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# Can We Investigate Direct Time Reversal Violation?

E. Di Salvo<sup>1,2\*</sup>, Z. J. Ajaltouni<sup>1†</sup>

<sup>1</sup> Laboratoire de Physique Corpusculaire de Clermont-Ferrand,  
IN2P3/CNRS Université Blaise Pascal, F-63177 Aubière Cedex, France

<sup>2</sup> Dipartimento di Fisica and I.N.F.N. - Sez. Genova,  
Via Dodecaneso, 33, 16146 Genova, Italy

## Abstract

We illustrate a theoretical condition under which a two-body decay of a resonance violates time reversal invariance. Moreover, we examine some cases for which this condition looks particularly simple. As a consequence, we deduce tests of time reversal violation for weak decays involving spinning particles.

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\*Elvio.Disalvo@ge.infn.it

†ziad@clermont.in2p3.fr

# 1 Introduction

The lively interest shown in the last years by high-energy physicists in CP violations[1-14] is mainly due to recent hints[15-26] for New Physics (NP). Indeed, such violations - and especially those involving  $b \rightarrow s$  transitions[27, 28] - constitute a promising door to physics beyond the Standard Model[29-34] (SM), unsatisfactory under several aspects (for recent reviews see [35, 36]), although consistent with a wealth of data[37].

Time Reversal Violation (TRV) is commonly regarded as the counterpart of CP violation, in view of the CPT theorem, valid under very mild assumptions and not contradicted by any experiments, even supported by stringent tests[38]. However direct TRV has been observed only in the CPLEAR experiment[39], by comparing  $K^0 \rightarrow \bar{K}^0$  to  $\bar{K}^0 \rightarrow K^0$  transition. In fact, it is generally quite difficult to realize experimentally the inverse process of a given decay; this is why people give up showing directly such a kind of violations. Alternatively, TRV may be revealed by the presence, in a hadronic two-body weak decay amplitude, of a "weak" phase, besides the one produced by strong Final State Interactions (FSI)[40, 41, 42, 43]. However also in this case experimental uncertainties of the "strong" phases create serious problems in singling out the "weak" one[40, 44]. Incidentally, in the SM the "weak" phase is provided by the Cabibbo-Kobayashi-Maskawa (CKM) scheme.

The aim of the present note is to illustrate a theoretical condition corresponding to TRV and to suggest tests for detecting it. Such tests are feasible in the framework of experiments like those recently suggested or realized[1-14] for CP violations and are suitable for two-body decays of resonances discovered in relatively recent times - like  $B$ ,  $B_s$  and  $\Lambda_b$  -, characterized by higher masses and nonzero spins of the decay products, and therefore by a greater number of amplitudes. Section 2 is devoted to deducing and illustrating the theoretical condition for TRV. In section 3 we suggest tests to be applied to some particular decays, while in section 4 we draw a short conclusion.

## 2 A condition for TRV

We focus on hadronic two-body decays of the type

$$R_0 \rightarrow R_1 R_2, \quad (1)$$

where  $R_0$  is the original resonance and  $R_1$  and  $R_2$  the decay products, with spin  $J$ ,  $s_1$  and  $s_2$  respectively.

Our condition for TRV is derived by extending the standard treatment of Time Reversal Invariance (TRI) for two-body decays[40, 41] to the case where more than one non-leptonic decay mode is involved[45]. If (1) is a weak decay, the relative, rotationally invariant amplitude reads, at first order in the weak coupling constant,

$$A_{\lambda_1\lambda_2}^J = \langle f^{out} | H_w | JM \rangle, \quad (2)$$

where  $H_w$  is the weak hamiltonian,  $|f^{out}\rangle$  a shorthand notation for the final two-body angular momentum eigenstate  $|JM\lambda_1\lambda_2\rangle$ ,  $M$  the component of the spin of  $R_0$  along the  $z$ -axis of a given frame and  $\lambda_1$  and  $\lambda_2$  the helicities of, respectively,  $R_1$  and  $R_2$  in the rest frame of  $R_0$ . Assume  $H_w$  to be TRI, *i. e.*,

$$TH_wT^\dagger = H_w, \quad (3)$$

where  $T$  is the Time Reversal (TR) operator. Then, taking into account the antilinear character of  $T$  and the rotational invariance of the amplitude, we get[46]

$$A_{\lambda_1\lambda_2}^J = \langle f^{in} | H_w | JM \rangle^*. \quad (4)$$

Inserting a complete set of "out" states yields

$$A_{\lambda_1\lambda_2}^J = \sum_n \langle f^{in} | n^{out} \rangle^* \langle n^{out} | H_w | JM \rangle^*. \quad (5)$$

The only terms which survive in this sum correspond to the decay modes of  $R_0$ ; furthermore the non-leptonic decay modes give the main contribution, since they involve a much greater coupling constant than the semi-leptonic decay modes. Relaxing the limitation of the state  $|f^{in}\rangle$  to a two-body one, and expressing the "out" states in

terms of the  $S$ -matrix - which is unitary and, owing to  $\text{TRI}^\ddagger$ , also symmetric with respect to angular momentum eigenstates[46] -, eq. (5) can be rewritten as

$$A_m = \sum_n S_{mn} A_n^*. \quad (6)$$

Here, omitting spin and helicity indices,

$$A_m = \langle m^{out} | H_w | R_0 \rangle \quad (7)$$

and

$$S_{mn} = \langle m^{in} | S | n^{in} \rangle. \quad (8)$$

It is worth noting that eq. (6) coincides with eq. (12) of ref. [45]. The  $S$ -matrix is block-diagonal[43], since not all hadronic states are strongly connected to one another. In particular, such blocks are characterized by flavor[43].

The most general solution to eq. (6) can be obtained by diagonalizing the  $S$ -matrix. To this end we recall a result by Suzuki[45], that is,

$$S_{mn} = \sum_k O_{mk} e^{2i\delta_k} O_{kn}^T, \quad (9)$$

where  $O$  is an orthogonal matrix and the  $\delta_k$  are strong phase-shifts. Then the solution to eq. (6) is given by

$$A_m = \sum_n O_{mn} a_n e^{i\delta_n}, \quad (10)$$

where  $a_n$  are real amplitudes representing the effects of weak interactions on the decay process. Obviously complex values of one or more such amplitudes - that is, “weak” phases - would imply  $\text{TRV}$ , analogously to the case when only elastic scattering is allowed between the decay products[40].

Before considering applications of our model independent result, an important remark is in order. If a local field theory is assumed - like the SM -, a nontrivial phase of at least one  $a_n$  implies also CP violation, owing to CPT symmetry. In particular, in the SM this phase is related to the phase of the CKM matrix.

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<sup>‡</sup>We neglect weak contributions to scattering.

### 3 Possible tests for TRV

Unfortunately the result found in the previous section is generally not useful for detecting TRV. Indeed, first of all, the elements of the matrix  $O$  are not known: at best one can elaborate models, as in ref. 45. Secondly, the amplitudes  $A_n$  may be determined, at best, up to a phase per decay mode. In fact, a decay of the type (1), with spinning and unstable decay products, allows to determine, through angular distribution, polarizations and polarization correlations[10, 32, 42, 47, 48, 49], all products of the type  $A_{\lambda_1\lambda_2}A_{\lambda'_1\lambda'_2}^*$ . In particular we recall a previous paper[49], where we showed a method for extracting such products from the observables of the decay

$$\Lambda_b \rightarrow \Lambda J/\psi. \quad (11)$$

These products, in turn, amount to inferring all moduli of the amplitudes and their phases relative to a given amplitude, taken as a reference. This is a too poor piece of information, since it is not sufficient for solving the linear system (10) with respect to the products  $a_n e^{i\delta_n}$ . Therefore generally we cannot elaborate tests for TRV through the method described.

However, relation (10) may be considerably simplified, provided we choose suitable decay modes. Let us consider, for example, the decay (11) or the following decay modes of  $B^+$  and  $B^0$ , already studied, both theoretically[10, 11, 12, 13, 14, 15, 19, 30, 31, 32, 42, 47, 48, 50, 51, 52] and experimentally[1-9]:

$$B^+ \rightarrow (J/\psi K^{*+}); \quad (12)$$

$$B^0 \rightarrow (J/\psi K^{*0}); \quad (13)$$

$$B_s \rightarrow (J/\psi \phi), \quad (J/\psi \bar{K}^{*0}). \quad (14)$$

We point out that the decay modes chosen do not involve isotopic spin, in order to avoid interference among different isospin amplitudes.

We examine the scattering corresponding to FSI between the decay products in decays of the type just illustrated. Here the two decay products constitute the initial hadrons. The momentum of such hadrons in the center-of-mass system is 1.6 to 1.7  $GeV/c$ , therefore the scattering involves at least 15 partial waves. Since the decays

considered imply orbital angular momenta not greater than 2, we are faced essentially with central collisions. These give rise mainly to inelastic scattering, whose shadow constitutes the largest part of elastic scattering. Instead, anelastic reactions - that is, with excitation of one or both decay products to higher mass states - are suppressed, as well as spin-flip elastic scattering, since these processes occur only in peripheral collisions, so as to keep coherence with the initial state.

But deep inelastic collisions imply a complete loss of coherence with respect to the initial particles. Indeed, any such process includes, initially, an exchange of gluons between the two hadrons, then hard collisions among partons, followed by parton fragmentations and/or recombinations to final hadrons, and, lastly, again gluon exchange among final hadrons. Therefore the sum (10) consists of quite a lot of terms. Moreover it appears logical to assume for the phases in (10) a very rapid dependence on the index  $n$  ( $n \neq m$ ), that is, a sudden variation of the phase-shift from state to state. On the contrary, there is no reason for the terms  $a_n$  (weak amplitudes) and  $O_{mn}$  (kinematic quantities) to depend so strongly on  $n$ . Then, for  $n \neq m$ , the sum can be approximated by an integral, whose integrand consists in the product of a slowly varying function times a rapidly varying phase. Therefore only the term with  $n = m$  survives in that sum, *i. e.*,

$$A_m = O_{mm} a_m e^{i\delta_m}. \quad (15)$$

This result was obtained also by Wolfenstein[44] with a different line of reasoning (see also refs. [53, 54]).

Eq. (15) can be tested by comparing the decays considered with the  $CP$ -conjugate ones and assuming the  $CPT$  symmetry. Indeed, under this assumption,  $\bar{a}_{\bar{m}}$  differs from  $a_m$  just by a phase, while  $O_{mm}$  and  $\delta_m$  are  $CP$ -invariant, since they depend only on strong interactions. Then we have

$$|A_m| = |\bar{A}_{\bar{m}}|, \quad (16)$$

the barred amplitude referring to the  $CP$ -conjugate process, obviously with opposite helicities, denoted synthetically by  $\bar{m}$ . We stress that this test is not mandatory for the method we are going to suggest for detecting TRV.

In order to state tests for TRV, we define, preliminarily, a particular observable that we can extract from analyses of decays, that is, for a given decay mode,

$$\Phi_m = \arg(A_m) - \arg(A_{m_0}). \quad (17)$$

Here, as explained before,  $A_{m_0}$  is conventionally taken to be real. Then we define the following asymmetries:

$$\mathcal{A}_{CP} = \frac{\Phi_m - \bar{\Phi}_m}{\Phi_m + \bar{\Phi}_m}, \quad \mathcal{A}_C = \frac{\Phi_m - \bar{\Phi}_m}{\Phi_m + \bar{\Phi}_m}, \quad \mathcal{A}_P = \frac{\Phi_m - \bar{\Phi}_m}{\Phi_m + \bar{\Phi}_m}. \quad (18)$$

Here the barred quantities refer to the phases of the  $C$ -conjugate amplitudes. Given the wealth of  $b - \bar{b}$  pairs to be produced at LHC per year ( $10^{12}$ ), such tests appear to be not so unrealistic.

## 4 Conclusion

We have illustrated a theoretical condition under which TR is violated. For certain types of decays this condition looks particularly simple and suitable for stating experimental tests for TRV, to be applied to non-leptonic two-body decays involving spinning particles. The tests may be realized by means of standard analyses of decay products. The condition found is rather general, independent of any specific model; therefore, in principle, comparing the results of the suggested tests with the SM predictions could help detecting NP effects. Lastly we observe that the condition we require for obtaining evidence for TRV is opposite to the one demanded for detecting CP violations, which needs interference among amplitudes, and therefore, necessarily, more terms in the sum (10)[43].

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