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Abstract

We establish the interrelation between the QCD scalar response of the nuclear medium and its response to a scalar probe coupled to nucleons, such as the scalar meson responsible for the nuclear binding. The relation that we derive applies at the nucleonic as well as at the nuclear levels. Non trivial consequences follow. In particular it opens the possibility of relating medium effects in the scalar meson exchange or three-body forces of nuclear physics to QCD lattice studies of the nucleon mass.

Pacs: 24.85.+p 11.30.Rd 12.40.Yx 13.75.Cs 21.30.-x

1 Introduction

The spectrum of scalar-isoscalar excitations is quite different in the vacuum and in the nuclear medium. In the second case it includes low lying nuclear excitations and also two quasi-pion states, *i.e.*, pions dressed by particle hole-excitations. All these lie at lower energies than the vacuum scalar excitations which start at $2\,m_\pi$. We have shown in previous works [1, 2, 3] that this produces a large increase of the magnitude of the scalar QCD susceptibility over its vacuum value. We have expressed the origin of this increase as arising from the mixing of the nuclear response to a scalar probe coupled to nucleonic scalar density fluctuations into the QCD scalar response.

It is natural to investigate also the reciprocal problem of the influence of the QCD scalar response to a probe which couples to the quark density fluctuations on the ordinary nuclear scalar response of nuclear physics, which is the object of the present work. We will study this influence not only for what concerns the nuclear excitations but also for a single nucleon for which only nucleonic excitations are involved. If this influence indeed exists, does it lead to non-trivial observable consequences ? We will show that this is the case, with one main application. It is the possibility to infer medium effects in the propagation of the scalar meson which binds the nucleus from QCD results, such as the lattice ones on the evolution of the nucleon mass with the pion mass.

Our article is organized as follows. In section **2** we illustrate the mixing notion of the nuclear response into the QCD one, and vice-versa in the framework of a nuclear chiral model with a scalar and vector meson exchange. We show that this mutual influence also exists at the nucleonic level. In section **3** we discuss the influence of the quark structure of the nucleon on the scalar response of nuclear physics in a framework which also incorporates confinement effects.

2 Mutual influence of the scalar QCD response and nuclear physics response

2.1 Study in a nuclear chiral model

We first remind how the usual nuclear physics response to a scalar field enters in the QCD susceptibility. For this, following ref. [1], we start from the expression of the modification of the quark condensate in the nuclear medium, $\Delta\langle\bar{q}q\rangle(\rho) = \langle\bar{q}q\rangle(\rho) - \langle\bar{q}q\rangle_{vac}$. We first use, as in ref. [1], its expression for a collection of independent nucleons :

$$\Delta\langle\bar{q}q\rangle(\rho) = Q_S\,\rho_S, \tag{1}$$

where ρ_S is the nucleon scalar density. We have introduced the scalar charge of the nucleon, $Q_S = \sigma_N/2\,m_q$, which represents the scalar number of quarks of the nucleon. The susceptibility of the nuclear medium, χ_S^A, is the derivative of the quark scalar density with respect to the quark mass. We define it in such a way that it represents a purely nuclear contribution with the vacuum susceptibility substracted off :

$$\chi_S^A = \left(\frac{\partial\Delta\langle\bar{q}q(\rho)}{\partial m_q}\right)_\mu = \left(\frac{\partial(Q_S\,\rho_S)}{\partial m_q}\right)_\mu . \tag{2}$$

Here the derivatives are taken at constant chemical potential. This expression contains two terms. One arises from the derivative of Q_S, which by definition is the free nucleon QCD scalar susceptibility, $\chi_S^N = \partial Q_S/\partial m_q$. The second one involves the derivative of the nucleon density ρ_S. This last contribution is itself built of two pieces, one involves antinucleon excitations and is small [1]. The other one involves, as shown in ref. [1], the nuclear response $\Pi_0 = -2M_N p_F/\pi^2$. In this case it is the free Fermi gas one since no interactions between nucleons have been introduced. The result of this derivation is summarized in the following equation:

$$\chi_S^A = \rho_S\,\chi_N^S\, +\, 2\,Q_S^2\,\Pi_0\,. \tag{3}$$

It says that the nuclear susceptibility is, as expected, the sum of a term arising from the individual nucleon response, *i.e.*, from the nucleonic excitations, and of a term linked to the nuclear excitations. This decomposition survives the introduction of the interactions between the nucleons, as will be shown next. The previous result has been generalized in ref. [2] to an assembly of nucleons interacting through a scalar and a vector meson exchanges, working at the mean field level as in relativistic mean field theories. The original point with respect to standard relativistic theories is that, following our suggestion of ref. [4], the nuclear scalar field is identified with a scalar field of the linear sigma model. Rather than the sigma field, chiral parner of the pion, this is a chiral invariant, denoted S, associated with the chiral circle radius. Nevertheless the nuclear scalar field influences the condensate. Ignoring pion loops the distinction between the two scalar fields, the chiral invariant nuclear one and the non chiral invariant sigma one, wil be ignored.

In ref. [2], the condensate was obtained as the derivative of the grand potential with respect to the quark mass (Feynman-Hellmann theorem) and the susceptibility as the derivative of the condensate, both being taken at constant chemical potential. The result for the susceptibility, as given in ref. [2], reads :

$$\chi_S = \left(\frac{\partial\langle\bar{q}q\rangle}{\partial m_q}\right)_\mu \simeq -2\,\frac{\langle\bar{q}q\rangle^2_{vac}}{f_\pi^2}\,\left(\frac{\partial\bar{S}}{\partial c}\right)_\mu . \tag{4}$$

$\bar{S} \equiv f_\pi + \bar{s}$ is the expectation value of the chiral invariant scalar field and $c = f_\pi\, m_\pi^2$ is the symmetry breaking parameter of the model used in [2]. The quantity $\left(\partial\bar{S}/\partial c\right)_\mu$ was shown in ref. [2] to be related to the in-medium sigma propagator :

$$\left(\frac{\partial\bar{S}}{\partial c}\right)_\mu = -D^*_\sigma(0) = \frac{1}{m^{*2}_\sigma} - \frac{g_S^2}{m^{*2}_\sigma}\,\Pi_S(0)\,\frac{1}{m^{*2}_\sigma} \tag{5}$$

where $\Pi_S(0)$ is the full RPA scalar polarization propagator and m^*_σ is the in-medium sigma mass, obtained from the second derivative of the energy density with respect to the order parameter :

$$m^{*2}_\sigma = \frac{\partial^2\varepsilon}{\partial\bar{s}^2} = V''(\bar{s}) + \frac{\partial\,(g_S)}{\partial\bar{s}}\,\rho_S = m^2_\sigma\left(1 + \frac{3\bar{s}}{f_\pi} + \frac{3}{2}\left(\frac{\bar{s}}{f_\pi}\right)^2\right) \tag{6}$$

where the potential V responsible for the spontaneous symmetry breaking is the standard quartic one of the linear sigma model, $V = (m_\sigma^2/2)\,(s^2 + s^3/f_\pi + s^4/(4f_\pi^2))$. At this stage the nucleons are sructureless and hence we ignore the medium renormalization of g_S, *i.e.* we take g_S to be independent of s. The mean scalar field $\bar{s}$ being negative, the term linear in $\bar{s}$ lowers the sigma mass by an appreciable amount ($\simeq 30$ % at ρ_0). This is the chiral dropping associated with chiral restoration [5] and arising from the 3σ interaction as depicted in fig 1.

Since we are interested only in the medium effects the vacuum value of the quantity $\left(\partial\bar{S}/\partial c\right)_\mu = 1/{m_\sigma}^2$ has to be subtracted off in eq. (5) and the purely nuclear suceptibility, χ_S^A, writes :

$$\chi_S^A = 2\,\frac{\langle\bar{q}q\rangle^2_{vac}}{f_\pi^2}\left[\frac{3\,\bar{s}/f_\pi + \frac{3}{2}\,(\bar{s}/f_\pi)^2}{m^{*2}_\sigma} + \frac{g_S^2}{m^{*2}_\sigma}\,\Pi_S(0)\,\frac{1}{m^{*2}_\sigma}\right] . \tag{7}$$

We see that χ_S^A receives two types of contributions, the second denoted as $(\chi_S^A)^{nuclear}$ being proportionnal to the full RPA scalar response Π_S (the response to the scalar nuclear field is $g_S^2\,\Pi_S$). The corresponding proportionality factor r between this second contribution and $g_S^2\,\Pi_S$ writes, to leading order, *i.e.*, neglecting the medium modification of the sigma mass :

$$r = \frac{(\chi_S^A)^{nuclear}}{g_S^2\,\Pi_S(0)} = 2\,\frac{\langle\bar{q}q\rangle^2_{vac}}{f_\pi^2\,m_\sigma^4} \simeq 2\,\frac{(Q_S^s)^2}{g_S^2} \tag{8}$$

where we have introduced the nucleon scalar charge Q_S^s from the scalar field, defined below. In the sigma model the free nucleon sigma commutator is the sum of two contributions, one arising from the pion cloud, which depends on the mean value of the squared pion field, *i.e.*, on the scalar number of pions in the nucleonic cloud. In the mean field approximation where pion loops are ignored this term does not appear. The other one, Q_S^s, arises from the scalar meson [6, 7, 8]. It is linear in the σ field :

$$Q_S^s = \frac{\sigma_N^s}{2m_q} = -\frac{\langle\bar{q}q\rangle_{vac}}{f_\pi}\int d^3r\,\langle N|\sigma(\vec{r})|N\rangle = -\frac{\langle\bar{q}q\rangle_{vac}}{f_\pi}\,\frac{g_S}{m_\sigma^2} \tag{9}$$

which establishes relation (8) if we ignore the in-medium modification of Q_S^s, *i.e.*, the difference beween m_σ^* and m_σ.

We now turn to the first part of χ_S^A which depends on the mean scalar field $\bar{s}$. We will show that it provides an information on the nucleon susceptibility. For this we investigate the low density limit of eq. (7). In this case, $\bar{s}$ reduces to $\bar{s} = -g_S\,\rho_S/m_\sigma^2$, and we can ignore the term in $\bar{s}^2$ as well as the difference beween m_σ^* and m_σ. In this limit the first term in the expression (7) of χ_S^A is linear in the density. In the decomposition of eq. (3) for χ_S^A, it obviously belongs to the individual nucleon contribution, $\rho_S\,\chi_S^N$, to the nuclear susceptibility. Writing the linear term explicitly in eq. (7) we deduce the free nucleon scalar susceptibility from the scalar field, $(\chi_S^N)^s$:

$$(\chi_S^N)^s \;=\; -2\,\frac{\langle\bar{q}q\rangle_{vac}^2}{f_\pi^3}\,\frac{3\,g_S}{m_\sigma^4}, \tag{10}$$

which is negative. The existence of a contribution to the nucleon susceptibility from the scalar field as given by the expression 10 is a new information provided by this study with interacting nucleons. We have obtained it from the low density expression of χ_S^A. Another way to derive it is from the derivative with respect to the quark mass of the scalar charge Q_S^s of eq. (9) :

$$(\chi_S^N)^s \;=\frac{\partial Q_S^s}{\partial m_q} = \frac{\partial}{\partial m_q}\left(-\,\frac{\langle\bar{q}q\rangle_{vac}}{f_\pi}\,\frac{g_S}{m_\sigma^2}\right). \tag{11}$$

Using the fact that, in the model, $\langle\bar{q}q\rangle_{vac}/f_\pi$ does not depend on m_q, only the derivative of the sigma mass with respect to m_q enters which, according to the Feynman-Hellmann theorem, is linked to the sigma commutator, σ_σ, of the σ. In the linear sigma model the derivative with respect to the quark mass is replaced by the derivative with respect to the symmetry breaking parameter, $c = f_\pi\,m_\pi^2$, keeping the other original parameters of the model, λ and v, constant. The result is :

$$\sigma_\sigma = m_q\,\frac{\partial m_\sigma}{\partial m_q} = \frac{3}{2}\,\frac{m_\pi^2}{m_\sigma}. \tag{12}$$

When inserted in eq. (11), it leads for $(\chi_S^N)^s$ to the expression of eq. (10).

We have seen that the nuclear part of the susceptibility is related to the nuclear response to the scalar field by the relation 8. Similarly we will show the nucleonic piece of the susceptibility, $(\chi_S^N)^s$, is related to the scattering amplitude of the scalar meson on the nucleon. Indeed, in the expression(6) of m_σ^{*2} there is a term linear in density which is obtained from the low density expression : $3\,\bar{s}\,m_\sigma^{*2} \simeq -(3\,g_S/f_\pi)\,\rho_S$. This term represents an optical potential for the scalar meson propagation. The corresponding σN scattering amplitude, $T_{\sigma N}$, which can also be evaluated directly from the graph of fig. 1, is equal to :

$$T_{\sigma N} = -3\,g_S/f_\pi. \tag{13}$$

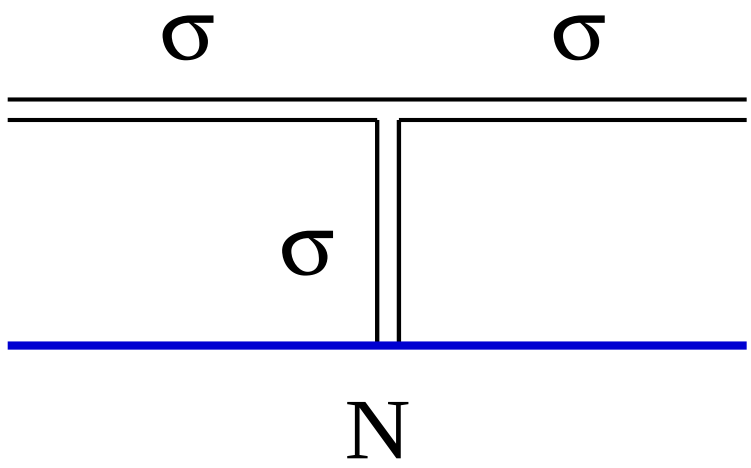

Figure 1: Contribution to the sigma-nucleon scattering amplitude responsible for the lowering of the sigma mass in the medium.

We are now in a situation to relate the nucleon scalar susceptibility (eq. (10)) to the sigma-nucleon amplitude of eq. (13), with the result :

$$r' = \frac{(\chi_S^N)^s}{T_{\sigma N}} = \frac{2\,(Q_S^s)^2}{g_S^2}. \tag{14}$$

We observe that the proportionality factor between $T_{\sigma N}$ and $(\chi_S^N)^s$ which is $2\,(Q_S^s)^2/g_S^2$, is identical to the one which involves the purely nuclear excitations. The quantity g_S which appears in the present factor is due to the σNN coupling constant. Adding now the two effects from the nucleonic and nuclear excitations the total QCD scalar susceptibility of the nuclear medium (vacuum value substracted) can therefore be related to the total response, T^A, to the nuclear scalar field through :

$$\chi_S^A = \frac{2\,(Q_S^s)^2}{g_S^2} T^A \tag{15}$$

where the two members include both the individual nucleon contribution and the one arising from the nuclear excitations,with :

$$T^A = \rho_S\, T_{\sigma N} \,+\, g_S^2\, \Pi_S. \tag{16}$$

The last term on the r.h.s. represents the influence of the Born part of the σN amplitude (in-medium corrected in particular for the Pauli effect) while the first piece arises from the non-Born part linked to nucleonic excitations.

3 Connection with lattice data

Since we have introduced QCD quantities such as the QCD scalar response, it is now interesting to connect our results to lattice simulations of the evolution of the nucleon mass with the pion mass, equivalently the quark mass. At present they do not cover the physical region but only the region beyond $m_\pi \simeq 400\,MeV$ and in order to reach the physical nucleon mass an extraplolation has to be performed. The pion cloud contribution to the nucleon self.energy has a non-analytic behavior in the quark mass, preventing a polynomial expansion in this quantity. For that reason Thomas et al [9] have separated out this contribution. This is done in a model dependent way with different cut-off forms for the pion loops (gaussian, dipole, monopole) with an adjustable parameter Λ. They expand the remaining part in terms of m_π^2 as follows:

$$M_N(m_\pi^2) = a_0 \,+\, a_2\, m_\pi^2 \,+ a_4\, m_\pi^4 \,+\, \Sigma_\pi(m_\pi, \Lambda). \tag{17}$$

The best fit value of the parameter a_4 which fixes the susceptibility shows little sensitivity to the shape of the form factor, with a value $a_4 \simeq -0.5\,GeV^{-3}$, while $a_2 \simeq 1.5\,GeV^{-1}$ (in a previous work [10] smaller values of a_2 and a_4 were given : $a_2 \simeq 1\,GeV^{-1}$ and $a_4 \simeq -0.23\,GeV^{-3}$). From the expansion of eq. (17) we can therefore infer the non-pionic pieces of the sigma commutator and of the susceptibility :

$$\sigma_N^{non-pion} = m_\pi^2\, \frac{\partial M}{\partial m_\pi^2} \;=\; a_2\, m_\pi^2 \,+\, 2\, a_4\, m_\pi^4 \;\simeq 29\,MeV\,. \tag{18}$$

It is largely dominated by the a_2 term. The corresponding value for $a_2 \simeq 1\,GeV^{-1}$ is $\sigma_N^{non-pion} = 20\,MeV$. In turn the nucleon susceptibility is :

$$\chi_S^{N,non-pion} \;= 2\,\frac{\langle\bar{q}q\rangle_{vac}^2}{f_\pi^4}\frac{\partial}{\partial m_\pi^2}\left(\frac{\sigma_N^{non-pion}}{m_\pi^2}\right) \;=\; \frac{\langle\bar{q}q\rangle_{vac}^2}{f_\pi^4}\, 4\, a_4 \;\simeq -5.4\,GeV^{-1} \tag{19}$$

The non-pionic susceptibility is found with a negative sign, as expected from the scalar meson term. In ref. [9] however, the negative sign is interpreted differently. It is attributed to possible deviations from the Gellman-Oakes-Renner (GOR) relation which links quark and pion masses. Here instead we assume the validity of the GOR relation.

It is then interesting to test if the empirical values from the lattice are compatible with our previous linear sigma model results. We thus tentatively make the following identifications :

$$Q_S^s \;= -\frac{\langle\bar{q}q\rangle_{vac}}{f_\pi}\,\frac{g_S}{m_\sigma^2} \;=\; \frac{\sigma_N^{non-pion}}{(2\, m_q)} \;\simeq -\frac{\langle\bar{q}q\rangle_{vac}}{f_\pi} a_2 = -2.4, \tag{20}$$

with $2\, m_q = 12\,MeV$ (taking $a_2 \simeq 1\,GeV^{-1}$ one would get $Q_S^s = 1.66$). It is interesting to translate this number into the value of the mean scalar field in the nuclear medium which, to leading order in density, is :

$$-\bar{s} = \frac{g_s\, \rho_S}{m_\sigma^2} = \frac{Q_S^s\, f_\pi\, \rho_S}{\langle\bar{q}q\rangle_{vac}} = \frac{\sigma_N^{non-pion}}{(2\, m_q)}\;\frac{f_\pi\, \rho_S}{\langle\bar{q}q\rangle_{vac}} = \frac{a_2 \,+\, a_4\, m_\pi^2}{f_\pi}\,\rho_S\;. \tag{21}$$

At normal density the value is $|\bar{s}(\rho_0)| \simeq 21\,MeV$, compatible with nuclear phenomenology. The second identification concerns the susceptibility. Identifying the value of the linear sigma model with the lattice one, we should have :

$$(\chi_S^N)^{non-pion} \;=\; -\,\frac{2\,(Q_S^s)^2}{g_S^2}\,\frac{3\, g_S}{f_\pi} \;=\; \frac{\langle\bar{q}q\rangle_{vac}^2}{f_\pi^4}\, 4\, a_4 \tag{22}$$

which, using the relation (20) between a_2 and Q_S^s leads to :

$$-a_4 = \frac{3}{2}\frac{(\sigma_N^{non-pion})^2}{g_S\, f_\pi\, m_\pi{}^4} \simeq \frac{3}{2}\frac{a_2^2}{g_S f_\pi} = \frac{3}{g_S f_\pi} a_2^2 = 3.5\, GeV^{-3}, \tag{23}$$

much larger than the lattice value, $-a_4 = 0.5\,\mathrm{GeV}^{-3}$. For $a_2 \simeq 1\, GeV^{-1}$ one gets $-a_4 \simeq 1.2\, GeV^{-3}$, also larger than the corresponding lattice value $(0.23\, GeV^{-3})$. Thus the linear σ model leads to a too large absolute value of the nucleon scalar susceptibility. We remind at this stage that it also fails in another respect, concerning the saturation properties of nuclear matter. The 3σ coupling present in this model, which lowers the sigma mass in the medium, prevents saturation to occur and produces the collapse [11]. Said differently the sigma nucleon scattering amplitude, $T_{\sigma N}$, of the model is too attractive. In fact what we have shown in this work is that the two problems are linked since we have found that $T_{\sigma N}$ and $(\chi_S^N)^s$ are related. These two failures are coherent. Some mechanism must be at work to introduce in both a suppression effect. In a previous work [2] we have invoked the quark meson coupling model (QMC) [12, 13] and confinement as a source of cancellation. Indeed, for three valence quarks confined in a bag of radius R, Guichon [14] derived $(\chi_S^N)^{bag} \simeq +0.25\, R \simeq 1\, GeV^{-1}$, (for $R = 0.8\, fm$). Contrary to the other components which are negative (of paramagnetic nature), it has a positive sign (of the diamagnetic type, linked to quark-antiquark excitations). In ref ([2]) we have introduced phenomenologically in the nucleon mass evolution of the linear sigma model a parameter, κ_{NS} which embodies the scalar response of the nucleon from confinement:

$$M_N^* = M_N + g_S \bar{s} + \frac{1}{2}\kappa_{NS}\bar{s}^2 \tag{24}$$

It allows a proper description of the saturation properties on nuclear matter.It is then natural to extend the linear sigma model description so as to incorporate effects arising from confinement.

4 Illustration in a hybrid model of the nucleon

We now want to generalize our previous results so as to incorporate the confinement aspect. In the following we will introduce a model of the nucleon proposed in ref. [15] which is intermediate between the two extreme pictures : the bag one and the Nambu-Jona-Lasinio (NJL) one which generates a linear sigma model. We will study both the scalar susceptibility of the nucleon and the scattering amplitude of the scalar field on the nucleon and their relation. In this framework we retain two concepts that were contained in our previous approach in the linear sigma model : (i) the nuclear scalar field is identified with the chiral field associated with the quark condensate and (ii) part of the nucleon mass originates from this condensate. This model consists in the following. Three constituant quarks moving in a non-perturbative vacuum are kept together by a central force which mimicks confinement and the effect of the color string tension. The mass M of the constituants quarks originates from the chiral condensate as in the NJL model. The nucleon mass is not $3M$ but, because of the confining force, becomes $3E(M)$ where the M dependence is fixed by the type of force. For illustration we take for simplicity a harmonic force of the form: $((K/4)(1+\gamma_0)\, r^2$, which leads to analytical results. With this particular potential the nucleon mass is :

$$M_N = 3\,E = 3\left(M + \frac{3}{2}\sqrt{\frac{K}{E+M}}\right)\,. \tag{25}$$

It is increased as compared to the value, $3M$, for three independent constituant quarks. Although oversimplified the model gives an intuitive picture of the role played by confinement. Since we assume that the nuclear scalar field is related to the quark condensate, the presence of the mean scalar field in the medium which modifies the condensate with respect to its vacuum value also affects the mass M. The derivative, $\partial M/\partial \bar{s}$, has a non-vanishing value, given by the NJL model,

$$\frac{\partial M}{\partial \bar{s}} = g_q = \frac{M}{f_\pi}. \tag{26}$$

The nucleon scalar charge, Q_S, writes :

$$Q_S = \frac{3}{2}\frac{\partial E}{\partial m_q} = \frac{3}{2}\frac{\partial E}{\partial M}\frac{\partial M}{\partial m_q} \tag{27}$$

with :

$$\frac{\partial E}{\partial M} = c_S = \frac{E+3M}{3E+M}\,. \tag{28}$$

As $E > M$, $c_S < 1$, the nucleon scalar charge is reduced as compared to a collection of three independent quarks. The nucleon scalar susceptibility, χ_S^N, given by the next derivative, is composed of two terms arising respectively from the derivative of c_S and from that of $\partial M/\partial m_q$:

$$\chi_S^N = \frac{\partial Q_S}{\partial m_q} = \frac{3}{2}\left[\frac{\partial c_S}{\partial M}\left(\frac{\partial M}{\partial m_q}\right)^2 + c_S\,\frac{\partial^2 M}{\partial^2 m_q^2}\right]$$

with:

$$\frac{\partial c_S}{\partial M} = \frac{24\,(E^2 - M^2)}{(3E+M)^3}\,. \tag{29}$$

Notice that this last derivative is positive since $E > M$ and that it vanishes in the absence of confining force, when $E = M$. Therefore the first part of the expression of χ_S^N represents the part of the susceptibility originating in confinement and, as in QMC, it is positive. We find in the susceptibility written in eq. (29) the double aspect of the mass, part arising from the constituant quark mass, i.e., from the condensate and part from confinement. The expression (29) can descibe the two extreme situations. In the MIT bag model the confined quarks are the current ones, $M = m_q$, the second term of the susceptibility disappears, only the confinement part enters. In the NJL model instead where the constituant quarks are unconfined, $E = M, c_S = 1$ and only the second term survives.

For what concerns the coupling constant, g_S , of the nucleon to the scalar field, it is given by the derivative of the nucleon mass with respect to the mean scalar field $\bar{s}$:

$$g_S = 3\,\frac{\partial E}{\partial \bar{s}} = 3\,\frac{\partial E}{\partial M}\frac{\partial M}{\partial \bar{s}} = 3\,c_S\,g_q. \tag{30}$$

The nucleon response to the scalar field originating in confinement, κ_{NS}, follows from the eq. (24) as the second derivative of the nucleon mass with respect to the scalar field :

$$\kappa_{NS} = 3\,\frac{\partial^2 E}{\partial \bar{s}^2} = 3\,\frac{\partial c_S}{\partial M}\left(\frac{\partial M}{\partial \bar{s}}\right)^2 . \tag{31}$$

The ratio, r_m, between the part of the nucleon scalar susceptibility due to confinement and κ_{NS} is

$$r_m = \frac{1}{2}\,\frac{(\frac{\partial M}{\partial m_q})^2}{(\frac{\partial M}{\partial s})^2} = \frac{2\,Q_S^2}{g_S^2}, \tag{32}$$

the same ratio, r, as was previously found in the linear sigma model.

While the quantity κ_{NS} represents the effect of the nucleon internal quark structure, there is another component of the σN amplitude, which is the tadpole term , $T_{\sigma N}$, an effect of the mexican hat chiral potential. For each constituent quark the tadpole amplitude is $t_{\sigma N} = -3\,g_q/f_\pi$. As the scalar number of constituent quarks is $3\,c_S$, the tadpole amplitude for the nucleon writes ${T_{\sigma N}}^{tadpole} = -3\,g_S/f_\pi$, the same expression as in the linear sigma model. This quantity should be compared to the other component of the susceptibility, $\frac{3}{2}c_S\,\frac{\partial^2 M}{\partial^2 m_q^2}$. We define r'_m as the corresponding ratio through :

$$\frac{3}{2}\,c_S\,\frac{\partial^2 M}{\partial m_q^2} = r'_m\,\left(-\frac{3\,g_S}{f_\pi}\right) . \tag{33}$$

In the semi-bosonized version of the NJL model we have :

$$\frac{\partial M}{\partial m_q} = -2\,\frac{g_q\,\langle\bar{q}q\rangle_{vac}}{f_\pi\,m_\sigma^2} \tag{34}$$

and

$$\frac{\partial^2 M}{\partial m_q^2} = -\frac{2\,g_q\,{\langle\bar{q}q\rangle_{vac}}^2}{f_\pi^3\,m_\sigma^4} \tag{35}$$

in such a way that the ratio r'_m becomes :

$$r'_m = \frac{2\,\langle\bar{q}q\rangle_{vac}^2}{f_\pi^2\,m_\sigma^4} = \frac{2\,Q_S^2}{g_S^2} \equiv r_m. \tag{36}$$

Since the same ratio applies to the two parts, $r'_m \equiv r_m$, it can be factorized when we add the two pieces of the susceptibility so as to obtain the relation

$$\chi_S^N = r_m\,\kappa_{NS}\,+\,r'_m\,\left(-\frac{3\,g_S}{f\pi}\right) = \frac{2\,Q_S^2}{g_S^2}\,T_N^{total} \tag{37}$$

which thus also holds in the presence of confinement. Adding the nuclear excitations contribution (term in Π_S) on both sides of the above equation once multiplied by the density we recover the relation (7) between the nuclear values χ_S^A and T_A.

Numerically we have chosen a value of the ratio $E/M \simeq 2.1$, which leads to a reasonable value for g_A and gives $c_S \simeq 0.7$. It results in a value of the dimensionless parameter $C = (f_\pi/2g_S)\,\kappa_{NS} \simeq 0.1$, while the value needed to account for the saturation properties is $C \simeq 1$ [16]. In fact the nuclear phenomenology requires a strong suppression of the tadpole term in the σN amplitude (total cancellation occurs for $C = 1.5$). On the other hand the lattice results also require a nearly total cancellation of the nucleon scalar susceptibility from the scalar meson by the effect of confinement. For us the two cancellations have a unique origin and description since the total susceptibility and the total σNamplitude are related. The condition for a total cancellation between the two components of χ_S^N, as approximately required by the phenomenology, writes

$$\frac{\partial c_S}{\partial M} - \frac{c_S}{2 g_q f_\pi} = 0, \tag{38}$$

which should approximately hold for the physical value of M, but is not fulfilled with our particular form of c_S where the second part dominates. Even if our particular model fails to account for the numerical value of C it has the merit to confirm the validity of the relation between the QCD response and the one to the nuclear scalar field in a situation where confinement enters. The relation (37) is indeed general and does not depend on the particular form of $E(M)$.

5 Applications to nuclear physics

We can now turn to the quantitative applications of the relation (37) between T_N^{total} and the scalar susceptibility. The last quantity is known from the lattice expansion. On the other hand the scalar charge which enters the relation (37) is also determined by this expansion. Therefore the only model dependent quantity to determine the amplitude, T_N^{total}, from the lattice expansion is g_S but this is only a moderate uncertainty. The important point is that the resulting value of T_N^{total} is small.

The resulting medium effects in the propagation of the nuclear scalar field can be written, using the eq. (37) and (19), as :

$$-D_s^{-1} = m_\sigma^2 + \frac{g_S^2}{2\,Q_S^2}\,\chi_S^N\,\rho_S \approx m_\sigma^2 + g_S^2\,\frac{2\,a_4}{a_2^2}\,\rho_S; \tag{39}$$

Numerically, at normal nuclear density, and for a value of the coupling constant $g_S = 10$, the second term on the rhs of the second equation takes the value $0.06\,GeV^2$ (a similar value is found for the other set of parameters a_2 and a_4). For a sigma mass of $m_\sigma = 0.75\,GeV$, this represents at ρ_0 only a 6% decrease of the mass, much less that the chiral dropping alone and in much better agreement with the nuclear phenomenology [2, 16].

At this stage, conceptual questions naturally arise. As the lattice parameter a_4 is very small one may conclude that QCD effects related to the nucleon quark substructure and chiral symmetry are simply not visible in nuclear physics as they annihilate each other. This conclusion is however erroneous. They strongly show up at the level of the three-body forces, playing an important role in the saturation mechanism, as explained below. For that purpose we introduce a new scalar field $u = s + (\kappa_{NS}/2g_S)\, s^2$ in such a way that the expression of the in-medium nucleon effective mass reduces to a simpler form according to :

$$M_N^*(\bar{u}) = M_N \,+\, g_S\, \bar{s} \,+\, \frac{1}{2}\, \kappa_{NS}\, \bar{s}^2 \equiv M_N \,+\, g_S\, \bar{u}. \tag{40}$$

Expressed in term of the u field, the chiral mexican hat potential takes the form :

$$V^{chiral} = V = \frac{m_\sigma^2}{2}\left(s^2 + \frac{s^3}{f_\pi} + \frac{s^4}{4\, f_\pi^2}\right) = \frac{m_\sigma^2}{2}\left(u^2 + \frac{u^3}{f_\pi}(1-2C) + \frac{u^4}{4}(1-8C+20C^2)\right). \tag{41}$$

In the formulation with the u field the three body forces are contained in the u^3 term :

$$V^{three-body} = \frac{m_\sigma^2}{2}\,\frac{\bar{u}^3}{f_\pi}\,(1\,-\,2C). \tag{42}$$

We remind the definition of $C = (\kappa_{NS} f_\pi)/(2g_S)$. As $\bar{u} < 0$, this force is repulsive for $C > 1/2$, which is actually the case. Without confinement, *i.e.*, $C = 0$, the chiral potential alone leads to attractive 3-body forces. The important point is that the balance between the effects of the chiral potential and of confinement are not the same in the propagation of the scalar field and in the three body forces. In the first case the amplitude T_N^{total} which governs the sigma self-energy is $T_N^{total} = 3\, g_S/f_\pi + \kappa_{NS} = (3g_S/f_\pi)\,(1 - 2C/3)$, while in the three body forces the combination is $1 - 2C$. With C of the order unity, a strong cancellation occurs in T_N^{total} while there is an overcompensation in the three body potential which becomes repulsive. The existence of repulsive three body forces in relativistic theories is strongly supported by the nuclear phenomenology [17, 18]. Using the eq. (19),(20) and (37), we can express C in terms of the lattice parameters :

$$C = \frac{3}{2} - \frac{g_S\, f_\pi}{a_2^2}\, a_4 \;\simeq 1.3, \tag{43}$$

which leads to

$$V^{three-body} = -\frac{m_\sigma^2}{2}\,\frac{\bar{u}^3}{f_\pi}\,\left(\frac{g_S\, f_\pi}{a_2^2}\, a_4 \,+\, 2\right) \tag{44}$$

Notice that since a_4 is small the term 2 dominates the parenthesis on the r.h.s. The equation of motion gives $\bar{u} \simeq -g_S\, \rho_S/m_\sigma^2$.

Numerically, for the phenomenological value $C = 1$, the contribution of the three-body forces to the energy per nucleon is :

$$\left(\frac{E}{A}\right)^{three-body} = \frac{V^{three-body}}{\rho} 20\,\left(\frac{\rho}{\rho_0}\right)^2_{,}\, MeV. \tag{45}$$

With the lattice value. $C = 1.3$, the result is $\simeq 50\%$ larger.

In the QMC approach of ref. [12, 13], the chiral aspect is not considered and hence the higher order terms in the mexican hat potential are absent. The three-body potential originates only from confinement through the nucleon scalar response :

$$V^{three-body} = \frac{m_\sigma^2}{2}\frac{\bar{u}^3}{f_\pi}(-2C^{QMC}). \tag{46}$$

Comparing the expressions (42) and (46), one sees that, numerically, our phenomenological value $C \simeq 1$ is equivalent to $C^{QMC} = 0.5$, which is close to the actual value of the QMC model.

6 Conclusion

In summary we have studied in this work the interplay between the two nuclear responses to probes which couple either to nucleon or to quark scalar density fluctuations. We have found that the two responses are reflected in each other. The scaling coefficient involves the nucleon scalar charge. Both responses incorporate the individual nucleon contributions to the nuclear response in such a way that our result holds not only at the level of the nuclear excitations but also for the nucleonic ones. The response of a nucleon to the nuclear scalar field is linked to its QCD scalar susceptibility. We have first established this results in the linear sigma model, adopting the view that the scalar field is the chiral invariant scalar field of this model, in which case it is linked to the quark condensate.

However the linear sigma model has serious shortcomings, first in nuclear physics where it makes nuclear matter collapse instead of saturate. In QCD as well it fails to account for the value of the nucleon scalar susceptibility, for which it predicts too large a magnitude as compared to the lattice result. In our views the two problems are not distinct but they are automatically linked and we have attributed them to the absence of confinement in the description. In a second step we have improved our approach to incorporate this effect. For the nucleon we have adopted a hybrid image of three constituant quarks sitting in a non perturbative vacuum and kept together by a confining potential. The nucleon mass thus originates in part from the quark condensate and in part from confinement. We have retained the concept that the nuclear scalar field has a relation to the scalar field associated with the chiral quark condensate. In this situation the presence of the nuclear scalar mean field affects the condensate and hence the constituent quark mass. We have shown that, in this model, the relation between the scalar meson self-energy and the nucleon QCD scalar susceptibility remains the same as in the linear sigma model.

The existence of relations between nuclear physics parameters (such as the opticel potential for the propagation of thr scalar field, the three-body potential) and those of QCD opens the possibility of a description of the properties of nuclear matter using as inputs the parameters of the lattice expansion of QCD. We have adopted this approach in ref.([16]) and it has been successful. With parameters close to those provided by the lattice expansion we have been able to reproduce the saturation properties of nuclear matter. This coherence, which it is not a priori acquired, suggests the validity of such an approach. It supports the idea that a part of the nucleon mass originates in the quark condensate and that the nuclear scalar field plays a role in the restoration of chiral symmetry in nuclei. We have found, both in the lattice expansion results and in the nuclear phenomenology, the need for a strong cancellation of the chiral effects by confinement in the sigma propagation. It follows that the sigma mass remains stable in the medium. Confinement nevertheless shows up very neatly in the three body potential where it dominates the attractive chiral effects, giving rise to repulsive three-body forces.

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