / 3 . \tag{11}
\end{equation*}
$$

However, data on the $R^{n p}$ ratio from SLAC [18], CERN [19-21], and Fermilab [22] disagree with this $\mathrm{SU}(6)$ prediction. The data show that $R^{n p}(x)$ is a straight line starting with $\left.R^{n p}\right|_{x \rightarrow 0} \approx 1$ and dropping to below $1 / 2$ as $x \rightarrow 1$. In addition, $A_{1}^{p}(x)$ is small at low $x[23-25]$. The fact that $\left.R^{n p}\right|_{x \rightarrow 0} \approx 1$ may be explained by the presence of a dominant amount of sea quarks in the low- $x$ region and the fact that $\left.A_{1}^{p}\right|_{x \rightarrow 0} \approx 0$ could be because these sea quarks are not highly polarized. At large $x$, however, there are few sea quarks and the deviation from $\operatorname{SU}(6)$ prediction indicates a problem with the wave function described by Eq. (7). In fact, $S U(6)$ symmetry is known to be broken [26] and the detail of possible $\mathrm{SU}(6)$ breaking mechanisms is an important open issue in hadronic physics.

## B. SU(6) breaking and hyperfine perturbed relativistic CQM

A possible explanation for the $\mathrm{SU}(6)$ symmetry breaking is the one-gluon exchange interaction which dominates the quark-quark interaction at short distances. This interaction was used to explain the behavior of $R^{n p}$ near $x \rightarrow 1$ and the $\approx 300 \mathrm{MeV}$ mass shift between the nucleon and the $\Delta(1232)$ [26]. Later this was described by an interaction term proportional to $\vec{S}_{i} \cdot \vec{S}_{j} \delta^{3}\left(\vec{r}_{i j}\right)$, with $\vec{S}_{i}$ the spin of the $i$ th quark, and hence is also called the hyperfine interaction or chromomagnetic interaction among the quarks [27]. The effect of this perturbation on the wave function is to lower the energy of the $S=0$ diquark state, causing the first term of Eq. (7), $\left|d \uparrow(d u)_{000}\right\rangle^{n}$, to become more stable and to dominate the high-energy tail of the quark momentum distribution that is probed as $x \rightarrow 1$. Since the struck quark in this term has its spin parallel to that of the nucleon, the dominance of this term as $x \rightarrow 1$ implies $(\Delta d / d)^{n} \rightarrow 1$ and $(\Delta u / u)^{n} \rightarrow-1 / 3$ for the neutron, while for the proton one has

$$
\begin{equation*}
\Delta u / u \rightarrow 1 \text { and } \Delta d / d \rightarrow-1 / 3 \text { as } x \rightarrow 1 \tag{12}
\end{equation*}
$$

One also obtains

$$
\begin{equation*}
R^{n p} \rightarrow 1 / 4 \text { as } x \rightarrow 1, \tag{13}
\end{equation*}
$$

which could explain the deviation of $R^{n p}(x)$ data from the $\mathrm{SU}(6)$ prediction. Based on the same mechanism, one can make the following predictions:

$$
\begin{equation*}
A_{1}^{p} \rightarrow 1 \text { and } A_{1}^{n} \rightarrow 1 \text { as } x \rightarrow 1 \tag{14}
\end{equation*}
$$

The hyperfine interaction is often used to break $\mathrm{SU}(6)$ symmetry in the relativistic CQM (RCQM). In this model, the constituent quarks have nonzero OAM which carries $\approx 25 \%$ of the nucleon spin [3]. The use of RCQM to predict the large $x$ behavior of the nucleon structure functions can be justified by the valence quark dominance, i.e., in the large $x$ region almost all quantum numbers, momentum, and the spin of the nucleon are carried by the three valence quarks, which can therefore be identified as constituent quarks. Predictions of $A_{1}^{n}$ and $A_{1}^{p}$ in the large $x$ region using the hyperfineperturbed RCQM have been achieved [28].

## C. Perturbative QCD and hadron helicity conservation

In the early 1970s, in one of the first applications of perturbative QCD, it was noted that as $x \rightarrow 1$, the scattering is from a high-energy quark and thus the process can be treated perturbatively [29]. Furthermore, when the quark OAM is assumed to be zero, the conservation of angular momentum requires that a quark carrying nearly all the momentum of the nucleon (i.e., $x \rightarrow 1$ ) must have the same helicity as the nucleon. This mechanism is called hadron helicity conservation (HHC), and is referred to as the leading-order PQCD in this paper. In this picture, quark-gluon interactions cause only the $S=1, S_{z}=1$ diquark spin projection component rather than the full $S=1$ diquark system to be suppressed as $x \rightarrow 1$, which gives

$$
\begin{gather*}
\Delta u / u \rightarrow 1 \text { and } \Delta d / d \rightarrow 1 \text { as } x \rightarrow 1  \tag{15}\\
R^{n p} \rightarrow \frac{3}{7}, \quad A_{1}^{p} \rightarrow 1, \text { and } A_{1}^{n} \rightarrow 1 \text { as } x \rightarrow 1 \tag{16}
\end{gather*}
$$

This is one of the few places where PQCD can make an absolute prediction for the $x$ dependence of the structure functions or their ratios. However, how low in $x$ and $Q^{2}$ this picture works is uncertain. HHC has been used as a constraint in a model to fit data on the first moment of the proton $g_{1}^{p}$, giving the Brodsky-Burkardt-Schmidt (BBS) parametrization [30]. The $Q^{2}$ evolution was not included in this calculation. Later in the Leader-Sidorov-Stamenov (BBS) [LSS(BBS)] parametrization [31], both proton and neutron $A_{1}$ data were fitted directly and the $Q^{2}$ evolution was carefully treated. Predictions for $A_{1}^{n}$ using both BBS and LSS(BBS) parametrizations have been made, as shown in Figs. 1 and 2 in Sec. II G.

HHC is based on the assumption that the quark OAM is zero. Recent experimental data on the tensor polarization in elastic $e-^{2} \mathrm{H}$ scattering [32], neutral pion photoproduction [33], and the proton electromagnetic form factors [34,35] disagree with the HHC predictions [36]. It has been suggested that effects beyond leading-order PQCD, such as the quark OAM [37-40], might play an important role in processes involving quark spin flips.

## D. Predictions from next-to-leading-order QCD fits

In a next-to-leading-order (NLO) QCD analysis of the world data [41], parametrizations of the polarized and unpolarized PDF's were performed without the HHC constraint. Predictions of $g_{1}^{p} / F_{1}^{p}$ and $g_{1}^{n} / F_{1}^{n}$ were made using these parametrizations, as shown in Figs. 1 and 2 in Sec. II G.

In a statistical approach, the nucleon is viewed as a gas of massless partons (quarks, antiquarks, and gluons) in equilibrium at a given temperature in a finite volume, and the parton distributions are parametrized using either Fermi-Dirac or Bose-Einstein distributions. Based on this statistical picture of the nucleon, a global NLO QCD analysis of unpolarized and polarized DIS data was performed [42]. In this calculation $\Delta u / u \approx 0.75, \Delta d / d \approx-0.5$, and $A_{1}^{p, n}<1$ at $x \rightarrow 1$.

## E. Predictions from chiral soliton and instanton models

While PQCD works well in high-energy hadronic physics, theories suitable for hadronic phenomena in the nonperturbative regime are much more difficult to construct. Possible approaches in this regime are quark models, chiral effective theories, and the lattice QCD method. Predictions for $A_{1}^{n, p}$ have been made using chiral soliton models $[43,44]$ and the results of Ref. [44] give $A_{1}^{n}<0$. The prediction that $A_{1}^{p}<0$ has also been made in the instanton model [45].

## F. Other predictions

Based on quark-hadron duality [46], one can obtain the structure functions and their ratios in the large $x$ region by summing over matrix elements for nucleon resonance transitions. To incorporate $\mathrm{SU}(6)$ breaking, different mechanisms consistent with duality were assumed and data on the structure function ratio $R^{n p}$ were used to fit the $\mathrm{SU}(6)$ mixing parameters. In this picture, $A_{1}^{n, p} \rightarrow 1$ as $x \rightarrow 1$ is a direct result. Duality predictions for $A_{1}^{n, p}$ using different $\mathrm{SU}(6)$-breaking mechanisms were performed in Ref. [47]. There also exist

TABLE I. Previous measurements of $A_{1}^{n}$.

| Experiment | Beam | Target | $x$ | $Q^{2}$ <br> $(\mathrm{GeV} / c)^{2}$ |
| :---: | :---: | :---: | :---: | :---: |
| E142 [51] | $19.42,22.66,25.51 \mathrm{GeV} ; e^{-}$ | ${ }^{3} \mathrm{He}$ | $0.03-0.6$ | 2 |
| E154 [52] | $48.3 \mathrm{GeV} ; e^{-}$ | ${ }^{3} \mathrm{He}$ | $0.014-0.7$ | $1-17$ |
| HERMES [50] | $27.5 \mathrm{GeV} ; e^{-}$ | ${ }^{3} \mathrm{He}$ | $0.023-0.6$ | $1-15$ |
| E143 [25] | $9.7,16.2,29.1 \mathrm{GeV} ; e^{-}$ | $\mathrm{NH}_{3}, \mathrm{ND}_{3}$ | $0.024-0.75$ | $0.5-10$ |
| E155 [53] | $48.35 \mathrm{GeV} ; e^{-}$ | $\mathrm{NH}_{3}, \mathrm{LiD}_{3}$ | $0.014-0.9$ | $1-40$ |
| SMC [49] | $190 \mathrm{GeV} ; \mu^{-}$ | $\mathrm{C}_{4} \mathrm{H}_{10} \mathrm{O}, \mathrm{C}_{4} \mathrm{D}_{10} \mathrm{O}$ | $0.003-0.7$ | $1-60$ |



FIG. 1. Previous data on $A_{1}^{n}$ [25,49-53] and various theoretical predictions: $A_{1}^{n}$ from $\mathrm{SU}(6)$ symmetry (solid line at zero) [17], hyperfine-perturbed RCQM (shaded band) [28], BBS parametrization at $Q^{2}=4(\mathrm{GeV} / c)^{2}$ (higher solid) [30], LSS(BBS) parametrization at $Q^{2}=4(\mathrm{GeV} / c)^{2}$ (dashed) [31], statistical model at $Q^{2}$ $=4(\mathrm{GeV} / c)^{2}$ (long-dashed) [42], quark-hadron duality using two different $\operatorname{SU}(6)$-breaking mechanisms (dash-dot-dotted and dash-dot-dot-dotted) [47], and nonmeson cloudy bag model (dash-dotted) [48]; $g_{1}^{n} / F_{1}^{n}$ from LSS2001 parametrization at $Q^{2}=5(\mathrm{GeV} / c)^{2}$ (lower solid) [41] and from chiral soliton models [43] at $Q^{2}$ $=3(\mathrm{GeV} / c)^{2}$ (long-dash-dotted) and [44] at $Q^{2}=4.8(\mathrm{GeV} / c)^{2}$ (dotted).
predictions from bag models [48], as shown in Figs. 1 and 2 in the next section.

## G. Previous measurements of $A_{1}^{n}$

A summary of previous $A_{1}^{n}$ measurements is given in Table I. The data on $A_{1}^{n}$ and $A_{1}^{p}$ are plotted in Figs. 1 and 2 along with theoretical calculations described in previous sections. Since the $Q^{2}$ dependence of $A_{1}$ is small and $g_{1} / F_{1}$ $\approx A_{1}$ in DIS, data for $g_{1}^{n} / F_{1}^{n}$ and $g_{1}^{p} / F_{1}^{p}$ are also shown and all data are plotted without evolving in $Q^{2}$. As becomes obvious in Fig. 1, the precision of previous $A_{1}^{n}$ data at $x>0.4$ from SMC [49], HERMES [50], and SLAC [25,51,52] is not sufficient to distinguish among different predictions.

## III. THE EXPERIMENT

We report on an experiment [55] carried out at the Hall A of Thomas Jefferson National Accelerator Facility (Jefferson Lab, or JLab). The goal of this experiment was to provide precise data on $A_{1}^{n}$ in the large $x$ region. We have measured the inclusive deep inelastic scattering of longitudinally polarized electrons off a polarized ${ }^{3} \mathrm{He}$ target, with the latter being used as an effective polarized neutron target. The scattered electrons were detected by the two standard high-resolution spectrometers (HRS's). The two HRS's were configured at the same scattering angles and momentum settings to double


FIG. 2. World data on $A_{1}^{p}$ [23-25,53,54] and predictions for $g_{1}^{p} / F_{1}^{p}$ at $Q^{2}=5(\mathrm{GeV} / c)^{2}$ from the E155 experimental fit (long-dash-dot-dotted) [53] and a new fit as described in Sec. VI B (long-dash-dot-dot-dotted). The solid curve corresponds to the prediction for $g_{1}^{n} / F_{1}^{n}$ from LSS(2001) parametrization at $Q^{2}=5(\mathrm{GeV} / c)^{2}$. Other curves are the same as in Fig. 1 except that there is no prediction for the proton from BBS and LSS(BBS) parametrizations.
the statistics. Data were collected at three $x$ points as shown in Table II. Both longitudinal and transverse electron asymmetries were measured, from which $A_{1}, A_{2}, g_{1} / F_{1}$, and $g_{2} / F_{1}$ were extracted using Eqs. (A22)-(A25).

## A. Polarized ${ }^{3} \mathrm{He}$ as an effective polarized neutron

As shown in Fig. 1, previous data on $A_{1}^{n}$ did not have sufficient precision in the large $x$ region. This is mainly due to two experimental limitations. First, high polarization and luminosity required for precision measurements in the large $x$ region were not available previously. Second, there exists no free dense neutron target suitable for a scattering experiment, mainly because of the neutron's short lifetime ( $\approx 886$ s). Therefore polarized nuclear targets such as ${ }^{2} \overrightarrow{\mathrm{H}}$ or ${ }^{3} \overrightarrow{\mathrm{He}}$ are commonly used as effective polarized neutron targets. Consequently, nuclear corrections need to be applied to extract neutron results from nuclear data.

For a polarized deuteron, approximately half of the deuteron spin comes from the proton and the other half comes from the neutron. Therefore the neutron results extracted from the deuteron data have a significant uncertainty coming from the error in the proton data. The advantage of using ${ }^{3} \overrightarrow{\mathrm{He}}$ is that the two protons' spins cancel in the dominant $S$ state of the ${ }^{3} \mathrm{He}$ wave function; thus the spin of the ${ }^{3} \mathrm{He}$ comes mainly ( $>87 \%$ ) from the neutron $[56,57]$, as illustrated in Fig. 3. As a result, there is less model dependence in the procedure of extracting the spin-dependent observables of the neutron from ${ }^{3} \mathrm{He}$ data. At large $x$, the advantage of using a polarized ${ }^{3} \mathrm{He}$ target is more prominent in the case of $A_{1}^{n}$. In

TABLE II. Kinematics of the experiment. The beam energy was $E=5.734 \mathrm{GeV} . E^{\prime}$ and $\theta$ are the nominal momentum and angle of the scattered electrons. $\langle x\rangle,\left\langle Q^{2}\right\rangle$, and $\left\langle W^{2}\right\rangle$ are values averaged over the spectrometer acceptance.

| $\langle x\rangle$ | 0.327 | 0.466 | 0.601 |
| :---: | :---: | :---: | :---: |
| $E^{\prime}$ | 1.32 | 1.72 | 1.455 |
| $\theta$ | $35^{\circ}$ | $35^{\circ}$ | $45^{\circ}$ |
| $\left\langle Q^{2}\right\rangle(\mathrm{GeV} / c)^{2}$ | 2.709 | 3.516 | 4.833 |
| $\left\langle W^{2}\right\rangle(\mathrm{GeV})^{2}$ | 6.462 | 4.908 | 4.090 |

this region almost all calculations show that $A_{1}^{n}$ is much smaller than $A_{1}^{p}$; therefore the $A_{1}^{n}$ results extracted from nuclear data are more sensitive to the uncertainty in the proton data and the nuclear model being used.

In the large $x$ region, the cross sections are small because the parton densities drop dramatically as $x$ increases. In addition, the Mott cross section, given by Eq. (A3), is small at large $Q^{2}$. To achieve a good statistical precision, high luminosity is required. Among all laboratories that are equipped with a polarized ${ }^{3} \mathrm{He}$ target and are able to perform a measurement of the neutron spin structure, the polarized electron beam at JLab, combined with the polarized ${ }^{3} \mathrm{He}$ target in Hall A, provides the highest polarized luminosity in the world [58]. Hence it is the best place to study the large $x$ behavior of the neutron spin structure.

## B. The accelerator and the polarized electron source

JLab operates a continuous-wave electron accelerator that recirculates the beam up to five times through two superconducting linear accelerators. Polarized electrons are extracted from a strained GaAs photocathode [59] illuminated by circularly polarized light, providing a polarized beam of (7080) $\%$ polarization and $\approx 200 \mu \mathrm{~A}$ maximum current to experimental Halls A, B, and C. The maximum beam energy available at JLab so far is 5.7 GeV , which was also the beam energy used during this experiment.

## C. Hall A overview

The basic layout of Hall A during this experiment is shown in Fig. 4. The major instrumentation [60] includes beamline equipment, the target, and two HRS's.


FIG. 3. An illustration of the ${ }^{3} \mathrm{He}$ wave function. The $S, S^{\prime}$, and $D$ state contributions are from calculations using the AV18 twonucleon interaction and the Tucson-Melbourne three-nucleon force, as given in Ref. [56].


FIG. 4. (Color online) Top view of the experimental Hall A (not to scale).

The beamline starts after the arc section of the accelerator where the beam is bent into the hall, and ends at the beam dump. The arc section can be used for beam energy measurement, as will be described in Sec. III D. After the arc section, the beamline is equipped with a Compton polarimeter, two beam current monitors (BCM's), an Unser monitor for absolute beam current measurement, a fast raster, the eP device for beam energy measurement, a Møller polarimeter, and two beam position monitors (BPM's). These beamline elements, together with spectrometers and the target, will be described in detail in the following sections.

## D. Beam energy measurement

The energy of the beam was measured absolutely by two independent methods: ARC and eP $[60,61]$. Both methods can provide a precision of $\delta E_{\text {beam }} / E_{\text {beam }} \approx 2 \times 10^{-4}$. For the ARC method $[60,62]$, the deflection of the beam in the arc section of the beamline is used to determine the beam energy. In the eP measurement $[60,63]$ the beam energy is determined by the measurement of the scattered electron angle $\theta_{e}$ and the recoil proton angle $\theta_{p}$ in ${ }^{1} \mathrm{H}\left(e, e^{\prime} p\right)$ elastic scattering.

## E. Beam polarization measurement

Two methods were used during this experiment to measure the electron beam polarization. The Møller polarimeter [60] measures Møller scattering of the polarized electron beam off polarized atomic electrons in a magnetized foil. The cross section of this process depends on the beam and target polarizations. The polarized electron target used by the Møller polarimeter was a ferromagnetic foil, with its polarization determined from foil magnetization measurements. The Møller measurement is invasive and typically takes an hour, providing a statistical accuracy of about $0.2 \%$. The systematic error comes mainly from the error in the foil target polarization. An additional systematic error is due to the
fact that the beam current used during a Møller measurement $(\approx 0.5 \mu \mathrm{~A})$ is lower than that used during the experiment. The total relative systematic error was $\approx 3.0 \%$ during this experiment.

During a Compton polarimeter $[60,64]$ measurement, the electron beam is scattered off a circularly polarized photon beam and the counting rate asymmetry of the Compton scattered electrons or photons between opposite beam helicities is measured. The Compton polarimeter measures the beam polarization concurrently with the experiment running in the hall.

The Compton polarimeter consists of a magnetic chicane which deflects the electron beam away from the scattered photons, a photon source, an electromagnetic calorimeter, and an electron detector. The photon source was a 200 mW laser amplified by a resonant Fabry-Pérot cavity. During this experiment the maximum gain of the cavity reached $G_{\max }$ $=7500$, leading to a laser power of 1500 W inside the cavity. The circular polarization of the laser beam was greater than $99 \%$ for both right and left photon helicity states. The asymmetry measured in Compton scattering at JLab with a 1.165 eV photon beam and the 5.7 GeV electron beam used by this experiment had a mean value of $\approx 2.2 \%$ and a maximum of $9.7 \%$. For a $12 \mu \mathrm{~A}$ beam current, one hour was needed to reach a relative statistical accuracy of $\left(\Delta P_{b}\right)_{\text {stat }} / P_{b} \approx 1 \%$. The total systematic error was $\left(\Delta P_{b}\right)_{\text {sys }} / P_{b} \approx 1.6 \%$ during this experiment.

The average beam polarization during this experiment was extracted from a combined analysis of 7 Møller and 53 Compton measurements. A value of $(79.7 \pm 2.4) \%$ was used in the final DIS analysis.

## F. Beam helicity

The helicity state of electrons is regulated every 33 ms at the electron source. The time sequence of the electrons' helicity state is carried by helicity signals, which are sent to experimental halls and the data acquisition (DAQ) system. Since the status of the helicity signal (H+ or $\mathrm{H}-$ pulses) has either the same or the opposite sign as the real electron helicity, the absolute helicity state of the beam needs to be determined by other methods, as will be described later.

There are two modes-toggle and pseudorandom-which can be used for the pulse sequence of the helicity signal. In the toggle mode, the helicity alternates every 33 ms . In the pseudorandom mode, the helicity alternates randomly at the beginning of each pulse pair, of which the two pulses must have opposite helicities in order to equalize the numbers of the $\mathrm{H}+$ and $\mathrm{H}-$ pulses. The purpose of the pseudorandom mode is to minimize any possible time-dependent systematic errors. Figure 5 shows the helicity signals and the helicity states of the DAQ system for the two regulation modes.

There is a half-wave plate at the polarized source which can be inserted to reverse the helicity of the laser illuminating the photocathode and hence reverse the helicity of the electron beam. During the experiment this half-wave plate was inserted for half of the statistics to minimize possible systematic effects related to the beam helicity.

The scheme described above was used to monitor the relative changes of the helicity state. The absolute sign of the


FIG. 5. Helicity signal and the helicity status of DAQ in toggle (top) and pseudorandom (bottom) modes.
electrons' helicity states during each of the $\mathrm{H}+$ and $\mathrm{H}-$ pulses was confirmed by measuring a well known asymmetry and comparing the measured asymmetry with its prediction, as will be presented in Secs. V B and V C.

## G. Beam charge measurement and charge asymmetry feedback

The beam current was measured by the BCM system located upstream of the target on the beamline. The BCM signals were fed to scaler inputs and were inserted in the data stream.

Possible beam charge asymmetry measured at Hall A can be caused by the timing asymmetry of the DAQ system, or by the timing and the beam intensity asymmetries at the polarized electron source. The beam intensity asymmetry originates from the intensity difference between different helicity states of the circularly polarized laser used to strike the photocathode. Although the charge asymmetry can be corrected for to first order, there may exist unknown nonlinear effects which can cause a systematic error in the measured asymmetry. Thus the beam charge asymmetry should be minimized. This was done by using a separate DAQ system initially developed for the parity-violation experiments [65], called the parity DAQ. The parity DAQ used the measured charge asymmetry in Hall A to control the orientation of a rotatable half-wave plate located before the photocathode at the source, such that intensities for each helicity state of the polarized laser used to strike the photocathode were adjusted accordingly. The parity DAQ was synchronized with the two HRS DAQ systems so that the charge asymmetry in the two different helicity states could be monitored for each run. The charge asymmetry was typically controlled to be below 2 $\times 10^{-4}$ during this experiment.

## H. Raster and beam position monitor

To protect the target cell from being damaged by the effect of beam-induced heating, the beam was rastered at the target. The raster consists of a pair of horizontal and vertical


FIG. 6. (Color online) Schematic layout of the left HRS and detector package (not to scale).
air-core dipoles located upstream of the target on the beamline, which can produce either a rectangular or an elliptical pattern. We used a raster pattern distributed uniformly over a circular area with a radius of 2 mm .

The position and the direction of the beam at the target were measured by two BPM's located upstream of the target [60]. The beam position can be measured with a precision of $200 \mu \mathrm{~m}$ with respect to the Hall A coordinate system. The beam position and angle at the target were recorded for each event.

## I. High-resolution spectrometers

The Hall A high-resolution spectrometer systems were designed for detailed investigations of the structure of nuclei and nucleons. They provide high resolution in momentum and in angle reconstruction of the reaction product as well as being able to be operated at high luminosity. For each spectrometer, the vertically bending design includes two quadrupoles followed by a dipole magnet and a third quadrupole. All quadrupoles and the dipole are superconducting. Both HRS's can provide a momentum resolution better than 2 $\times 10^{-4}$ and a horizontal angular resolution better than 2 mrad with a design maximum central momentum of $4 \mathrm{GeV} / c$ [60]. By convention, the two spectrometers are identified as the left and the right spectrometers based on their position when viewed looking downstream.

The basic layout of the left HRS is shown in Fig. 6. The detector package is located in a large steel and concrete detector hut following the last magnet. For this experiment the detector package included (1) two scintillator planes S1 and S2 to provide a trigger to activate the DAQ electronics; (2) a set of two vertical drift chambers (VDC's) [66] for particle tracking; (3) a gas Cerenkov detector to provide particle identification (PID) information; and (4) a set of lead glass counters for additional PID. The layout of the right HRS is almost identical except for a slight difference in the geometry of the gas Čerenkov detector and the lead glass counters.

## J. Particle identification

For this experiment the largest background came from photoproduced pions. We refer to PID in this paper as the


FIG. 7. (Color online) Summed ADC signal of the left HRS gas Čerenkov detector without cuts, after lead glass counters electron cut, and after pion cut. The vertical line shows a cut of ADC sum higher than 400 channels applied to select electrons.
identification of electrons from pions. PID for each HRS was accomplished by a $\mathrm{CO}_{2}$ threshold gas Čerenkov detector and a double-layered lead glass shower detector.

The two Čerenkov detectors, one on each HRS, were operated with $\mathrm{CO}_{2}$ at atmospheric pressure. The refraction index of the $\mathrm{CO}_{2}$ gas was 1.00041 , giving a threshold momentum of $\approx 17 \mathrm{MeV} / c$ for electrons and $\approx 4.8 \mathrm{GeV} / c$ for pions. The incident particles on each HRS were also identified by their energy deposits in the lead glass shower detector.

Since Cerenkov detectors and lead glass shower detectors are based on different mechanisms and their PID efficiencies are not correlated [67], we extracted the PID efficiency of the lead glass counters by using electron events selected by the Čerenkov detector, and vice versa. Figure 7 shows a spectrum of the summed analog-to-digital converter (ADC) signal of the left HRS gas Čerenkov detector, without a cut on the lead glass signal and after applying such lead glass electron and pion cuts. The spectrum from the right HRS is similar.

Figure 8 shows the distribution of the energy deposit in the two layers of the right HRS lead glass counters, without a Čerenkov cut, and after Čerenkov electron and pion cuts.

Detailed PID analysis was done both before and during the experiment. The PID performance of each detector is characterized by the electron detection efficiency $\eta_{e}$ and the pion rejection factor $\eta_{\pi, \text { rej }}$, defined as the number of pions needed to cause one pion contamination event. In the HRS central momentum range of $0.8<p_{0}<2.0 \mathrm{GeV} / c$, the PID efficiencies for the left HRS were found to be

Gas Čerenkov: $\eta_{\pi, \text { rej }}>770$ at $\eta_{e}=99.9 \%$,
Lead glass counters: $\eta_{\pi, \text { re }} \approx 38$ at $\eta_{e}=98 \%$,
Combined: $\eta_{\pi, \text { rej }}>3 \times 10^{4}$ at $\eta_{e}=98 \%$,
and for the right HRS were
Gas Čerenkov: $\eta_{\pi, \text { rej }}=900$ at $\eta_{e}=99 \%$,
Lead glass counters: $\eta_{\pi, \text { rej }} \approx 182$ at $\eta_{e}=98 \%$,
Combined: $\eta_{\pi, \text { rej }}>1.6 \times 10^{5}$ at $\eta_{e}=97 \%$.


FIG. 8. (Color online) Energy deposited in the first layer (preshower) vs that in the second layer (shower) of lead glass counters in the right HRS. The two blobs correspond to the spectrum with a tight gas Čerenkov ADC electron cut and with a pion cut applied. The lines show the boundary of the two-dimensional cut used to select electrons in the data analysis.

## K. Data acquisition system

We used the CEBAF Online Data Acquisition (CODA) system [68] for this experiment. In the raw data file, data from the detectors, from the beamline equipment, and from the slow control software were recorded. The total volume of data accumulated during the two-month running period was about 0.6 terabytes. Data from the detectors were processed using an analysis package called Experiment Scanning Program for Hall A Collaboration Experiments (ESPACE) [69]. ESPACE was used to filter raw data, to make histograms for reconstructed variables, to export variables into $n$-tuples for further analysis, and to calibrate experiment-specific detector constants. It also provided the possibility to apply conditions on the incoming data. The information from scaler events was used to extract beam charge and DAQ dead-time corrections.

## IV. THE POLARIZED TARGET

Polarized ${ }^{3} \mathrm{He}$ targets are widely used at SLAC, DESY, MAINZ, MIT-Bates, and JLab to study the electromagnetic structure and the spin structure of the neutron. There exist two major methods to polarize ${ }^{3} \mathrm{He}$ nuclei. The first one uses the metastable-exchange optical pumping technique [70]. The second method is based on optical pumping [71] and spin exchange [72]. It has been used at JLab since 1998 [73], and was used here.

The ${ }^{3} \overrightarrow{\mathrm{He}}$ target at JLab Hall A uses the same design as the SLAC ${ }^{3} \mathrm{He}$ target [74]. The first step to polarize ${ }^{3} \mathrm{He}$ nuclei is to polarize an alkali metal vapor (rubidium was used at JLab as well as at SLAC) by optical pumping [71] with circularly polarized laser light. Depending on the photon helicity, the electrons in the Rb atoms will accumulate at either the $F$ $=3, m_{F}=3$ or the $F=3, m_{F}=-3$ level (here $F$ is the atom's total spin and $m_{F}$ is its projection along the magnetic field axis). The polarization is then transferred to the ${ }^{3} \mathrm{He}$ nuclei


FIG. 9. (Color online) JLab target cell; geometries are given in millimeters for cell 2 used in this experiment.
through the spin exchange mechanism [72] during collisions between Rb atoms and the ${ }^{3} \mathrm{He}$ nuclei. Under operating conditions the ${ }^{3} \mathrm{He}$ density is about $10^{20}$ nuclei/ $\mathrm{cm}^{3}$ and the Rb density is about $10^{14}$ atoms $/ \mathrm{cm}^{3}$.

To minimize depolarization effects caused by the unpolarized light emitted from decay of the excited electrons, $\mathrm{N}_{2}$ buffer gas was added to provide a channel for the excited electrons to decay to the ground state without emitting photons [71]. In the presence of $\mathrm{N}_{2}$, electrons decay through collisions between the Rb atoms and $\mathrm{N}_{2}$ molecules, which is usually referred to as nonradiative quenching. The number density of $\mathrm{N}_{2}$ was about $1 \%$ of that of ${ }^{3} \mathrm{He}$.

## A. Target cells

The target cells used for this experiment were $25-\mathrm{cm}$-long pressurized glass cells with $\approx 130-\mu \mathrm{m}$-thick end windows. The cell consisted of two chambers, a spherical upper chamber which holds the Rb vapor and in which the optical pumping occurs, and a long cylindrical chamber where the electron beam passes through and interacts with the polarized ${ }^{3} \mathrm{He}$ nuclei. Two cells were used for this experiment. Figure 9 is a picture of the second cell with dimensions shown in mm . Table III gives the cell volumes and densities.

## B. Target setup

Figure 10 is a schematic diagram of the target setup. There were two pairs of Helmholtz coils to provide a 25 G main holding field, with one pair oriented perpendicular and the other parallel to the beamline (only the perpendicular pair is shown). The holding field could be aligned in any horizontal direction with respect to the incident electron beam. The coils were excited by two power supplies in the constant voltage mode. The coil currents were continuously measured and recorded by the slow control system.

The cell was held at the center of the Helmholtz coils with its pumping chamber mounted inside an oven heated to $170^{\circ} \mathrm{C}$ in order to vaporize the Rb . The lasers used to polarize the Rb were three 30 W diode lasers tuned to a wave-

TABLE III. Target cell characteristics. Symbols are $V_{p}$, pumping chamber volume in $\mathrm{cm}^{3} ; V_{t}$, target chamber volume in $\mathrm{cm}^{3} ; V_{\mathrm{tr}}$, transfer tube volume in $\mathrm{cm}^{3} ; V_{0}$, total volume in $\mathrm{cm}^{3} ; L_{\mathrm{tr}}$, transfer tube length in $\mathrm{cm} ; n_{0},{ }^{3} \mathrm{He}$ density in amagats at room temperature ( 1 amagat $=2.69 \times 10^{-19} / \mathrm{cm}^{3}$, which corresponds to the gas density at the standard pressure and $T=0^{\circ} \mathrm{C}$ ); lifetime is in hours.

| Name | $V_{p}$ | $V_{t}$ | $V_{\mathrm{tr}}$ | $V_{0}$ | $L_{\mathrm{tr}}$ | $n_{0}$ | Lifetime |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cell 1 | 116.7 | 51.1 | 3.8 | 171.6 | 6.574 | 9.10 | 49 |
| Cell 2 | 116.1 | 53.5 | 3.9 | 173.5 | 6.46 | 8.28 | 44 |
| Uncertainty | 1.5 | 1.0 | 0.25 | 1.8 | 0.020 | $2 \%$ | 1 |

length of 795 nm . The target polarization was measured by two independent methods-NMR (nuclear magnetic resonance) $[60,73,75]$ and the EPR (electron paramagnetic resonance) [58,60,73,76] polarimetry. The NMR system consisted of one pair of pickup coils (one on each side of the cell target chamber), one pair of RF coils, and the associated electronics. The RF coils were placed at the top and the bottom of the scattering chamber, oriented in the horizontal plane, as shown in Fig. 10. The EPR system shared the RF coils with the NMR system. It consisted of one additional RF coil to induce light signal emission from the pumping chamber, a photodiode and the related optics to collect the light, and associated electronics for signal processing.

## C. Laser system

The laser system used during this experiment consisted of seven diode lasers-three for longitudinal pumping, three for transverse pumping, and one spare. To protect the diode lasers from radiation damage from the electron beam, as well as to minimize the safety issues related to the laser hazard, the diode lasers and the associated optics system were located in a concrete laser hut located on the right side of the beamline at $90^{\circ}$, as shown in Fig. 4. The laser optics had seven individual lines, each associated with one diode laser. All seven optical lines were identical and were placed one on


FIG. 10. (Color online) Target setup overview (schematic).
top of the other on an optics table inside the laser hut. Each optical line consisted of one focusing lens to correct the angular divergence of the laser beam, one beam splitter to linearly polarize the lasers, two mirrors to direct them, three quarter-wave plates to convert linear polarization to circular polarization, and two half-wave plates to reverse the laser helicity. Figure 11 shows a schematic diagram of one optics line.

Under the operating conditions for either longitudinal or transverse pumping, the original beam of each diode laser was divided into two by the beam splitter. Therefore there were a total of six polarized laser beams entering the target. The diameter of each beam was about 5 cm which approximately matched the size of the pumping chamber. The target was about 5 m away from the optical table. For the pumping of the transversely polarized target, all these laser beams went directly toward the pumping chamber of the cell through a window on the side of the target scattering cham-


FIG. 11. (Color online) Laser polarizing optics setup (schematic) for the Hall A polarized ${ }^{3} \mathrm{He}$ target.
ber enclosure. For longitudinal pumping, they were guided toward the top of the scattering chamber, and then were reflected twice and finally reached the cell pumping chamber.

## D. NMR polarimetry

The polarization of the ${ }^{3} \mathrm{He}$ was determined by measuring the ${ }^{3} \mathrm{He}$ nuclear magnetic resonance signal. The principle of NMR polarimetry is the spin reversal of ${ }^{3} \mathrm{He}$ nuclei using the adiabatic fast passage (AFP) [77] technique. At resonance this spin reversal will induce an electromagnetic field and a signal in the pickup coil pair. The signal magnitude is proportional to the polarization of the ${ }^{3} \mathrm{He}$ and can be calibrated by performing the same measurement on a water sample, which measures the known thermal polarization of protons in water. The systematic error of the NMR measurement was about $3 \%$, dominated by the error in the water calibration [75].

## E. EPR polarimetry

In the presence of a magnetic field, the Zeeman splitting of Rb , characterized by the electron paramagnetic resonance frequency $\nu_{\mathrm{EPR}}$, is proportional to the field magnitude. When ${ }^{3} \mathrm{He}$ nuclei are polarized $(P \approx 40 \%)$, their spins generate a small magnetic field $B{ }^{3} \mathrm{He}$ of the order of $\approx 0.1 \mathrm{G}$, superimposed on the main holding field $B_{H}=25 \mathrm{G}$. During an EPR measurement [76] the spin of the ${ }^{3} \mathrm{He}$ is flipped by AFP; hence the direction of $B^{3}{ }_{\mathrm{He}}$ is reversed and the change in the total field magnitude causes a shift in $\nu_{\text {EPR }}$. This frequency shift $\delta \nu_{\text {EPR }}$ is proportional to the ${ }^{3} \mathrm{He}$ polarization in the pumping chamber. The ${ }^{3} \mathrm{He}$ polarization in the target chamber is calculated using a model which describes the polarization diffusion from the pumping chamber to the target chamber. The value of the EPR resonance frequency $\nu_{\text {EPR }}$ can also be used to calculate the magnetic field magnitude. The systematic error of the EPR measurement was about 3\%, which came mainly from uncertainties in the cell density and temperature, and from the diffusion model.

## F. Target performance

The target polarizations measured during this experiment are shown in Fig. 12. Results from the two polarimetries are in good agreement and the average target polarization in beam was $(40.0 \pm 2.4) \%$. In a few cases the polarization measurement itself caused an abrupt loss in the polarization. This phenomenon may be the so-called "masing effect" [74] due to nonlinear couplings between the ${ }^{3} \mathrm{He}$ spin rotation and conducting components inside the scattering chamber, e.g., the NMR pickup coils, and the "Rb ring" formed by the rubidium condensed inside the cell at the join of the two chambers. This masing effect was later suppressed by adding coils to produce an additional field gradient.

## V. DATA ANALYSIS

In this section we present the analysis procedure leading to the final results in Sec. VI. We start with the analysis of elastic scattering, the $\Delta(1232)$ transverse asymmetry, and the


FIG. 12. Target polarization, starting June 1 of 2001, as measured by EPR and NMR polarimetries.
check for false asymmetry. Next, the DIS analysis and radiative corrections are presented. Finally we describe nuclear corrections which were used to extract neutron structure functions from the ${ }^{3} \mathrm{He}$ data.

## A. Analysis procedure

The procedure to extract the electron asymmetries from our data is outlined in Fig. 13.

From the raw data one first obtains the helicity-dependent electron yield $N^{ \pm}$using acceptance and PID cuts. The efficiencies associated with these cuts are not helicity dependent; hence they are not corrected for in the asymmetry analysis. The yield is then corrected for the helicitydependent integrated beam charge $Q^{ \pm}$and the live time of the DAQ system $\eta_{\mathrm{LT}}^{ \pm}$. The asymmetry of the corrected yield is the raw asymmetry $A_{\text {raw }}$. Next, to go from $A_{\text {raw }}$ to the physics asymmetries $A_{\|}$and $A_{\perp}$, four factors need to be taken into account: the beam polarization $P_{b}$, the target polarization $P_{t}$, the nitrogen dilution factor $f_{\mathrm{N}_{2}}$ due to the unpolarized nitrogen nuclei mixed with the polarized ${ }^{3} \mathrm{He}$ gas, and a sign based on the knowledge of the absolute state of the electron helicity and the target spin direction:

$$
\begin{equation*}
A_{\|, \perp}= \pm \frac{A_{\mathrm{raw}}}{f_{\mathrm{N}_{2}} P_{b} P_{t}} \tag{17}
\end{equation*}
$$

The results of the beam and the target polarization measurements have been presented in previous sections. The nitrogen dilution factor is obtained from data taken with a reference cell filled with nitrogen. The sign of the asymmetry is described by "the sign convention." The sign convention for parallel asymmetries was obtained from the elastic scattering asymmetry and that for perpendicular asymmetries was from the $\Delta(1232)$ asymmetry analysis, as will be described in Secs. V B and V C. The physics asymmetries $A_{\|}$and $A_{\perp}$, after corrections for radiative effects, were used to calculate $A_{1}$ and $A_{2}$ and the structure function ratios $g_{1} / F_{1}$ and $g_{2} / F_{1}$ using Eqs. (A22)-(A25). Then the last step is to apply nuclear corrections in order to extract the neutron asymme-


FIG. 13. Procedure for asymmetry analysis.
tries and the structure function ratios from the ${ }^{3} \mathrm{He}$ results, as will be described in Sec. V F.

Although the main goal of this experiment was to provide precise data on the asymmetries, cross sections were also extracted from the data. The procedure for the cross section analysis is outlined in Fig. 14. One first determines the absolute yield of $\vec{e}-{ }_{-}^{3} \overrightarrow{\mathrm{He}}$ inclusive scattering from the raw data. Unlike in the asymmetry analysis, corrections need to be made for the detector and PID efficiencies and the spectrometer acceptance. A Monte Carlo simulation is used to calculate the spectrometer acceptance based on a transport model for the HRS [60] with radiative effects taken into account. One then subtracts the yield of $e-\mathrm{N}$ scattering caused by the $\mathrm{N}_{2}$ nuclei in the target. The clean $\vec{e}^{-}{ }^{3} \mathrm{He}$ yield is then corrected for the helicity-averaged beam charge and the DAQ live time to give cross section results. Using world fits for the unpolarized structure functions (form factors) of ${ }^{3} \mathrm{He}$, one can calculate the expected DIS (elastic) cross section from the Monte Carlo simulation and compare to the data.

## B. Elastic analysis

Data for $\vec{e}^{-}{ }^{3} \overrightarrow{\mathrm{He}}$ elastic scattering were taken on a longitudinally polarized target with a beam energy of 1.2 GeV . The scattered electrons were detected at an angle of $20^{\circ}$. The formalism for the cross sections and asymmetries is summarized in Appendix B. Results for the elastic asymmetry were used to check the product of beam and target polarizations, as well as to determine the sign convention for different beam helicity states and target spin directions.

The raw asymmetry was extracted from the data by


FIG. 14. Procedure for cross section analysis.

$$
\begin{equation*}
A_{\mathrm{raw}}=\frac{N^{+} / Q^{+} \eta_{\mathrm{LT}}^{+}-N^{-} / Q^{-} \eta_{\mathrm{LT}}^{-}}{N^{+} / Q^{+} \eta_{\mathrm{LT}}^{+}+N^{-} Q^{-} \eta_{\mathrm{LT}}^{-}} \tag{18}
\end{equation*}
$$

with $N^{ \pm}, Q^{ \pm}$, and $\eta_{\mathrm{LT}}^{ \pm}$the helicity-dependent yield, beam charge, and live-time correction, respectively. The elastic asymmetry is

$$
\begin{equation*}
A_{\|}^{\mathrm{el}}= \pm \frac{A_{\mathrm{raw}}}{f_{\mathrm{N}_{2}} f_{\mathrm{QE}} P_{b} P_{t}} \tag{19}
\end{equation*}
$$

with $f_{\mathrm{N}_{2}}=0.975 \pm 0.003$ the $\mathrm{N}_{2}$ dilution factor determined from data taken with a reference cell filled with nitrogen, and


FIG. 15. (Color online) Elastic parallel asymmetry results for the two HRS's. The kinematics are $E=1.2 \mathrm{GeV}$ and $\theta=20^{\circ}$. A cut in the invariant mass $\left|W-M^{3}{ }_{\mathrm{He}}\right|<6 \mathrm{MeV}$ was used to select elastic events. Data from runs with beam half-wave plate inserted are shown as triangles. The error bars shown are total errors including a $4.5 \%$ systematic uncertainty, which is dominated by the error of the beam and target polarizations. The combined asymmetry and its total error from $\approx 20$ runs are shown by the horizontal solid and dashed lines, respectively, as well as the solid circle as labeled [58].


FIG. 16. (Color online) Elastic cross section results for the two HRS's. The kinematics were $E=1.2 \mathrm{GeV}$ and $\theta=20^{\circ}$. A systematic error of $67 \%$ was assigned to each data point, which was dominated by the uncertainty in the target density and the HRS transport functions [58].
$P_{b}$ and $P_{t}$ the beam and target polarizations, respectively. A cut in the invariant mass $\left|W-M^{3}{ }_{\mathrm{He}}\right|<6(\mathrm{MeV})$ was used to select elastic events. Within this cut there are a small amount of quasielastic events and $f_{\mathrm{QE}}>0.99$ is the quasielastic dilution factor used to correct for this effect.

The sign on the right hand side of Eq. (19) depends on the configuration of the beam half-wave plate, the spin precession of electrons in the accelerator, and the target spin direction. It was determined by comparing the sign of the measured raw asymmetries with the calculated elastic asymmetry. We found that for this experiment the electron helicity was aligned to the beam direction during $\mathrm{H}+$ pulses when the beam half-wave plate was not inserted. Since the electron spin precession in the accelerator can be well calculated using quantum electrodynamics and the results showed that the beam helicity during H+ pulses was the same for the two beam energies used for elastic and DIS measurements, the above convention also applies to the DIS data analysis.

A Monte Carlo simulation was performed which took into account the spectrometer acceptance, the effect of the quasielastic scattering background, and radiative effects. Results for the elastic asymmetry and the cross section are shown in Figs. 15 and 16, respectively, along with the expected values from the simulation. The data show good agreement with the simulation within the uncertainties.

## C. $\Delta(1232)$ transverse asymmetry

Data on the $\Delta(1232)$ resonance were taken on a transversely polarized target using a beam energy of 1.2 GeV . The scattered electrons were detected at an angle of $20^{\circ}$ and


FIG. 17. (Color online) Measured raw $\Delta$ (1232) transverse asymmetry, with beam half-wave plate inserted and target spin pointing to the left side of the beamline. The kinematics are $E=1.2 \mathrm{MeV}$, $\theta=20^{\circ}$, and $E^{\prime}=0.8 \mathrm{GeV} / c$. The dashed lines show the expected value obtained from previous ${ }^{3} \mathrm{He}$ data extrapolated in $Q^{2}$.
the central momentum of the spectrometers was set to $0.8 \mathrm{GeV} / c$. The transverse asymmetry defined by Eq. (A15) was extracted from the raw asymmetry using Eq. (17).

A cut in the invariant mass $|W-1232|<20 \mathrm{MeV}$ was used to select $\Delta(1232)$ events. The sign on the right hand side of Eq. (17) depends on the beam half-wave plate status, the spin precession of electrons in the accelerator, the target spin direction, and in which (left or right) HRS the asymmetry is measured. Since data from a previous experiment [73] in a similar kinematic region showed that $A_{\|}^{\Delta}<0$ and $A_{\perp}^{\Delta}>0$ [78], $A_{\perp}^{\Delta}$ can be used to determine the sign convention of the measured transverse asymmetries. The raw $\Delta(1232)$ transverse asymmetry measured during this experiment was positive on the left HRS, as shown in Fig. 17, with the beam half-wave plate inserted and the target spin pointing to the left side of the beamline. Also shown is the expected value obtained from previous ${ }^{3} \mathrm{He}$ data extrapolated in $Q^{2}$. Similar to the longitudinal configuration, this convention applied to both the $\Delta(1232)$ and DIS measurements.

## D. False asymmetry and background

False asymmetries were checked by measuring the asymmetries from a polarized beam scattering off an unpolarized ${ }^{12} \mathrm{C}$ target. The results show that the false asymmetry was less than $2 \times 10^{-3}$, which was negligible compared to the statistical uncertainties of the measured ${ }^{3} \mathrm{He}$ asymmetries. To estimate the background from pair production $\gamma \rightarrow e^{-}+e^{+}$, the positron yield was measured at $x=0.33$, which is expected to have the highest pair production background. The positron cross section was found to be $\approx 3 \%$ of the total cross section at $x=0.33$, and the positron contribution at $x$

TABLE IV. Total radiation length $X_{0}$ and thickness $d$ of the material traversed by incident (before interaction) and scattered (after interaction) electrons. The cell is made of glass GE180 which has $X_{0}$ $=7.04 \mathrm{~cm}$ and density $\rho=2.77 \mathrm{~g} / \mathrm{cm}^{3}$. The radiation length and thickness after interaction are given by left (l) or right (r) depending on by which HRS the electrons were detected.

| $x$ | $0.33,0.48$ | 0.61 | 0.61 |
| :---: | :---: | :---: | :---: |
| $\theta$ | $35^{\circ}$ | $45^{\circ}$ | $45^{\circ}$ |
| Cell | 2 | 2 | 1 |
| Cell window $(\mu \mathrm{m})$ | 144 | 144 | 132 |
| $X_{0}$ (before) | 0.00773 | 0.00773 | 0.00758 |
| $d\left(\mathrm{~g} / \mathrm{cm}^{2}\right.$, before $)$ | 0.23479 | 0.23479 | 0.23317 |
| Cell wall (mm) | $1.44(1) / 1.33(\mathrm{r})$ | $1.44(1) / 1.33(\mathrm{r})$ | $1.34(1) / 1.43(\mathrm{r})$ |
| $X_{0}$ (after) | $0.0444(1) / 0.0416(\mathrm{r})$ | $0.0376(1) / 0.0354(\mathrm{r})$ | $0.0356(1) / 0.0374(\mathrm{r})$ |
| $d\left(\mathrm{~g} / \mathrm{cm}^{2}\right.$, after $)$ | $0.9044(1) / 0.8506(\mathrm{r})$ | $0.7727(1) / 0.7293(\mathrm{r})$ | $0.7336(1) / 0.7687(\mathrm{r})$ |

$=0.48$ and 0.61 should be even smaller. The effect of pair production asymmetry is negligible compared to the statistical uncertainties of the measured ${ }^{3} \mathrm{He}$ asymmetries and is not corrected for in this analysis.

## E. DIS analysis

The longitudinal and transverse asymmetries defined by Eqs. (A13) and (A15) for DIS were extracted from the raw asymmetries as

$$
\begin{equation*}
A_{\|, \perp}= \pm \frac{A_{\mathrm{raw}}}{f_{\mathrm{N}_{2}} P_{b} P_{t}} \tag{20}
\end{equation*}
$$

where the sign on the right hand side was determined by the procedure described in Secs. V B and V C. The $\mathrm{N}_{2}$ dilution factor, extracted from runs where a reference cell was filled with pure $\mathrm{N}_{2}$, was found to be $f_{\mathrm{N}_{2}}=0.938 \pm 0.007$ for all three DIS kinematics.

Radiative corrections were performed for the ${ }^{3} \mathrm{He}$ asymmetries $A_{\|}^{{ }^{3} \mathrm{He}}$ and $A_{\perp}^{{ }^{3} \mathrm{He}}$. We denote by $A^{\text {obs }}$ the observed asymmetry, $A^{\text {Born }}$ the nonradiated (Born) asymmetry, $\Delta A^{\text {ir }}$ the correction due to internal radiation effects, and $\Delta A^{\text {er }}$ the one due to external radiation effects. One has $A^{\text {Born }}=A^{\mathrm{obs}}$ $+\Delta A^{\mathrm{ir}}+\Delta A^{\mathrm{er}}$ for a specific target spin orientation.

Internal corrections were calculated using an improved version of POLRAD 2.0 [79]. External corrections were calculated with a Monte Carlo simulation based on the procedure first described by Mo and Tsai [80]. Since the theory of radiative corrections is well established [80], the accuracy of the radiative correction depends mainly on the structure functions used in the procedure. To estimate the uncertainty of both corrections, five different fits [81-85] were used for

TABLE V. Internal radiative corrections to $A_{\|}^{{ }^{3} \mathrm{He}}$ and $A_{\perp}^{{ }^{3} \mathrm{He}}$.

| $x$ | $\Delta A_{\\|}^{\mathrm{ir},{ }^{3} \mathrm{He}}\left(\times 10^{-3}\right)$ | $\Delta A_{\perp}^{\mathrm{ir},{ }^{3} \mathrm{He}}\left(\times 10^{-3}\right)$ |
| :---: | :---: | :---: |
| 0.33 | $-5.77 \pm 0.47$ | $2.66 \pm 0.03$ |
| 0.48 | $-3.28 \pm 0.13$ | $1.47 \pm 0.05$ |
| 0.61 | $-2.66 \pm 0.15$ | $1.28 \pm 0.07$ |

the unpolarized structure function $F_{2}$ and two fits $[86,87]$ were used for the ratio $R$. For the polarized structure function $g_{1}$, in addition to those used in POLRAD 2.0 [88,89], we fitted to world $g_{1}^{p} / F_{1}^{p}$ and $g_{1}^{n} / F_{1}^{n}$ data including the new results from this experiment. Both fits will be presented in Sec. VI B. For $g_{2}$ we used both $g_{2}^{\text {WW }}$ and an assumption that $g_{2}$ $=0$. The variation in the radiative corrections using the fits listed above was taken as the full uncertainty of the corrections.

For external corrections the uncertainty also includes the contribution from the uncertainty in the target cell wall thickness. The total radiation length and thickness of the material traversed by the scattered electrons are given in Table IV for each kinematic setting. Results for the internal and external radiative corrections are given in Tables V and VI, respectively.

By measuring DIS unpolarized cross sections and using the asymmetry results, one can calculate the polarized cross sections and extract $g_{1}$ and $g_{2}$ from Eqs. (A5) and (A6). We used a Monte Carlo simulation to calculate the expected DIS unpolarized cross sections within the spectrometer acceptance. This simulation included internal and external radiative corrections. The structure functions used in the simulation were from the latest DIS world fits $[83,87]$ with the nuclear effects corrected [90]. The radiative corrections from the elastic and quasielastic processes were calculated in the peaking approximation [91] using the world proton and neutron form factor data [92-94]. The DIS cross section results agree with the simulation at a level of $10 \%$. Since this is not a dedicated cross section experiment, we obtained the values for $g_{1}$ and $g_{2}$ by multiplying our $g_{1} / F_{1}$ and $g_{2} / F_{1}$ results by

TABLE VI. External radiative corrections to $A_{\|}{ }_{\|} \mathrm{He}$ and $A_{\perp}^{{ }^{3} \mathrm{He}}$. Errors are from uncertainties in the structure functions and in the cell wall thickness.

| $x$ | $\Delta A_{\\|}^{\text {er, }^{3} \mathrm{He}}\left(\times 10^{-3}\right)$ | $\Delta A_{\perp}^{\mathrm{er,}^{3} \mathrm{He}}\left(\times 10^{-3}\right)$ |
| :---: | :---: | :---: |
| 0.33 | $-0.67 \pm 0.10$ | $-0.05 \pm 0.11$ |
| 0.48 | $-1.16 \pm 0.15$ | $0.80 \pm 0.46$ |
| 0.61 | $-0.39 \pm 0.03$ | $0.29 \pm 0.04$ |

TABLE VII. Results for ${ }^{3} \mathrm{He}$ asymmetries $A_{1}^{{ }^{3} \mathrm{He}}$ and $A_{2}^{{ }^{3} \mathrm{He}}$, structure function ratios $g_{1}^{{ }^{3} \mathrm{He}} / F_{1}{ }^{3} \mathrm{He}$ and $g_{2}^{3} \mathrm{He} / F_{1}{ }^{3} \mathrm{He}$, and polarized structure functions $g_{1}{ }^{3} \mathrm{He}$ and $g_{2}{ }^{3} \mathrm{He}$. Errors are given as $\pm$ statistical $\pm$ systematic.

| $\langle x\rangle$ | 0.33 | 0.47 | 0.60 |
| :---: | ---: | ---: | ---: |
| $\left\langle Q^{2}\right\rangle(\mathrm{GeV} / c)^{2}$ | 2.71 | 3.52 | 4.83 |
| $A_{\\|}{ }^{3} \mathrm{He}$ | $-0.020 \pm 0.005 \pm 0.001$ | $-0.008 \pm 0.005 \pm 0.000$ | $0.007 \pm 0.007 \pm 0.001$ |
| $A_{\perp}{ }^{3} \mathrm{He}$ | $0.000 \pm 0.010 \pm 0.000$ | $0.016 \pm 0.008 \pm 0.001$ | $-0.010 \pm 0.016 \pm 0.001$ |
| $A_{1}^{3} \mathrm{He}$ | $-0.024 \pm 0.006 \pm 0.001$ | $-0.019 \pm 0.006 \pm 0.001$ | $0.010 \pm 0.009 \pm 0.001$ |
| $A_{2}^{{ }^{3} \mathrm{He}}$ | $-0.004 \pm 0.014 \pm 0.001$ | $0.020 \pm 0.012 \pm 0.001$ | $-0.013 \pm 0.023 \pm 0.001$ |
| $g_{1}{ }^{3} \mathrm{He} / F_{1}{ }^{3} \mathrm{He}$ | $-0.022 \pm 0.005 \pm 0.001$ | $-0.008 \pm 0.008 \pm 0.001$ | $0.003 \pm 0.009 \pm 0.001$ |
| $g_{2}{ }_{2}{ }_{2 e} / F_{1}{ }^{3} \mathrm{He}$ | $0.010 \pm 0.036 \pm 0.002$ | $0.050 \pm 0.022 \pm 0.003$ | $-0.028 \pm 0.038 \pm 0.002$ |
| $g_{1}{ }^{3} \mathrm{He}$ | $-0.024 \pm 0.006 \pm 0.001$ | $-0.004 \pm 0.004 \pm 0.000$ | $0.001 \pm 0.002 \pm 0.000$ |
| $g_{2}{ }^{3} \mathrm{He}$ | $0.011 \pm 0.039 \pm 0.001$ | $0.026 \pm 0.012 \pm 0.002$ | $-0.006 \pm 0.009 \pm 0.001$ |

the world fits for unpolarized structure functions $F_{1}[83,87]$, instead of the $F_{1}$ from this analysis.

## F. From ${ }^{3} \mathrm{He}$ to neutron

Properties of protons and neutrons embedded in nuclei are expected to be different from those in free space because of a variety of nuclear effects, including those from spin depolarization, binding, and Fermi motion, the off-shell nature of the nucleons, the presence of non-nucleonic degrees of freedom, and nuclear shadowing and antishadowing. A coherent and complete picture of all these effects for the ${ }^{3} \mathrm{He}$ structure function $g_{1}{ }^{\text {He }}$ in the range of $10^{-4} \leqslant x \leqslant 0.8$ was presented in [97]. It gives

$$
\begin{align*}
g_{1}^{3} \mathrm{He}= & P_{n} g_{1}^{n}+2 P_{p} g_{1}^{p}-0.014\left[g_{1}^{p}(x)-4 g_{1}^{n}(x)\right]+a(x) g_{1}^{n}(x) \\
& +b(x) g_{1}^{p}(x) \tag{21}
\end{align*}
$$

where $P_{n}\left(P_{p}\right)$ is the effective polarization of the neutron (proton) inside ${ }^{3} \mathrm{He}$ [57]. The functions $a(x)$ and $b(x)$ are $Q^{2}$ dependent and represent the nuclear shadowing and antishadowing effects.

From Eq. (A12), the asymmetry $A_{1}$ is approximately the ratio of the spin structure function $g_{1}$ and $F_{1}$. Noting that shadowing and antishadowing are not present in the large $x$ region, using Eq. (21) one obtains

$$
\begin{equation*}
A_{1}^{n}=\frac{F_{2}^{3} \mathrm{He}\left[A_{1}^{3} \mathrm{He}-2\left(F_{2}^{p} / F_{2}^{3} \mathrm{He}\right) P_{p} A_{1}^{p}\left(1-0.014 / 2 P_{p}\right)\right]}{P_{n} F_{2}^{n}\left(1+0.056 / P_{n}\right)} \tag{22}
\end{equation*}
$$

The two terms $0.056 / P_{n}$ and $0.014 / 2 P_{p}$ represent the corrections to $A_{1}^{n}$ associated with the $\Delta(1232)$ component in the ${ }^{3} \mathrm{He}$ wave function. Both terms cause $A_{1}^{n}$ to increase in the $x$ range of this experiment, and to turn positive at lower values of $x$ compared to the situation when the effect of the $\Delta(1232)$ is ignored. For $F_{2}^{n}$ and $F_{2}^{3} \mathrm{He}$, we used the world proton and deuteron $F_{2}$ data and took into account the nuclear effects [90]. We used the world proton asymmetry data for $A_{1}^{p}$. The effective nucleon polarizations $P_{n, p}$ can be calculated using ${ }^{3} \mathrm{He}$ wave functions constructed from $\mathrm{N}-\mathrm{N}$ interactions, and their uncertainties were estimated using various nuclear models [56,57,98,99], giving

$$
\begin{equation*}
P_{n}=0.86_{-0.02}^{+0.036} \text { and } P_{p}=-0.028_{-0.004}^{+0.009} \tag{23}
\end{equation*}
$$

Equation (22) was also used for extracting $A_{2}^{n}, g_{1}^{n} / F_{1}^{n}$, and $g_{2}^{n} / F_{1}^{n}$ from our ${ }^{3} \mathrm{He}$ data. The uncertainty in $A_{1}^{n}$ due to the uncertainties in $F_{2}^{p, d}$, in the correction for nuclear effects, in $A_{1}^{p}$ data, and in $P_{n, p}$ is given in Table X below. Compared to the convolution approach [98] used by previous ${ }^{3} \mathrm{He}$ experiments [50-52], in which only the first two terms on the right hand side of Eq. (21) are present, the values of $A_{1}^{n}$ extracted

TABLE VIII. Results for the asymmetries and spin structure functions for the neutron. Errors are given as $\pm$ statistical $\pm$ systematic.

| $\langle x\rangle$ | 0.33 | 0.47 | 0.60 |
| :---: | ---: | ---: | ---: |
| $\left\langle Q^{2}\right\rangle(\mathrm{GeV} / c)^{2}$ | 2.71 | 3.52 | 4.83 |
| $A_{1}^{n}$ | $-0.048 \pm 0.024_{-0.016}^{+0.015}$ | $-0.006 \pm 0.027_{-0.019}^{+0.019}$ | $0.175 \pm 0.048_{-0.019}^{+0.026}$ |
| $A_{2}^{n}$ | $-0.004 \pm 0.063_{-0.005}^{+0.005}$ | $0.117 \pm 0.055_{-0.021}^{+0.012}$ | $-0.034 \pm 0.124_{-0.014}^{+0.014}$ |
| $g_{1}^{n} / F_{1}^{n}$ | $-0.043 \pm 0.022_{-0.009}^{+0.009}$ | $0.040 \pm 0.035_{-0.011}^{+0.011}$ | $0.124 \pm 0.045_{-0.011}^{+0.017}$ |
| $g_{2}^{n} / F_{1}^{n}$ | $0.034 \pm 0.153_{-0.010}^{+0.010}$ | $0.207 \pm 0.103_{-0.010}^{+0.022}$ | $-0.190 \pm 0.204_{-0.021}^{+0.022}$ |
| $g_{1}^{n}$ | $-0.012 \pm 0.006_{-0.003}^{+0.003}$ | $0.005 \pm 0.004_{-0.001}^{+0.001}$ | $0.006 \pm 0.002_{-0.001}^{+0.001}$ |
| $g_{2}^{n}$ | $0.009 \pm 0.043_{-0.003}^{+0.003}$ | $0.026 \pm 0.013_{-0.003}^{+0.003}$ | $-0.009 \pm 0.009_{-0.001}^{+0.001}$ |

TABLE IX. Experimental systematic errors for the $A_{1}^{n}$ result.

| Source | Error |
| :--- | ---: |
| Beam energy $E_{b}$ | $\Delta E_{b} / E_{b}<5 \times 10^{-4}$ |
| HRS central momentum $p_{0}$ | $\Delta E_{e} / E_{e}<5 \times 10^{-4}[103]$ |
| HRS central angle $\theta_{0}$ | $\Delta \theta_{0}<0.1^{\circ}[104]$ |
| Beam polarization $P_{b}$ | $\Delta P_{b} / P_{b}<3 \%$ |
| Target polarization $P_{t}$ | $\Delta P_{t} / P_{t}<4 \%$ |
| Target spin direction $\alpha_{t}$ | $\Delta \alpha_{t}<1^{\circ}$ |

from Eq. (22) are larger by $0.01-0.02 \%$ in the region 0.2 $<x<0.7$.

## G. Resonance contributions

Since there are a few nucleon resonances with masses above 2 GeV and our measurement at the highest- $x$ point has an invariant mass close to 2 GeV , the effect of possible contributions from baryon resonances was evaluated. This was done by comparing the resonance contribution to $g_{1}^{n}$ with that to $F_{1}^{n}$. For our kinematics at $x=0.6$, data on the unpolarized structure function $F_{2}$ and $R$ [95] show that the resonance


FIG. 18. Results for the ${ }^{3} \mathrm{He}$ asymmetry $A_{1}^{{ }^{3} \mathrm{He}}$ and the structure functions $g_{1}{ }^{3} \mathrm{He}$ as a function of $x$, along with previous data from SLAC [51,100] and HERMES [101]. Error bars of the results from this work include both statistical and systematic uncertainties.
contribution to $F_{1}$ is less than $5 \%$. The resonance asymmetry was estimated using the MAID model [96] and was found to be approximately 0.10 at $W=1.7 \mathrm{GeV}$. Since the resonance structure is more evident at smaller $W$, we took this value as an upper limit of the contribution at $W=2 \mathrm{GeV}$. The resonance contribution to our $A_{1}^{n}$ and $g_{1}^{n} / F_{1}^{n}$ results at $x=0.6$ was then estimated to be at most 0.008 , which is negligible compared to their statistical errors.

## VI. RESULTS

## A. ${ }^{3} \mathrm{He}$ results

Results of the electron asymmetries for $\vec{e}-{ }^{3} \overrightarrow{\mathrm{He}}$ scattering, $A_{\|}^{3} \mathrm{He}$ and $A_{\perp}{ }^{3} \mathrm{He}$, the virtual photon asymmetries $A_{1}^{3} \mathrm{He}$ and $A_{2}^{3} \mathrm{He}$, structure function ratios $g_{1}{ }^{3} \mathrm{He} / F_{1}{ }^{3} \mathrm{He}$ and $g_{2}^{3} \mathrm{He} / F_{1}{ }^{3} \mathrm{He}$, and polarized structure functions $g_{1}^{3} \mathrm{He}$ and $g_{2}^{3} \mathrm{He}$ are given in Table VII. Results for $g_{1,2}{ }^{3} \mathrm{He}$ were obtained by multiplying the $g_{1,2}^{3} / F_{1}{ }^{3} \mathrm{He}$ results by the unpolarized structure function $F_{1}^{3} \mathrm{He}$, which were calculated using the latest world fits of DIS data $[83,87]$ and with nuclear effects corrected [90]. Results for $A_{1}^{{ }^{3}} \mathrm{He}$ and $g_{1}{ }^{3} \mathrm{He}$ are shown in Fig. 18 along with SLAC $[51,100]$ and HERMES [101] data.


FIG. 19. Our $A_{1}^{n}$ results along with theoretical predictions and previous world data obtained from polarized ${ }^{3} \mathrm{He}$ targets [50-52]. Curves: predictions of $A_{1}^{n}$ from $\mathrm{SU}(6)$ symmetry ( $x$ axis at zero) [17], constituent quark model (shaded band) [28], statistical model at $Q^{2}=4(\mathrm{GeV} / c)^{2}$ (long-dashed) [42], quark-hadron duality using two different $\mathrm{SU}(6)$-breaking mechanisms (dash-dot-dotted and dash-dot-dot-dotted), and nonmeson cloudy bag model (dashdotted) [48]; predictions of $g_{1}^{n} / F_{1}^{n}$ from PQCD HHC-based BBS parametrization at $Q^{2}=4(\mathrm{GeV} / c)^{2}$ (higher solid) [30] and LSS(BBS) parametrization at $Q^{2}=4(\mathrm{GeV} / c)^{2}$ (dashed) [31], LSS 2001 NLO polarized parton densities at $Q^{2}=5(\mathrm{GeV} / c)^{2}$ (lower solid) [41], and chiral soliton models [43] at $Q^{2}=3(\mathrm{GeV} / c)^{2}$ (long-dash-dotted) and [44] at $Q^{2}=4.8(\mathrm{GeV} / c)^{2}$ (dotted).


FIG. 20. Results for $g_{1}^{n} / F_{1}^{n}$ along with previous world data from SLAC [25,53]. The curves are the prediction for $g_{1}^{n} / F_{1}^{n}$ from the LSS 2001 NLO polarized parton densities at $Q^{2}=5(\mathrm{GeV} / c)^{2}$ [41], the E155 experimental fit at $Q^{2}=5(\mathrm{GeV} / c)^{2}$ (long-dash-dotdotted) [53], and the new fit as described in the text (long-dash-dot-dot-dotted).

## B. Neutron results

Results for the neutron asymmetries $A_{1}^{n}$ and $A_{2}^{n}$, structure function ratios $g_{1}^{n} / F_{1}^{n}$ and $g_{2}^{n} / F_{1}^{n}$, and polarized structure functions $g_{1}^{n}$ and $g_{2}^{n}$ are given in Table VIII.

The $A_{1}^{n}, g_{1}^{n} / F_{1}^{n}$, and $g_{1}^{n}$ results are shown in Figs. 19-21, respectively. In the region of $x>0.4$, our results have improved the world data precision by about an order of magnitude, and will provide valuable inputs to parton distribution function parametrizations. Our data at $x=0.33$ are in good agreement with previous world data. For the $A_{1}^{n}$ results, this is the first time that the data show a clear trend that $A_{1}^{n}$ turns to positive values at large $x$. As $x$ increases, the agreement between the data and the predictions from the constituent quark model becomes better. This is within the expectation since the CQM is more likely to work in the valence quark


FIG. 21. Results for $g_{1}^{n}$ along with previous world data from SLAC [51-53] and HERMES [50].

TABLE X. Total uncertainties for $A_{1}^{n}$.

| $\langle x\rangle$ | 0.33 | 0.47 | 0.60 |
| :--- | ---: | ---: | ---: |
| Statistics | 0.024 | 0.027 | 0.048 |
| Experimental syst. | 0.004 | 0.003 | 0.004 |
| $\Delta A_{1}^{n \text { ir }}$ | 0.012 | 0.013 | 0.015 |
| $\Delta A_{1}^{n, \text { er }}$ | 0.002 | 0.002 | 0.003 |
| $F_{2}^{p}, F_{2}^{d}$ | 0.006 | 0.008 | +0.005 |
|  |  |  | -0.010 |
| Nuclear effect | 0.001 | 0.000 | 0.009 |
| $A_{1}^{p}$ | 0.001 | 0.005 | 0.011 |
| $P_{n}, P_{p}$ | +0.005 | +0.009 | +0.018 |
|  | -0.012 | -0.020 | -0.037 |

region. It also indicates that $A_{1}^{n}$ will go to higher values at $x>0.6$. However, the trend of the $A_{1}^{n}$ results does not agree with the BBS and LSS(BBS) parametrizations, which are from leading-order PQCD analyses based on hadron helicity conservation. This indicates that there might be a problem in the assumption that quarks have zero orbital angular momentum, which is used by HHC.

The sources for the experimental systematic uncertainties are listed in Table IX.

Systematic uncertainties for the $A_{1}^{n}$ results include that from experimental systematic errors, uncertainties in internal radiative corrections $\Delta A_{1}^{n, \text { ir }}$ and external radiative corrections $\Delta A_{1}^{n, \text { er }}$ as derived from the values in Tables V and VI, and that from nuclear corrections as described in Sec. V F. Table X gives these systematic uncertainties for the $A_{1}^{n}$ results along with their statistical uncertainties. The total uncertainties are dominated by the statistical uncertainties.

We used five functional forms $x^{\alpha} P_{n}(x)\left(1+\beta / Q^{2}\right)$ to fit our $g_{1}^{n} / F_{1}^{n}$ results combined with data from previous experiments [25,53]. Here $P_{n}$ is the $n$ th-order polynomial, $n=1,2$ for a finite $\alpha$ or $n=1,2,3$ if $\alpha$ is fixed to be 0 . The total number of parameters is limited to $\leqslant 5$. For the $Q^{2}$ dependence of $g_{1} / F_{1}$, we used a term $1+\beta / Q^{2}$ as in the E155 experimental fit [53]. No constraints were imposed on the fit concerning the behavior of $g_{1} / F_{1}$ as $x \rightarrow 1$. The function which gives the smallest $\chi^{2}$ value is $g_{1}^{n} / F_{1}^{n}=\left(a+b x+c x^{2}\right)\left(1+\beta / Q^{2}\right)$. The new fit is shown in Fig. 20. Results for the fit parameters are given in Table XI and the covariance error matrix is

$$
\epsilon=\left[\begin{array}{cccc}
1.000 & -0.737 & 0.148 & 0.960 \\
-0.737 & 1.000 & -0.752 & -0.581 \\
0.148 & -0.752 & 1.000 & -0.039 \\
0.960 & -0.581 & -0.039 & 1.000
\end{array}\right]
$$

Similar fits were performed to the proton world data $[25,53,54]$ and the function $g_{1}^{p} / F_{1}^{p}=x^{\alpha}(a+b x)\left(1+\beta / Q^{2}\right)$ was found to give the smallest $\chi^{2}$ value. The new fit is shown in Fig. 2 of Sec. II G. Results for the fit parameters are given in Table XII and the covariance error matrix is

$$
\epsilon=\left[\begin{array}{cccc}
1.000 & 0.908 & -0.851 & 0.723 \\
0.908 & 1.000 & -0.967 & 0.401 \\
-0.851 & -0.967 & 1.000 & -0.369 \\
0.723 & 0.401 & -0.369 & 1.000
\end{array}\right]
$$

TABLE XI. Result of the fit $g_{1}^{n} / F_{1}^{n}=\left(a+b x+c x^{2}\right)\left(1+\beta / Q^{2}\right)$.
$a=-0.049 \pm 0.052$
$b=-0.162 \pm 0.217$
$c=0.698 \pm 0.345$
$\beta=0.751 \pm 2.174$

Figures 22 and 23 show the results for $A_{2}^{n}$ and $x g_{2}^{n}$, respectively. The precision of our data is comparable to that of the data from the E155x experiment at SLAC [102], which is so far the only experiment dedicated to measuring $g_{2}$ with published results.

To evaluate the matrix element $d_{2}^{n}$, we combined our $g_{2}^{n}$ results with the E155x data [102]. The average $Q^{2}$ of the E155x data set is about $5(\mathrm{GeV} / c)^{2}$. Following a similar procedure as used in Ref. [102], we assumed that $\bar{g}_{2}\left(x, Q^{2}\right)$ is independent of $Q^{2}$ and $\bar{g}_{2} \propto(1-x)^{m}$ with $m=2$ or 3 for $x$ $\gtrsim 0.78$ beyond the measured region of both experiments. We obtained from Eq. (6)

$$
\begin{equation*}
d_{2}^{n}=0.0062 \pm 0.0028 \tag{24}
\end{equation*}
$$

Compared to the value published previously [102], the uncertainty on $d_{2}^{n}$ has been improved by about a factor of 2 . The large decrease in uncertainty despite the small number of our data points arises from the $x^{2}$ weighting of the integral which emphasizes the large $x$ kinematics. The uncertainties on the integrand have been improved in the region $x>0.4$ due to our $g_{2}^{n}$ results at the two higher- $x$ points being more precise than that of E155x. While a negative value was predicted by lattice QCD [11] and most other models [12-14], the new result for $d_{2}^{n}$ suggests that the higher-twist contribution is positive.

## C. Flavor decomposition using the quark-parton model

Assuming the strange quark distributions $s(x), \bar{s}(x), \Delta s(x)$, and $\Delta \bar{s}(x)$ to be small in the region $x>0.3$, and ignoring any $Q^{2}$ dependence of the ratio of structure functions, one can extract polarized quark distribution functions based on the quark-parton model as

$$
\begin{equation*}
\frac{\Delta u+\Delta \bar{u}}{u+\bar{u}}=\frac{4 g_{1}^{p}\left(4+R^{d u}\right)}{15 F_{1}^{p}}-\frac{g_{1}^{n}\left(1+4 R^{d u}\right)}{15 F_{1}^{n}} \tag{25}
\end{equation*}
$$

and

TABLE XII. Result of the fit $g_{1}^{p} / F_{1}^{p}=x^{\alpha}(a+b x)\left(1+\beta / Q^{2}\right)$.
$\alpha=0.813 \pm 0.049$
$a=1.231 \pm 0.122$
$b=-0.413 \pm 0.216$
$\beta=0.030 \pm 0.124$


FIG. 22. Results for $A_{2}^{n}$ along with the best previous world data [102]. The curve gives the twist-2 contribution at $Q^{2}=4(\mathrm{GeV} / c)^{2}$ calculated using the E155 experimental fit [53] and $g_{2}^{\mathrm{WW}}$ of Eq. (5).

$$
\begin{equation*}
\frac{\Delta d+\Delta \bar{d}}{d+\bar{d}}=\frac{4 g_{1}^{n}\left(1+4 R^{d u}\right)}{15 F_{1}^{n} R^{d u}}-\frac{g_{1}^{p}\left(4+R^{d u}\right)}{15 F_{1}^{p} R^{d u}} \tag{26}
\end{equation*}
$$

with $R^{d u} \equiv(d+\bar{d}) /(u+\bar{u})$. Results for $(\Delta u+\Delta \bar{u}) /(u+\bar{u})$ and $(\Delta d+\Delta \bar{d}) /(d+\bar{d})$ are given in Table XIII. As inputs we used our own results for $g_{1}^{n} / F_{1}^{n}$, the world data on $g_{1}^{p} / F_{1}^{p}$ [58], and the ratio $R^{d u}$ extracted from proton and deuteron unpolarized structure function data [105]. In a similar manner as for Eqs. (25) and (26) and ignoring nuclear effects, one can also add the world data on $g_{1}^{2} \mathrm{H} / F_{1}^{2} \mathrm{H}$ to the fitted data set and extract these polarized quark distributions. The results are, however, consistent with those given in Table XIII and have very similar error bars because the data on the deuteron in general have poorer precision than the data on the proton and the neutron data from this experiment. The results presented here have changed compared to the values published previously in Ref. [15] due to an error discovered in our fitting of $R^{d u}$ from Ref. [105]. The analysis procedure is consistent with what was used in Ref. [15].

Figure 24 shows our results along with semi-inclusive data on $(\Delta q+\Delta \bar{q}) /(q+\bar{q})$ obtained from recent results for $\Delta q$ and $\Delta \bar{q}$ [106] by the HERMES Collaboration, and the


FIG. 23. Results for $x g_{2}^{n}$ along with the best previous world data [102]. The curve gives the twist- 2 contribution at $Q^{2}=4(\mathrm{GeV} / c)^{2}$ calculated using the E155 experimental fit [53] and $g_{2}^{\mathrm{WW}}$ of Eq. (5).

TABLE XIII. Results for the polarized quark distributions. The three uncertainties are those due to the $g_{1}^{n} / F_{1}^{n}$ statistical error, the $g_{1}^{n} / F_{1}^{n}$ systematic uncertainty, and the uncertainties of the $g_{1}^{p} / F_{1}^{p}$ data, the $R^{d u}$ fit, and the correction for $s$ and $c$ quark contributions.

| $\langle x\rangle$ | $(\Delta u+\Delta \bar{u}) /(u+\bar{u})$ | $(\Delta d+\Delta \bar{d}) /(d+\bar{d})$ |
| :---: | ---: | ---: |
| 0.33 | $0.545 \pm 0.004 \pm 0.002_{-0.025}^{+0.024}$ | $-0.352 \pm 0.035 \pm 0.014_{-0.031}^{+0.017}$ |
| 0.47 | $0.649 \pm 0.006 \pm 0.002_{-0.058}^{+0.058}$ | $-0.393 \pm 0.063 \pm 0.020_{-0.044}^{+0.041}$ |
| 0.60 | $0.728 \pm 0.006 \pm 0.002_{-0.114}^{+0.114}$ | $-0.440 \pm 0.092 \pm 0.035_{-0.142}^{+0.107}$ |

CTEQ6M unpolarized PDF [107]. To estimate the effect of the $s$ and $\bar{s}$ contributions, we used two unpolarized PDF sets, CTEQ6M [107] and MRST2001 [108], and three polarized PDF sets, AAC2003 [109], BB2002 [110], and GRSV2000 [111]. For $c$ and $\bar{c}$ contributions we used the two unpolarized PDF sets $[107,108]$ and the positivity conditions that $|\Delta c / c| \leqslant 1$ and $|\Delta \bar{c} / \bar{c}| \leqslant 1$. To compare with the RCQM pre-


FIG. 24. Results for $(\Delta u+\Delta \bar{u}) /(u+\bar{u})$ and $(\Delta d+\Delta \bar{d}) /(d+\bar{d})$ in the quark-parton model, compared with semi-inclusive data from HERMES [106] and CTEQ unpolarized PDF [107] as described in the text, the RCQM predictions (dash-dotted) [28], predictions from LSS 2001 NLO polarized parton densities at $Q^{2}=5(\mathrm{GeV} / c)^{2}$ (solid) [41], the statistical model at $Q^{2}=4(\mathrm{GeV} / c)^{2}$ (long-dashed) [42], the PQCD-based predictions with the HHC constraint (dashed) [31], the duality model using two different $\mathrm{SU}(6)$-breaking mechanisms (dash-dot-dotted and dash-dot-dot-dotted) [47], and predictions from chiral soliton model at $Q^{2}=4.8(\mathrm{GeV} / c)^{2}$ (dotted) [44]. The error bars of our data include the uncertainties given in Table XIII. The shaded band near the horizontal axis shows the difference between $\Delta q_{V} / q_{V}$ and $(\Delta q+\Delta \bar{q}) /(q+\bar{q})$ that needs to be added to the data when comparing with the RCQM calculation.
dictions, which are given for valence quarks, the difference between $\Delta q_{V} / q_{V}$ and $(\Delta q+\Delta \bar{q}) /(q+\bar{q})$ was estimated using the two unpolarized PDF sets $[107,108]$ and the three polarized PDF sets [109-111] and is shown as the shaded band near the horizontal axis of Fig. 24. Here $q_{V}\left(\Delta q_{V}\right)$ is the unpolarized (polarized) valence quark distribution for $u$ or $d$ quark. Results shown in Fig. 24 agree well with the predictions from the RCQM [28] and the LSS 2001 NLO polarized parton densities [41]. The results agree reasonably well with the statistical model calculation [42]. But results for the $d$ quark do not agree with the predictions from the leadingorder PQCD LSS(BBS) parametrization [31] assuming hadron helicity conservation.

## VII. CONCLUSIONS

We have presented precise data on the neutron spin asymmetry $A_{1}^{n}$ and the structure function ratio $g_{1}^{n} / F_{1}^{n}$ in the deep inelastic region at large $x$ obtained from a polarized ${ }^{3} \mathrm{He}$ target. These results will provide valuable inputs to the QCD parametrizations of parton densities. The new data show a clear trend that $A_{1}^{n}$ becomes positive at large $x$. Our results for $A_{1}^{n}$ agree with the LSS 2001 NLO QCD fit to the previous data and the trend of the $x$ dependence of $A_{1}^{n}$ agrees with the hyperfine-perturbed RCQM predictions. Data on the transverse asymmetry and structure function $A_{2}^{n}$ and $g_{2}^{n}$ were also obtained with a precision comparable to the best previous world data in this kinematic region. Combined with previous world data, the matrix element $d_{2}^{n}$ was evaluated and the new value differs from zero by more than two standard deviations. This result suggests that the higher-twist contribution is positive. Combined with the world proton data, the polarized quark distributions $(\Delta u+\Delta \bar{u}) /(u+\bar{u})$ and $(\Delta d+\Delta \bar{d}) /(d$ $+\bar{d})$ were extracted based on the quark-parton model. While results for $(\Delta u+\Delta \bar{u}) /(u+\bar{u})$ agree well with predictions from various models and fits to the previous data, results for $(\Delta d+\Delta \bar{d}) /(d+\bar{d})$ agree with the predictions from RCQM and from the LSS 2001 fit, but do not agree with leading-order PQCD predictions that use hadron helicity conservation. Since hadron helicity conservation is based on the assumption that quarks have negligible orbital angular momentum, the new results suggest that the quark orbital angular momentum, or other effects beyond leading-order PQCD, may play an important role in this kinematic region.

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## APPENDIX A: FORMALISM FOR ELECTRON DEEP INELASTIC SCATTERING

The fundamental quark and gluon structure of strongly interacting matter is studied primarily through experiments that emphasize hard scattering from the quarks and gluons at sufficiently high energies. One important way of probing the distribution of quarks and antiquarks inside the nucleon is electron scattering, where an electron scatters from a single quark or antiquark inside the target nucleon and transfers a large fraction of its energy and momentum via exchanged photons. In the single photon exchange approximation, the electron interacts with the target nucleon via only one photon, as shown in Fig. 25 [6], and probes the quark structure of the nucleon with a spatial resolution determined by the four-momentum transfer squared of the photon, $Q^{2} \equiv-q^{2}$. Moreover, if a polarized electron beam and a polarized target are used, the spin structure of the nucleon becomes accessible.

In the following we denote the incident electron energy by $E$, the energy of the scattered electron by $E^{\prime}$ (thus the energy transfer of the photon is $\nu=E-E^{\prime}$ ), and the three-momentum transfer from the electron to the target nucleus by $\vec{q}$.

## 1. Structure functions

In the case of unpolarized electrons scattering off an unpolarized target, the differential cross section for detecting the outgoing electron in a solid angle $d \Omega$ and an energy range $\left(E^{\prime}, E^{\prime}+d E^{\prime}\right)$ in the laboratory frame can be written as

$$
\begin{equation*}
\frac{d^{2} \sigma}{d \Omega d E^{\prime}}=\left(\frac{d \sigma}{d \Omega}\right)_{\mathrm{Mott}} \times\left[\frac{1}{\nu} F_{2}\left(x, Q^{2}\right)+\frac{2}{M} F_{1}\left(x, Q^{2}\right) \tan ^{2} \frac{\theta}{2}\right], \tag{A1}
\end{equation*}
$$

where $\theta$ is the scattering angle of the electron in the laboratory frame. The four-momentum transfer $Q^{2}$ is given by


FIG. 25. (Color online) Electron scattering in the one-photon exchange approximation.

$$
\begin{equation*}
Q^{2}=4 E E^{\prime} \sin ^{2} \frac{\theta}{2} \tag{A2}
\end{equation*}
$$

and the Mott cross section

$$
\begin{equation*}
\left(\frac{d \sigma}{d \Omega}\right)_{\text {Mott }}=\frac{\alpha^{2} \cos ^{2}(\theta / 2)}{4 E^{2} \sin ^{4}(\theta / 2)}=\frac{\alpha^{2}\left[\cos ^{2}(\theta / 2)\right]}{Q^{4}} \frac{E^{\prime}}{E} \tag{A3}
\end{equation*}
$$

with $\alpha$ the fine structure constant is the cross section for scattering relativistic electrons from a spin-0 pointlike infinitely heavy target. $F_{1}\left(x, Q^{2}\right)$ and $F_{2}\left(x, Q^{2}\right)$ are the unpolarized structure functions of the target, which are related to each other as

$$
\begin{equation*}
F_{1}\left(x, Q^{2}\right)=\frac{F_{2}\left(x, Q^{2}\right)\left(1+\gamma^{2}\right)}{2 x\left[1+R\left(x, Q^{2}\right)\right]} \tag{A4}
\end{equation*}
$$

with $\gamma^{2}=(2 M x)^{2} / Q^{2}$. Here $R$ is defined as $R \equiv \sigma_{L} / \sigma_{T}$ with $\sigma_{L}$ and $\sigma_{T}$ the longitudinal and transverse virtual photon cross sections, which can also be expressed in terms of $F_{1}$ and $F_{2}$.

Note that for a nuclear target, there exists an alternative per nucleon definition (e.g., as used in Ref. [83]) which is $1 / A$ times the definition used in this paper; here $A$ is the number of nucleons inside the target nucleus.

A review of doubly polarized DIS was given in Ref. [112]. When the incident electrons are longitudinally polarized, the cross section difference between scattering off a target with its nuclear (or nucleon) spins aligned antiparallel and parallel to the incident electron momentum is

$$
\begin{align*}
\frac{d^{2} \sigma_{\uparrow \Downarrow}}{d \Omega d E^{\prime}}-\frac{d^{2} \sigma_{\uparrow \Uparrow}}{d \Omega d E^{\prime}}= & \frac{4 \alpha^{2} E^{\prime}}{\nu E Q^{2}} \times\left[\left(E+E^{\prime} \cos \theta\right) g_{1}\left(x, Q^{2}\right)\right. \\
& \left.-2 M x g_{2}\left(x, Q^{2}\right)\right] \tag{A5}
\end{align*}
$$

where $g_{1}\left(x, Q^{2}\right)$ and $g_{2}\left(x, Q^{2}\right)$ are the polarized structure functions. If the target nucleons are transversely polarized, then the cross section difference is given by

$$
\begin{align*}
\frac{d^{2} \sigma_{\uparrow \Rightarrow}}{d \Omega d E^{\prime}}-\frac{d^{2} \sigma_{\uparrow \Leftarrow}}{d \Omega d E^{\prime}}= & \frac{4 \alpha^{2} E^{\prime 2}}{\nu E Q^{2}} \sin \theta \times\left[g_{1}\left(x, Q^{2}\right)\right. \\
& \left.+\frac{2 M E}{\nu} g_{2}\left(x, Q^{2}\right)\right] \tag{A6}
\end{align*}
$$

## 2. Bjorken scaling and its violation

A remarkable feature of the structure functions $F_{1}, F_{2}, g_{1}$, and $g_{2}$ is their scaling behavior. In the Bjorken limit [113] ( $Q^{2} \rightarrow \infty$ and $\nu \rightarrow \infty$ at a fixed value of $x$ ), the structure functions become independent of $Q^{2}$ [114]. Moreover, in this limit $\sigma_{L}$ vanishes [6]; hence $R=0$ and Eq. (A4) reduces to $F_{2}(x)=2 x F_{1}(x)$, known as the Callan-Gross relation [115].

At finite $Q^{2}$, the scaling of structure functions is violated due to the radiation of gluons by both initial and scattered quarks. These gluon radiative corrections cause a logarithmic $Q^{2}$ dependence in the structure functions, which has been verified by experimental data [116] and can be precisely calculated in PQCD using the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations [117].

## 3. From bjorken limit to finite $Q^{2}$ using the operator product expansion

In order to calculate observables at finite values of $Q^{2}$, a method called the operator product expansion (OPE) [7] can be applied to DIS, which can separate the nonperturbative part of an observable from its perturbative part. In the OPE, whether an operator is perturbative or not is characterized by the "twist" of the operator. At large $Q^{2}$ the leading-twist $t$ $=2$ term dominates, while at small $Q^{2}$ higher-twist operators need to be taken into account, which are sensitive to interactions beyond the quark-parton model, e.g., quark-gluon and quark-quark correlations [9].

## 4. Virtual photon-nucleon asymmetries

Virtual photon asymmetries are defined in terms of a helicity decomposition of the virtual photon absorption cross sections [118]. For the absorption of circularly polarized virtual photons with helicity $\pm 1$ by longitudinally polarized nucleons, the longitudinal asymmetry $A_{1}$ is defined as

$$
\begin{equation*}
A_{1}\left(x, Q^{2}\right) \equiv \frac{\sigma_{1 / 2}-\sigma_{3 / 2}}{\sigma_{1 / 2}+\sigma_{3 / 2}} \tag{A7}
\end{equation*}
$$

where $\sigma_{1 / 2(3 / 2)}$ is the total virtual photoabsorption cross section for the nucleon with a projection of $1 / 2(3 / 2)$ for the total spin along the direction of photon momentum.
$A_{2}$ is a virtual photon asymmetry given by

$$
\begin{equation*}
A_{2}\left(x, Q^{2}\right) \equiv \frac{2 \sigma_{L T}}{\sigma_{1 / 2}+\sigma_{3 / 2}} \tag{A8}
\end{equation*}
$$

where $\sigma_{L T}$ describes the interference between transverse and longitudinal virtual photon-nucleon amplitudes. Because of the positivity limit, $A_{2}$ is usually small in the DIS region and it has an upper bound given by [119]

$$
\begin{equation*}
A_{2}\left(x, Q^{2}\right) \leqslant \sqrt{\frac{R}{2}\left[1+A_{1}\left(x, Q^{2}\right)\right]} \tag{A9}
\end{equation*}
$$

These two virtual photon asymmetries, depending in general on $x$ and $Q^{2}$, are related to the nucleon structure functions $g_{1}\left(x, Q^{2}\right), g_{2}\left(x, Q^{2}\right)$, and $F_{1}\left(x, Q^{2}\right)$ via

$$
\begin{equation*}
A_{1}\left(x, Q^{2}\right)=\frac{g_{1}\left(x, Q^{2}\right)-\gamma^{2} g_{2}\left(x, Q^{2}\right)}{F_{1}\left(x, Q^{2}\right)} \tag{A10}
\end{equation*}
$$

and

$$
\begin{equation*}
A_{2}\left(x, Q^{2}\right)=\frac{\gamma\left[g_{1}\left(x, Q^{2}\right)+g_{2}\left(x, Q^{2}\right)\right]}{F_{1}\left(x, Q^{2}\right)} . \tag{A11}
\end{equation*}
$$

At high $Q^{2}$, one has $\gamma^{2} \ll 1$ and

$$
\begin{equation*}
A_{1}\left(x, Q^{2}\right) \approx \frac{g_{1}\left(x, Q^{2}\right)}{F_{1}\left(x, Q^{2}\right)} \tag{A12}
\end{equation*}
$$

In QCD the asymmetry $A_{1}$ is expected to have less $Q^{2}$ dependence than the structure functions themselves because of the similar leading-order $Q^{2}$ evolution behavior of $g_{1}\left(x, Q^{2}\right)$ and $F_{1}\left(x, Q^{2}\right)$. Existing data on the proton and the neutron asymmetries $A_{1}^{p}$ and $A_{1}^{n}$ indeed show little $Q^{2}$ dependence [53].

## 5. Electron asymmetries

In an inclusive experiment covering a large range of excitation energies the virtual photon momentum direction changes frequently, and it is usually more practical to align the target spin longitudinally or transversely to the incident electron direction than to the momentum of the virtual photon. The virtual photon asymmetries can be related to the measured electron asymmetries through polarization factors, kinematic variables, and the ratio $R$ defined in Sec. A 1. The longitudinal electron asymmetry is defined by [120]

$$
\begin{equation*}
A_{\|} \equiv \frac{\sigma_{\downarrow \uparrow}-\sigma_{\uparrow \Uparrow}}{\sigma_{\downarrow \Uparrow}+\sigma_{\uparrow \Uparrow}}=\frac{(1-\epsilon) M^{3}}{(1-\epsilon R) \nu F_{1}}\left[\left(E+E^{\prime} \cos \theta\right) g_{1}-\frac{Q^{2}}{\nu} g_{2}\right], \tag{A13}
\end{equation*}
$$

where $\sigma_{\downarrow \Uparrow}\left(\sigma_{\uparrow \Uparrow}\right)$ is the cross section of scattering off a longitudinally polarized target, with the incident electron spin aligned antiparallel (parallel) to the target spin, and $\epsilon$ is the magnitude of the virtual photon's longitudinal polarization:

$$
\begin{equation*}
\epsilon=\left[1+2\left(1+1 / \gamma^{2}\right) \tan ^{2}(\theta / 2)\right]^{-1} \tag{A14}
\end{equation*}
$$

Similarly the transverse electron asymmetry is defined for a target polarized perpendicular to the beam direction as

$$
\begin{equation*}
A_{\perp} \equiv \frac{\sigma_{\downarrow \Rightarrow}-\sigma_{\uparrow \Rightarrow}}{\sigma_{\downarrow \Rightarrow}+\sigma_{\uparrow \Rightarrow}}=\frac{(1-\epsilon) E^{\prime} M^{3}}{(1-\epsilon R) \nu F_{1}}\left[g_{1}+\frac{2 E}{\nu} g_{2}\right] \cos \theta \tag{A15}
\end{equation*}
$$

where $\sigma_{\downarrow \Rightarrow}\left(\sigma_{\uparrow \Rightarrow}\right)$ is the cross section for scattering off a transversely polarized target with incident electron spin aligned antiparallel (parallel) to the beam direction, and the scattered electrons are detected on the same side of the beam as that to which the target spin is pointing [15]. The electron asymmetries can be written in terms of $A_{1}$ and $A_{2}$ as

$$
\begin{equation*}
A_{\|}=D\left(A_{1}+\eta A_{2}\right) \tag{A16}
\end{equation*}
$$

and

$$
\begin{equation*}
A_{\perp}=d\left(A_{2}-\xi A_{1}\right), \tag{A17}
\end{equation*}
$$

where the virtual photon polarization factor is given by

$$
\begin{equation*}
D=\frac{1-(1-y) \epsilon}{1+\epsilon R} \tag{A18}
\end{equation*}
$$

with $y \equiv \nu / E$ the fractional energy loss of the incident electron. The remaining kinematic variables are given by

$$
\begin{gather*}
\eta=\left(\epsilon \sqrt{Q^{2}}\right) /\left(E-E^{\prime} \epsilon\right),  \tag{A19}\\
\xi=\eta(1+\epsilon) /(2 \epsilon),  \tag{A20}\\
d=D \sqrt{2 \epsilon /(1+\epsilon)} . \tag{A21}
\end{gather*}
$$

## 6. Extracting polarized structure functions from asymmetries

From Eqs. (A16) and (A17) the virtual photon asymmetries $A_{1}$ and $A_{2}$ can be extracted from measured electron asymmetries as

$$
\begin{equation*}
A_{1}=\frac{1}{D(1+\eta \xi)} A_{\|}-\frac{\eta}{d(1+\eta \xi)} A_{\perp} \tag{A22}
\end{equation*}
$$

and

$$
\begin{equation*}
A_{2}=\frac{\xi}{D(1+\eta \xi)} A_{\|}+\frac{1}{d(1+\eta \xi)} A_{\perp} \tag{A23}
\end{equation*}
$$

If the unpolarized structure functions $F_{1}\left(x, Q^{2}\right)$ and $R\left(x, Q^{2}\right)$ are known, then the polarized structure functions can be extracted from measured asymmetries $A_{\|}$and $A_{\perp}$ as [112]

$$
\begin{equation*}
g_{1}\left(x, Q^{2}\right)=\frac{F_{1}\left(x, Q^{2}\right)}{D^{\prime}}\left[A_{\|}+A_{\perp} \tan (\theta / 2)\right] \tag{A24}
\end{equation*}
$$

and

$$
\begin{equation*}
g_{2}\left(x, Q^{2}\right)=\frac{y F_{1}\left(x, Q^{2}\right)}{2 D^{\prime} \sin \theta}\left[A_{\perp} \frac{E+E^{\prime} \cos \theta}{E^{\prime}}-A_{\|} \sin \theta\right], \tag{A25}
\end{equation*}
$$

with $D^{\prime}$ given by

$$
\begin{equation*}
D^{\prime}=\frac{(1-\epsilon)(2-y)}{y\left[1+\epsilon R\left(x, Q^{2}\right)\right]} \tag{A26}
\end{equation*}
$$

## APPENDIX B: FORMALISM FOR $e-{ }^{3} \mathrm{He}$ ELASTIC SCATTERING

The cross section for electron elastic scattering off an unpolarized ${ }^{3} \mathrm{He}$ target can be written as

$$
\begin{align*}
\left(\frac{d \sigma}{d \Omega}\right)^{u}= & \frac{\sigma_{\mathrm{Mott}}}{1-\tau}\left\{\frac{Q^{2}}{|\vec{q}|^{2}} F_{c}^{2}(Q)+\frac{\mu_{3_{\mathrm{H}}}^{2} Q^{2}}{2 M^{2}}\left[\frac{1}{2} \frac{Q^{2}}{|\vec{q}|^{2}}\right.\right. \\
& \left.\left.-\tan ^{2}(\theta / 2)\right] F_{m}^{2}(Q)\right\} \tag{B1}
\end{align*}
$$

where $\tau \equiv Q^{2} /\left(4 M_{t}^{2}\right)=\nu /\left(2 M_{t}\right)$ is the recoil factor, $M_{t}$ is the target $\left({ }^{3} \mathrm{He}\right)$ mass, $Q^{2}$ is calculated from Eq. (A2), $\vec{q}$ is the three-momentum transfer, $\mu^{3}{ }_{\mathrm{He}}$ is the ${ }^{3} \mathrm{He}$ magnetic moment, and $F_{c}$ and $F_{m}$ are the ${ }^{3} \mathrm{He}$ charge and magnetic form factors, which have been measured to a good precision [121]. The Mott cross section $\sigma_{\text {Mott }}$ for a target of charge $Z$ can be written as

$$
\begin{equation*}
\sigma_{\mathrm{Mott}} \equiv\left(\frac{d \sigma}{d \Omega}\right)_{\mathrm{Mott}}=\frac{Z^{2} \alpha^{2} \cos ^{2}(\theta / 2)}{4 E^{2} \sin ^{4}(\theta / 2)} \frac{E^{\prime}}{E} \tag{B2}
\end{equation*}
$$

with $E^{\prime}$ the energy of the outgoing electrons:

$$
\begin{equation*}
E^{\prime}=\frac{E}{1+(2 E / M t) \sin ^{2}(\theta / 2)} \tag{B3}
\end{equation*}
$$

The elastic cross section for a polarized target can be written as [122]

$$
\begin{equation*}
\left(\frac{d \sigma}{d \Omega}\right)^{h}=\left(\frac{d \sigma}{d \Omega}\right)^{u}+h \Delta\left(\theta^{*}, \phi^{*}, E, \theta, Q^{2}\right) \tag{B4}
\end{equation*}
$$

where $h$ is the helicity of the incident electron beam, $\Delta\left(\theta^{*}, \phi^{*}, E, \theta, Q^{2}\right)$ describes the helicity-dependent cross


FIG. 26. (Color online) Polar and azimuthal angles of the target spin.
section, $\theta^{*}$ is the polar angle, and $\phi^{*}$ is the azimuthal angle of the nucleon spin direction, as shown in Fig. 26. We write them explicitly for a target with spin parallel to the beam direction as [122]

$$
\begin{equation*}
\cos \theta^{*}=\left(E-E^{\prime} \cos \theta\right) /|\vec{q}| \tag{B5}
\end{equation*}
$$

$$
\begin{equation*}
\phi^{*}=0 . \tag{B6}
\end{equation*}
$$

The helicity-dependent part of the cross section can be written as

$$
\begin{align*}
\left(\frac{d \sigma}{d \Omega}\right)^{h=+1}-\left(\frac{d \sigma}{d \Omega}\right)^{h=-1}= & -\sigma_{\mathrm{Mott}}\left[V_{T^{\prime}} R_{T^{\prime}}\left(Q^{2}\right) \cos \theta^{*}\right. \\
& \left.+V_{T L^{\prime}} R_{T L^{\prime}}\left(Q^{2}\right) \sin \theta^{*} \cos \phi^{*}\right] \tag{B7}
\end{align*}
$$

with kinematic factors

$$
\begin{equation*}
V_{T^{\prime}} \equiv \tan \frac{\theta}{2} \sqrt{\frac{Q^{2}}{|\vec{q}|^{2}}+\tan ^{2} \frac{\theta}{2}} \tag{B8}
\end{equation*}
$$

and

$$
\begin{equation*}
V_{T L^{\prime}} \equiv-\frac{Q^{2}}{\sqrt{2}|\vec{q}|^{2}} \tan \frac{\theta}{2} \tag{B9}
\end{equation*}
$$

$R_{T^{\prime}}, R_{T L^{\prime}}$ can be related to the ${ }^{3} \mathrm{He}$ form factors $F_{c}, F_{m}$ as

$$
\begin{equation*}
R_{T^{\prime}}=\frac{2 \tau E^{\prime}}{E}\left(\mu_{3}{ }_{\mathrm{He}} F_{m}\right)^{2} \tag{B10}
\end{equation*}
$$

and

$$
\begin{equation*}
R_{T L^{\prime}}=-\frac{2 \sqrt{2 \tau(1+\tau)} E^{\prime}}{E}\left(Z F_{c}\right)\left(\mu_{3}{ }_{\mathrm{He}} F_{m}\right) . \tag{B11}
\end{equation*}
$$

The elastic asymmetry, defined by

$$
\begin{equation*}
A_{\|}^{\mathrm{el}} \equiv \frac{\left(\frac{d \sigma}{d \Omega d E^{\prime}}\right)^{h=+1}-\left(\frac{d \sigma}{d \Omega d E^{\prime}}\right)^{h=-1}}{\left(\frac{d \sigma}{d \Omega d E^{\prime}}\right)^{h=+1}+\left(\frac{d \sigma}{d \Omega d E^{\prime}}\right)^{h=-1}} \tag{B12}
\end{equation*}
$$

can therefore be calculated from Eq. (B1) and (B7) as

$$
\begin{aligned}
A_{\|}^{\mathrm{el}}= & -(1-\tau) \\
& \times \frac{\left(V_{T^{\prime}} R_{T^{\prime}} \cos \theta^{*}+V_{T L^{\prime}} R_{T L^{\prime}} \sin \theta^{*} \cos \phi^{*}\right)}{\left\{\left(Q^{2} /|\vec{q}|^{2}\right) F_{c}^{2}+\left(\mu^{2} Q^{2} / 2 M^{2}\right)\left[Q^{2} / 2|\vec{q}|^{2}-\tan ^{2}(\theta / 2)\right] F_{m}^{2}\right\}} .
\end{aligned}
$$

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