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December 10, 2010

Abstract

The role of Poincaré covariant space-time translations is investigated in the case of the pseudoscalar-meson charge form factors. It is shown that this role extends beyond the standard energy-momentum conservation, which is accounted for in all relativistic quantum mechanics calculations. It implies constraints that have been largely ignored until now but should be fulfilled to ensure the full Poincaré covariance. The violation of these constraints, which is more or less important depending on the form of relativistic quantum mechanics that is employed, points to the validity of using a single-particle current, which is generally assumed in calculations of form factors. In short, these constraints concern the relation of the momentum transferred to the constituents to the one transferred to the system. How to account for the related constraints, as well as restoring the equivalence of different relativistic quantum mechanics approaches in estimating form factors, is discussed. Some conclusions relative to the underlying dynamics are given in the pion case.

PACS numbers: 12.39.Ki, 13.40.Fn, 14.40.Aq
Keywords: Relativistic quark model; Form factors; Pseudoscalar mesons

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1 Introduction

There are many relativistic frameworks in which hadronic systems can be described. Each of them has its advantages and its drawbacks. Field theory is the most ambitious one but it implies an undetermined, possibly, infinite number of degrees of freedom. Lattice calculations can account in principle for the complexity of the QCD interaction but they are limited by their necessary finite size. Finally, relativistic quantum mechanics (RQM) offers the advantage to rely on a finite number of degrees of freedom. It is therefore very close to the simplest view of the surrounding world, with hadrons considered as a bound state of quarks but these ones can only be effective objects.

In this paper, we will be concerned with the application of the last framework to the calculation of form factors of pseudoscalar mesons (pion and kaon). Many approaches were proposed, depending on the symmetry properties of the surface on which physics is described [1]. This entails that some of the generators of the Poincaré algebra have a dynamical character while the other ones have a kinematic one. The construction of the algebra was first done by Bakamjian and Thomas [2] and extended later on to the front and point forms [3]. It relies on the introduction of a mass operator that is Poincaré invariant and can be used in any form. The different forms were employed for the calculation of form factors of various hadrons, mesons as well as baryons. In principle, results should not depend on the chosen form [4] but one of them may be more efficient than another one. Looking at the results of calculations, generally based on a single-particle current, it was found that they could strongly depend on the form for the same solution of the mass operator [5] or suppose quite different solutions of this operator if they were fitted to some measurements [6, 7, 8]. This is especially true for the pion charge form factor. In such a case, one learns nothing about the underlying dynamics of the system under consideration. The dependence on the form points to the role of interaction terms that are here or there depending on the choice of the underlying hypersurface.

Recently, it was found that constraints stemming from the Poincaré covariance of currents under space-time translations [9] could play an important role in discriminating between different approaches for the calculation of form factors [10, 11]. These constraints, go beyond the usual energy-momentum conservation which is assumed in all calculations and involve a relation between the squared momentum transferred to the system and the one transferred to the constituents. They imply that the current should necessarily contain many-particle terms. The only exception is the case of the front form with $q^+ = 0$. Moreover, an indirect way for accounting for these many-particle currents was found with the important result that all forms can now produce identical predictions for form factors from the same solution of a mass operator. However, these results were concerning a pion-like scalar system composed of scalar particles, which is not of much interest for physical systems made of quarks that have spin 1/2. It is noticed that form factors corresponding to the simplest triangle Feynman diagram [5] could be reproduced exactly [10, 11]. Those for the Wick-Cutkosky model [12, 13] could also be reproduced to a very good accuracy, [10, 11], pointing in this case to a rather good determination of the mass operator [14, 15, 5]. In both cases, the mass operator has a quadratic form.

In the present work, we want to extend the above work for scalar constituents to the case of pseudoscalar mesons that are considered as quark-antiquark systems with the goal of getting, ultimately, some information on the underlying mass operator. Many works have been done for the pion and kaon mesons. Though the distinction is not always clear as soon as approximations are made, they roughly fall in two groups based on field theory [16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32] and on relativistic quantum mechanics [33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 6, 8, 44, 45]. A somewhat different group contains lattice calculations [46, 47, 48, 49]. Within the RQM approach, which we are interested in here, most works have relied on the front-form approach with $q^+ = 0$ but there are also a few works relying on the instant- or point-form approaches as well as a front-form one with $q^+ \neq 0$. Most often, simple wave functions, with a Gaussian or a power-law expression, were used. For a given wave function, calculated form factors were found to be strongly dependent on the form that was used [45] and, *vice versa*, when a fit to measurements was done, quite different wave functions were obtained [6, 8]. There are only few works using a more founded wave function, obtained from a mass operator containing both confinement and an instantaneous one-gluon exchange interactions. In absence of quark form factors, the first ones [35, 36], based on the front form with $q^+ = 0$, tend to overestimate the measurements, forcing the authors to introduce some quark form factor and, thus, providing some information about this quantity. The other one [45] was motivated by reproducing the asymptotic pion charge form factor in both the front form with $q^+ = 0$ and the Breit-frame instant form. The parameters were not especially optimized on some data but this work tends also to overestimate data though not as much as the first works. Had an other form been used (point form, or front form with a parallel momentum configuration, or instant form with a large average momentum and also a parallel momentum configuration), the corresponding form factors would underestimate the measurements instead. Thus, between the determination of the best form, the role of the wave function (or the mass operator), the contribution of two-particle currents ensuring the right asymptotic behavior, the role of possible quark form factors, we believe that there is a sufficiently large number of reasons to look at the charge form factor of pseudoscalar mesons in RQM frameworks and possible information about the mass operator. Independently, for a somewhat academic system made of scalar constituents, it was found that accounting for the constraints from covariant space-time translations was allowing one to recover expressions of form factors obtained in a dispersion-relation approach [11]. The question arises of whether such a result holds for systems consisting of quarks.

The plan of the paper is as follows. In the second section, we discuss a number of ingredients relative to the determination of the mass operator, which we would like to check ultimately from the comparison of form factors calculated using its solutions to those actually measured. They concern in particular the linear or quadratic form of the mass operator, the normalization of the solutions and the consequences for form factors in the asymptotic domain. The third section is devoted to constraints stemming from the Poincaré covariance of currents under space-time translations while their implementation for form factors of pseudoscalar mesons is discussed in the fourth section. The role of these constraints is illustrated in the fifth section by using a Gaussian wave function that is approximately a solution of an interaction containing only confinement. Some observations are made in relation with solving some paradoxes and restoring fundamental symmetries. In the sixth section, we account for one-gluon exchange contributions to both the mass operator and the current. We provide values for ingredients (string tension, quark mass and QCD coupling) that allow one to approximately reproduce measurements in the pion case while paying attention to the charge radius and the pion decay constant. The seventh section contains a discussion of the results and the conclusion.

2 Mass operator

The construction of the Poincaré algebra in RQM approaches supposes that the mass operator fulfills some conditions such as independence of the total momentum, of the underlying hyperplane orientation, or of the angular momentum but, apart from these general properties, not much is known about it. In a two-body system, it depends on a 3-dimensional internal variable, denoted $\vec{k}$ here. The mass operator therefore looks very much like a non-relativistic interaction to which it can be identified in some cases. We nevertheless stress that the RQM approach is not a center of mass non-relativistic one with some relativistic corrections. It is characterized by a deep internal consistency, often ignored, which stems from the Bakamjian-Thomas transformation in the instant form [2] and its generalizations in the other cases [3].

The Bakamjian-Thomas construction [2] was originally involving a linear mass operator, $M = M_0 + U$, but a quadratic one, $M^2 = M_0^2 + V$, could be used as well [3]. This choice supposes that the corresponding interaction, V, be related to the linear-case one, U, by the relation $V = \{M_0, U\} + U^2$. A reason to prefer the quadratic expression is that M^2 can be identified to P^2, which is more likely the quantity to be considered rather than the quantity $M = \sqrt{P^2}$, but a stronger reason is provided by the examination of results for the simplest triangle Feynman diagram with scalar constituents [5]. This one represents a theoretical model that is free of dynamical uncertainty and, moreover, can be easily studied. It provides a minimal set of results that any RQM implementation of relativity should reproduce.

Looking at the expression of the charge, which can also be considered as a definition of the normalization in the present case, it is found that, in the case of unequal-mass constituents, it can be written as:

$$F_1(Q^2=0) = \frac{16\pi^2}{N}\int \frac{d\vec{k}}{(2\pi)^3}\,\frac{(e_{1k}+e_{2k})}{2\,e_{1k}\,e_{2k}}\;\tilde{\phi}^2(\vec{k}^2) = 1\,, \tag{1}$$

where the wave function $\tilde{\phi}(\vec{k}^2 = k^2)$ is given by:

$$\tilde{\phi}(\vec{k}^2) = \frac{1}{(e_{1k}+e_{2k})^2 - M^2}\,, \tag{2}$$

with $e_{1k} = \sqrt{m_1^2 + k^2}$, $e_{2k} = \sqrt{m_2^2 + k^2}$. The occurrence of the normalization front factor, $16\pi^2/N$, may look strange here but it provides some simplification of the normalization condition when the integral is performed over the Mandelstam variable s rather than the internal momentum variable k (see sect. 4). While the wave function, $\tilde{\phi}(\vec{k}^2)$, is more appropriate for expressing form factors, due to factorizing out typical relativistic quantities $1/e_{1k}, 1/e_{2k}$, a related expression could be more useful to make the relation with the solution of a Hermitian mass operator. The relation is given by:

$$\phi_0(\vec{k}) = \phi_0(|\vec{k}| = k) = \frac{\sqrt{e_{1k}+e_{2k}}}{\sqrt{2\,e_{1k}\,e_{2k}}}\;\tilde{\phi}(\vec{k}^2)\,, \tag{3}$$

which corresponds to the normalization condition:

$$F_1(Q^2=0) = \frac{16\pi^2}{N}\int \frac{d\vec{k}}{(2\pi)^3}\,\phi_0^2(\vec{k}) = \frac{8}{N}\int dk\,k^2\,\phi_0^2(k) = 1\,. \tag{4}$$

The expression of the wave function $\phi_0(\vec{k})$ obtained from eq. (2) together with eq. (3) obviously suggests that it can be the solution of an equation with a quadratic mass operator having the form:

$$(M^2-(e_{1k}+e_{2k})^2)\,\phi_0(\vec{k}) \;=\; -\int \frac{d\vec{k}'}{(2\,\pi)^3}\,\frac{\sqrt{e_{1k}+e_{2k}}}{\sqrt{2\,e_{1k}\,e_{2k}}}\,\frac{g^2_{eff}\;\sqrt{e_{1k'}+e_{2k'}}}{\sqrt{2\,e_{1k'}\,e_{2k'}}}\,\phi_0(\vec{k}')\,, \tag{5}$$

which, due to its separable form, can be easily solved. To make sense, the coupling g^2_{eff} at the r.h.s. should tend to 0 so that its product with the diverging integral $\int d\vec{k}'\,(\sqrt{e_{1k'}+e_{2k'}}/\sqrt{2\,e_{1k'}\,e_{2k'}})\,\phi_0(\vec{k}')$ be finite. This does not prevent one, however, to use the wave function, without any problem, for the calculation of form factors for the model under consideration.

It is interesting to contrast the above result with what would be obtained from a semi-relativistic Schrödinger equation in the center of mass:

$$(M-(e_{1k}+e_{2k}))\,\phi_0'(\vec{k}) \;=\; -\int \frac{d\vec{k}'}{(2\,\pi)^3}\,\frac{g^2_{eff}}{2\sqrt{e_{1k}\,e_{2k}}\quad 2\sqrt{e_{1k'}\,e_{2k'}}}\,\phi_0'(\vec{k}')\,. \tag{6}$$

The equation has also a separable form and, up to a numerical factor, its solution is given by:

$$\phi_0'(\vec{k}) = \frac{1}{2\sqrt{e_{1k}\,e_{2k}}\,(e_{1k}+e_{2k}-M)}\,. \tag{7}$$

Comparing this solution with the one for the quadratic mass operator, it is seen that they behave differently at large k. This feature has consequences for the convergence of the norm but, more importantly, for estimating the charge form factor at large momentum transfers, as far as the charge form factor extrapolates the expression of the norm at $Q^2\neq 0$. In the present case, the charge form factor so obtained would tend to overshoot the theoretical one, evidencing a wrong asymptotic power-law behavior.

The reasons of the above discrepancy have been analyzed in detail, what is made possible by the simplicity of the theoretical model [5, 50]. Actually, considering the center of mass time component of the charge current, one should add to the contribution due to positive-energy constituents:

$$\Delta J^+ = \frac{16\pi^2}{N}\int \frac{d\vec{k}}{(2\pi)^3}\,\frac{1}{4e_{1k}\,e_{2k}\,(e_{1k}+e_{2k}-M)^2}\,, \tag{8}$$

the one due to negative-energy constituents (double Z-type diagram):

$$\Delta J^- = -\frac{16\pi^2}{N}\int \frac{d\vec{k}}{(2\pi)^3}\,\frac{1}{4e_{1k}\,e_{2k}\,(e_{1k}+e_{2k}+M)^2}\,. \tag{9}$$

Considering the sum:

$$\Delta J^+ + \Delta J^- = 2M\,\frac{16\pi^2}{N}\int \frac{d\vec{k}}{(2\pi)^3}\,\frac{e_{1k}+e_{2k}}{2\,e_{1k}\,e_{2k}\,((e_{1k}+e_{2k})^2-M^2)^2} = 2M\,, \tag{10}$$

it is seen that it factorizes into a term that is identical to the norm one obtained in relation with the quadratic mass operator, eqs. (1, 2), and a factor $2M$, which is nothing but the value of the quantity appearing in the time component of the charge current, $E_i + E_f$, calculated in the center of mass. Thus, instead of a quadratic mass operator, one could as well use a linear one but this last choice would require adding further terms in the current so that to make results consistent with the predictions of the simplest triangle Feynman diagram. We do not think this is the most efficient way to proceed and we therefore discard this choice.

The choice of a quadratic mass operator has further advantages. The invariance of the charge under boosts is more naturally satisfied with the corresponding solution, eq. (2). In the instant form, for instance, the expression of the charge for a system with momentum $\vec{P}$ may read:

$$F_1(0) = \frac{16\pi^2}{N}\int \frac{d\vec{p}}{(2\pi)^3}\,\frac{e_1+e_2}{2\,e_1\,e_2\,((e_1{+}e_2)^2-E^2)^2}\,, \tag{11}$$

where $e_1=\sqrt{m_1^2+(\vec{P}-\vec{p})^2}$, $e_2=\sqrt{m_2^2+\vec{p}^{\,2}}$, $E=\sqrt{M^2+\vec{P}^2}$. In this case, one can easily verify that the quantities $d\vec{p}\,(e_1{+}e_2)/(2\,e_1\,e_2)$ and $(e_1{+}e_2)^2-E^2$ are Lorentz invariant, which is not so for the quantities generalizing to an arbitrary total momentum those given separately by eqs. (8) and (9). One could add that the suppression of the center of mass time component of the charge current with respect to the scalar one by a factor of the order $M/2m$ is more easily accounted for within a framework based on the quadratic mass operator. Actually, some of the above properties can be ascribed to a fully relativistic calculation which involves both forward and backward time-ordered processes, as given by eqs. (8) and (9) for instance, often offering convergence properties better than the one with retaining the first of these processes.

After making the choice of using a quadratic mass operator, the following question concerns the interaction itself. In the non-relativistic case, an instantaneous approximation is most often used (partly motivated by the success of the Coulomb interaction). Again, the consideration of a simple model, the Wick-Cutkosky one [12, 13], is instructive. To reproduce the ground-state energy using the instantaneous approximation, it was found that the coupling constant should be renormalized significantly. This renormalization takes into account contributions that, in a time-ordered diagram approach, would correspond to retardation effects in a first approximation. The charge and scalar form factors calculated in this way were found to roughly agree with those calculated from the original Wick-Cutkosky model, including the large Q^2 domain [5, 10, 11]. A better agreement, at the level of a few %, was obtained by improving the interaction. The corrections, of the order of k^2/e_k^2, were chosen so that the power-law behavior of the interaction in the high-momentum limit be not changed. Interestingly, the interaction in this high-momentum limit becomes closer to the bare one. The above results are important because they indicate how the interaction entering the mass operator should be chosen so that to reproduce results expected for form factors in the asymptotic domain.

The earlier results that we reminded for the scalar-constituent case may be useful for the pseudoscalar mesons we are considering here. There is however a significant gap between the two systems. The mesons involve spin-1/2 constituents instead of scalar ones and the exchanged boson has spin 1 instead of 0. While, for the Wick-Cutkosky model, we had to only consider the ladder diagram, for a realistic description of mesons, we have to consider both the ladder and crossed diagrams. Moreover, gluons that are exchanged between quarks carry some color. This prevents that the contributions due to crossed diagrams and to retardation effects cancel, as they do in the Coulomb case. One cannot therefore rely on the successful instantaneous character of the Coulomb interaction to justify *a priori* the use of a similar approximation for the gluon-exchange case. As a further source of uncertainty, we should add the confinement interaction and its interference with the gluon-exchange one. Thus, while the determination of the mass operator in the scalar-particle case seems to be on a good track, the determination in the QCD case is largely *terra incognita*. The first one can be at most a guide for the second one. As a starting point, one could consider that the solution of the mass operator, $\phi_0(k)$, is described by a Gaussian wave function for the confining part and by a term behaving asymptotically like $k^{-7/2}$ (in our conventions [45]) for the perturbative part produced by a one-gluon exchange. We expect that the comparison of theoretical predictions and measurements will be very instructive to reduce the uncertainty on the determination of the pseudoscalar-meson mass operator. This supposes that predictions do not depend on the chosen implementation of relativity however, what we consider in next section.

3 Constraints from Poincaré covariant space-time translations

It is well known that predictions of form factors in the single-particle current approximation, when fitted to measurements, lead to solutions of the mass operator that are strongly dependent on the form that has been used [6, 7, 8]. As a complementary information, and always in the same approximation of a single-particle current, predictions made from the same solution of a mass operator exhibit a strong dependence on the form employed to implement relativity [5]. This situation is quite unsatisfactory as a correct implementation of relativity should ensure that properties of the system under consideration should behave covariantly under transformations of the Poincaré group. These transformation properties are somewhat kinematic ones and should not depend on the underlying dynamics. As the dependence on a form implies a dependence on the underlying hypersurface on which physics is described and that changing one hypersurface for another one implies interaction effects, it can be inferred that the above calculations of form factors miss some interaction effects. In this respect, we notice that accounting for constraints related to the covariant transformations of currents under space-time translations could remove discrepancies for the scalar system composed of scalar constituents. The above constraints go beyond the usual overall energy-momentum conservation that is, evidently, assumed in all calculations. There is no special difficulty in generalizing to spin-1/2 constituents considered in this work results previously obtained for spin-0 ones. We therefore summarize below the main points, providing for some of them a presentation that complements the one given elsewhere [11].

Under Poincaré space-time translations, a vector or a scalar current (denoted $J^\nu(x)$ and $S(x)$ respectively) transforms as:

$$e^{iP\cdot a}\ J^\nu(x)\ (S(x))\ e^{-iP\cdot a} = J^\nu(x+a)\ (S(x+a)), \tag{12}$$

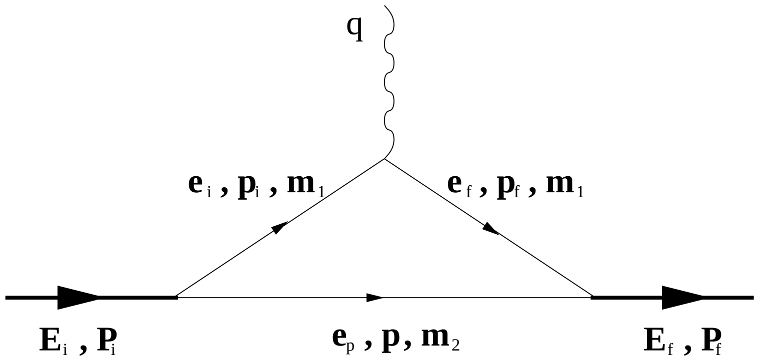

Figure 1: Graphical representation of a virtual photon absorption on a pseudoscalar meson together with kinematic definitions.

where P^μ represents the 4-momentum operator. Making the choice $a = -x$, one also obtains the relations:

$$J^\nu(x) \text{ (or } S(x)) = e^{iP\cdot x}\, J^\nu(0) \text{ (or } S(0))\, e^{-iP\cdot x}\,. \tag{13}$$

Considering the matrix element of the above relations between eigenstates of P^μ, one obtains the following relation:

$$< i\,|J^\nu(x) \text{ (or } S(x))|\, f >= e^{i(P_i-P_f)\cdot x}\; < i\,|J^\nu(0) \text{ (or } S(0))|\, f >\,, \tag{14}$$

which allows one to factorize the x dependence. Combined with the function $e^{iq\cdot x}$ describing the interaction with an external probe carrying momentum q^μ, and integrating over the x variable or assuming space-time translation invariance, one gets the usual energy-momentum conservation relation:

$$(P_f - P_i)^\mu = q^\mu\,. \tag{15}$$

This relation tells nothing about the relation of the momentum transferred to the system, q^μ, and to the constituents, $(p_f - p_i)^\mu$ (see fig. 1 for kinematic definitions). It tells nothing either on the single-particle character of the current, most often assumed, or its many-particle one. This is the place where further relations involving implicitly the covariant transformation properties of the current under translations given by eq. (12) can be useful. These ones, mentioned by Lev [9], involve the commutator of the current with the momentum operator P^μ on the one hand, the derivatives of the current with respect to x on the other hand. The simplest one is given by:

$$\begin{aligned}\Big[P^\mu\,,\; J^\nu(x)\Big] &= -i\partial^\mu\, J^\nu(x),\\ \Big[P^\mu\,,\; S(x)\Big] &= -i\partial^\mu\, S(x)\,.\end{aligned} \tag{16}$$

At the next order in P^μ, a particularly interesting relation is given by:

$$\begin{aligned}\Big[P_\mu\,,\Big[P^\mu\,,\; J^\nu(x)\Big]\Big] &= -\partial_\mu\,\partial^\mu\, J^\nu(x),\\ \Big[P_\mu\,,\Big[P^\mu\,,\; S(x)\Big]\Big] &= -\partial_\mu\,\partial^\mu\, S(x)\,.\end{aligned} \tag{17}$$

When the matrix element of this relation is taken between eigenstates of P^μ and assuming a single-particle current, one gets, after factorizing the x dependence as done in eq. (14):

$$< \,|q^2\, J^\nu(0)\ (\text{or}\ S(0))|\, >=< \,|(p_i - p_f)^2\, J^\nu(0)\ (\text{or}\ S(0))|\, > , \tag{18}$$

where q^2 represents the squared momentum transferred to the system and $(p_i - p_f)^2$ the one transferred to the constituents. We observe that the relation could be satisfied in a field-theory approach, where $(p_f - p_i)^\mu = q^\mu$, but, until recently, it was not checked within RQM approaches where it turns out to be generally violated. In this case, the violation shows that the assumption of a single-particle current is not supported by the above constraints. The current should then involve many-particle terms and it can be hoped that these ones contribute to restore the equivalence of different forms for the calculation of form factors [4]. Interestingly, it is found that eq. (18) is fulfilled in the front-form approach with $q^+ = 0$ (see below), providing in this case support for the assumption of a single-particle current. In all the other cases, the current should involve many-particle terms to satisfy the covariant character of translations evidenced by eq. (12). Calculating the contribution of many-particle terms is quite tedious and this has been done for a limited number of cases with the aim of accounting for current conservation [50] or getting the expected asymptotic behavior of the charge form factors for the pion [45] or for a system of scalar constituents in the point form-approach [50]. Moreover, if these extra terms restore the equivalence of predictions of different approaches, we expect that they should occur at all orders in the interaction. However, the fact that the form factor in the front form with $q^+ = 0$ satisfies eq. (18) with a single-particle current suggests that the task is not hopeless and that some simplification could occur.

4 Form factors with implementation of constraints

In this section, we provide information about various quantities that will be calculated in the next sections. They often represent a generalization of expressions given elsewhere for equal-mass constituents. This is also done with the aim to facilitate the comparison with other works, as far as notations or conventions are concerned.

We first remind the definitions that we are using for the charge and scalar form factors, $F_1(Q^2)$ and $F_0(Q^2)$ respectively, in the case of an interaction of constituent number 1 with the external probe:

$$\begin{aligned} \sqrt{2E_i\, 2E_f}\, \langle\, i\, |J^{(1)\mu}_{op.}|\, f\, \rangle &= (P_i + P_f)^\mu\, F_1^{(1)}(Q^2)\,, \\ \sqrt{2E_i\, 2E_f}\, \langle\, i\, |S^{(1)}_{op.}|\, f\, \rangle &= 4m_1\, F_0^{(1)}(Q^2)\,. \end{aligned} \tag{19}$$

The single-particle part of the quark operators $J^{(1)\mu}_{op.}$ and $S^{(1)}_{op.}$, we give here for fixing some relative overall factors, are given by $\bar{q}\gamma^\mu q$ and $\bar{q}q$ respectively. They determine the strength of possible many-particle contributions that should be considered, such as Z-type-diagram ones for instance. These contributions, which depend on the approach under consideration, have an interaction character.

To account for the constraints mentioned in the previous section, we proceed as follows. i) We assume that the current keeps the structure of a single-particle current otherwise

it would be difficult to recover the front-form results with $q^+ = 0$.
ii) The discrepancy between different approaches involves interaction effects that are here or there depending on its choice. Looking at the expression of wave functions entering calculations, it is found that they differ by interaction effects that are located in the coefficient of the momentum transfer, q, or some related quantity. Our proposal is to multiply this quantity by a factor α so that to fulfill eq. (18) and incorporate in this way interaction effects that have been missed. We thus obtain the equation:

$$\begin{aligned} q^2 &= \text{``}[(P_i - P_f)^2 + 2\,(\Delta_i - \Delta_f)\,(P_i - P_f)\cdot\xi + (\Delta_i - \Delta_f)^2\,\xi^2]\text{''} \\ &= \alpha^2 q^2 - 2\,\alpha\,\text{``}(\Delta_i - \Delta_f)\text{''}\; q\cdot\xi + \text{``}(\Delta_i - \Delta_f)^2\text{''}\;\xi^2\,, \end{aligned} \tag{20}$$

where the quantity Δ holds for an interaction effect, of which expression given in ref. [11] is reminded here:

$$\Delta = \frac{s - M^2}{\sqrt{(P\cdot\xi)^2 + (s - M^2)\,\xi^2} + P\cdot\xi}\,. \tag{21}$$

The 4-vector ξ^μ represents the orientation of the hyperplane on which physics is described. It appears in the equation that relates the momenta of the constituents, the total momentum of the system, and the quantity Δ [11]:

$$(p_1 + p_2)^\mu = P^\mu + \Delta\,\xi^\mu\,. \tag{22}$$

Expressions of ξ^μ for different approaches may be found in ref. [11][1]. The notation "$\cdots$" in eq. (20) reminds that the corresponding quantity should include the effect of the constraints. It is immediately seen that, in the front-form case where ξ^μ is often denoted ω^μ with $\omega^2 = 0$, this equation is satisfied for the momentum configuration $q^+ = \omega.q = 0$ with $\alpha = 1$. In this case, the equality of the squared momentum transferred to the system and to the constituents, eq. (18), is trivially fulfilled. In the other cases, one has to take into account the modification of the calculation given by the coefficient α, which is solution of eq. (20). Consistently with our intent, the coefficient α departs from the value 1 by interaction effects. While the coefficient α was given an approximate numerical value in earlier works, we stress that it is given here its exact algebraic expression as obtained from solving the above equation, similarly to what has already been done in ref. [11] for scalar constituents.

The practical implementation of the constraints discussed above for the form factors of pseudoscalar mesons does not differ much from the one for a scalar system composed of scalar constituents [11]. As for this system, there are two form factors corresponding to Lorentz 4-vector and scalar currents, $F_1(Q^2)$ and $F_0(Q^2)$ respectively, which sum up the contributions of constituents 1 and 2 with the corresponding charges. The main change concerns the introduction of the quark-spinor description. In short, this change amounts to multiply the integrand for scalar constituents, mostly given in ref. [11], by the ratio of the matrix elements of the free-particle currents for spin-1/2 and spin-0 constituents. The most general expressions of form factors for pseudoscalar mesons considered here are somewhat cumbersome. They involve results for different forms, without and with the

[1] In the case of the "instant form with the symmetry of the point form" [51] that has been used in many works, two 4-vectors ξ^μ_i and ξ^μ_f describing the velocity of the initial and final states must be introduced. In this approach, which is denoted here "P.F." and is different from the point form proposed by Dirac [52], the above equation becomes slightly more complicated but is still manageable.

effect of the implementation of the constraints and for different quark masses. Due to their technical character, we refer to a separate note [53] for details. In this note, it is also shown how charge form factors in different approaches, after implementing the effect of constraints, can be identified to a unique expression, which turns out to be the one obtained from a dispersion-relation approach.

First examples of a relationship between this last approach and a RQM one were mentioned for the front-form case with $q^+ = 0$ [54, 40]. Quite generally, such identities are non-trivial. On the one hand, the dispersion approach implies a two-dimensional integration over Mandelstam variables of the covariant interaction-free scattering amplitude of the constituents with the external field. As a result, the squared momentum transferred to the constituents, $(p_i - p_f)^2$, and to the system, q^2, are equal and eq. (18) is automatically fulfilled. On the other hand, the RQM approaches imply generally a three-dimensional integration over the momentum of one of the constituents. Different choices are possible for the matrix element of the current which is integrated over in this case (see ref. [5] for an example). Matrix elements used in the present work are consistent with what is expected in the interaction-free case. They nevertheless contain extra terms that have an interaction character and complete those accounted for by the factor α discussed above. These terms ensure that the form factor at $Q^2 = 0$, which is not affected by the above constraints, should be Lorentz invariant. This is illustrated in a particular case by the consideration of eqs. (8), (9) and (10) of sect. 2.

4.1 Expression of form factors in the dispersion-relation approach

As the dispersion-relation approach is the one which the other RQM approaches are expected to converge to after implementing the effect of constraints, we give here the corresponding results. Though we are mainly interested here in the charge form factor $F_1(Q^2)$, we also include the scalar form factor $F_0(Q^2)$ as the comparison of the two form factors shows features significantly different from the scalar constituent case, especially with respect to their asymptotic behavior. The expressions for the contributions of the constituent 1 read:

$$\begin{aligned}
F_1^{(1)}(Q^2) &= \frac{1}{N}\int d\bar{s}\; d(\frac{s_i-s_f}{Q})\;\phi(s_i)\;\phi(s_f) \\
&\quad\times\frac{\left[2s_i\,s_f-\Delta m^2(2\bar{s}+Q^2)-(m_1-m_2)^2(2\bar{s}-2\Delta m^2+Q^2)\right]\,\theta(\cdots)}{D\sqrt{D}\sqrt{s_i-(m_1-m_2)^2}\;\sqrt{s_f-(m_1-m_2)^2}}\,, \\
F_0^{(1)}(Q^2) &= \frac{1}{N}\int d\bar{s}\; d(\frac{s_i-s_f}{Q})\;\phi(s_i)\;\phi(s_f) \\
&\quad\times\frac{\left[2m_1\left(\bar{s}-(m_1-m_2)^2\right)+m_2\,Q^2\right]\,\theta(\cdots)}{2\sqrt{D}\,(2m_1)\,\sqrt{s_i-(m_1-m_2)^2}\;\sqrt{s_f-(m_1-m_2)^2}}\,,
\end{aligned} \tag{23}$$

where m_1 refers to the interacting constituent, m_2 refers to the spectator one and $\Delta m^2 = m_2^2 - m_1^2$. The quantities $\bar{s}$, D and $\theta(\cdots)$ are defined as:

$$\bar{s} = \frac{s_i + s_f}{2}\,,$$

$$
\begin{aligned}
& D = 4\bar{s} + Q^2 + \frac{(s_i - s_f)^2}{Q^2}\,, \\
& \theta(\cdots) = \theta\Big(\frac{s_i s_f}{D}\, c_{\Delta m^2} - m_2^2\Big)\,, \\
& \text{with}\;\; c_{\Delta m^2} = \Big(1 + \frac{\Delta m^2}{s_i}\Big)\Big(1 + \frac{\Delta m^2}{s_f}\Big) + \frac{\Delta m^2 (s_i - s_f)^2}{Q^2 s_i s_f}\,.
\end{aligned} \tag{24}
$$

In the equal-mass constituent case, where there is no difference between contributions of constituents 1 and 2, the above expressions simplify to read:

$$
\begin{aligned}
F_1(Q^2) &= \frac{1}{N}\int d\bar{s}\; d(\frac{s_i - s_f}{Q})\; \phi(s_i)\; \phi(s_f)\; \frac{2\sqrt{s_i\, s_f}\;\theta(\cdots)}{D\sqrt{D}}\,, \\
F_0(Q^2) &= \frac{1}{N}\int d\bar{s}\; d(\frac{s_i - s_f}{Q})\; \phi(s_i)\; \phi(s_f)\; \frac{(2\bar{s} + Q^2)\;\theta(\cdots)}{4\sqrt{s_i\, s_f}\sqrt{D}}\,.
\end{aligned} \tag{25}
$$

It is noticed that the above form factors differ from the ones for scalar constituents by making the exchange of factors $2\sqrt{s_i\, s_f}$ and $2\bar{s} + Q^2$ at the numerator. This immediately shows that the charge form factor will decrease asymptotically faster than the scalar one in the pion case compared to the scalar-constituent case. In this case, the ratio is given by a factor 2 which represents the large Q^2 limit of the ratio $2(2\bar{s} + Q^2)/D$.

We notice that the above expression for the charge form factor agrees with the one given by Melikhov [40] but disagrees with an other one given in ref. [41] for equal-mass constituents. The discrepancy factor in the integrand, $(s_i + s_f + Q^2)/(2\sqrt{s_i\, s_f})$, is the same as the factor found for scalar constituents [11]. In this case, expressions of form factors were checked by considering the simplest Feynman triangle diagram, including unequal constituent masses or different masses for the initial and final states. In the pion case, the comparison supposes to disentangle the effect of the Wigner rotation used in one of the approach [41] (see appendix A).

4.2 Expression of form factors in the front-form approach with $q^+ = 0$

As expressions of form factors in the front-form case with $q^+ = 0$ are not affected by constraints related to space-time translations, we also give their expressions. They read:

$$
\begin{aligned}
F_1^{(1)}(Q^2) &= \frac{1}{\pi\, N}\int d^2R \int_0^1 \frac{dx}{x(1-x)}\; \frac{I^0_\omega}{\tilde{I}^0_\omega}\; \phi(s_i)\; \phi(s_f)\,, \\
F_0^{(1)}(Q^2) &= \frac{1}{\pi\, N}\int d^2R \int_0^1 \frac{dx}{2x(1-x)^2}\; \frac{S}{\tilde{S}}\; \phi(s_i)\; \phi(s_f)\,,
\end{aligned} \tag{26}
$$

where the arguments, s_i and s_f, entering the wave functions may be written as:

$$
\begin{aligned}
s_i = (p+p_i)^2 &= \frac{m_1^2 + p_{i\perp}^2}{1-x} + \frac{m_2^2 + p_\perp^2}{x} - P_{i\perp}^2 = \frac{x\, m_1^2 + (1-x) m_2^2 + (\vec{R} - x\, \vec{P}_{i\perp})^2}{x\,(1-x)}\,, \\
s_f = (p+p_f)^2 &= \frac{m_1^2 + p_{f\perp}^2}{1-x} + \frac{m_2^2 + p_\perp^2}{x} - P_{f\perp}^2 = \frac{x\, m_1^2 + (1-x) m_2^2 + (\vec{R} - x\vec{P}_{f\perp})^2}{x\,(1-x)}\,.
\end{aligned} \tag{27}
$$

The ratios, $\frac{I^0_\omega}{\tilde{I}^0_\omega}$ and $\frac{S}{\tilde{S}}$ have been inserted in the expressions of form factors for the case of scalar constituents. They take into account that we are dealing here with spin-1/2 constituents instead of scalar ones. They read:

$$\begin{aligned}
\frac{I^0_\omega}{\tilde{I}^0_\omega} &= \frac{2(1-x)\Big(\bar{s}-(m_1-m_2)^2\Big) + xq^2}{2(1-x)\sqrt{s_i-(m_1-m_2)^2}\,\sqrt{s_f-(m_1-m_2)^2}}\,, \\
\frac{S}{\tilde{S}} &= \frac{2m_1\Big(\bar{s}-(m_1-m_2)^2\Big)-m_2\,q^2}{2m_1\sqrt{s_i-(m_1-m_2)^2}\,\sqrt{s_f-(m_1-m_2)^2}}\,,
\end{aligned} \tag{28}$$

The above expressions can be cast into the dispersion-relation ones by an appropriate change of variables, which allows one to reduce the 3-dimensional integration to a 2-dimensional one [11]. A different demonstration is given in ref. [40]

4.3 Normalization

The normalization is most often associated to a conserved current and, in absence of other candidate, it is taken as the charge, $F_1(0)$. Starting from eq. (23), one gets after some algebra of which detail is given elsewhere [53]:

$$F_1(0) = \frac{1}{N}\int d\bar{s}\;\phi^2(\bar{s})\,\frac{\sqrt{\bar{s}^2 - 2\bar{s}\,(m_2^2+m_1^2) + (m_2^2-m_1^2)^2}}{\bar{s}}\,. \tag{29}$$

It is noticed that the above expression is symmetrical in the exchange of the constituent masses, m_1 and m_2, as expected. The symmetry property reflects a similar one for the contribution of each constituent, allowing one to sum up their charges to get the total charge.

It is useful to make the relation of this expression with the expression of the norm in terms of the internal variable $\vec{k}$. This can be obtained as follows. Using the Bakamjian-Thomas transformation for unequal constituent masses [2], possibly generalized to any form [10][2], one can express the s variable entering the wave function $\phi(s)$ as:

$$s = (p_1+p_2)^2 = (e_{1k}+e_{2k})^2\,, \tag{30}$$

where $e_{1k} = \sqrt{m_1^2+k^2}$, $e_{2k} = \sqrt{m_2^2+k^2}$. Noticing that the above expression for s implies relations such as:

$$\begin{aligned}
2k &= \frac{\sqrt{s^2 - 2s\,(m_2^2+m_1^2) + (m_2^2-m_1^2)^2}}{\sqrt{s}}\,, \\
ds &= \frac{2k\,(e_{1k}+e_{2k})^2}{e_{1k}\,e_{2k}}\,dk\,,
\end{aligned} \tag{31}$$

as well as the relation for the wave function $\tilde{\phi}(k^2=\vec{k}^2)$ that is useful for our purpose:

$$\phi(s) = \tilde{\phi}\Big(\frac{s^2 - 2s\,(m_2^2+m_1^2) + (m_2^2-m_1^2)^2}{4s}\Big) = \tilde{\phi}(k^2=\vec{k}^2)\,, \tag{32}$$

[2]Factors e_k in eq. (2) of this reference should be replaced by $e_{1k,2k}$ depending on the particle and the factor $2e_k$ in the next equations (3, 4, 5) should be replaced by $e_{1k}+e_{2k}$.

the expression of the norm given by eq. (29) can be cast into the following ones in terms of the internal k variable:

$$F_1(0) = \frac{16\pi^2}{N} \int \frac{d\vec{k}}{(2\pi)^3}\, \tilde{\phi}^2(\vec{k}^2)\, \frac{e_{1k}+e_{2k}}{2\, e_{1k}\, e_{2k}}$$
$$= \frac{8}{N} \int dk\, k^2\, \tilde{\phi}^2(k^2)\, \frac{e_{1k}+e_{2k}}{2\, e_{1k}\, e_{2k}} = \frac{8}{N} \int dk\, k^2\; \phi_0^2(k)\,. \tag{33}$$

This last expression is a rather straightforward generalization of the norm for equal-mass constituents. Using the expression $k^2 = \vec{k}^2 = (s^2 - 2s\,(m_2^2+m_1^2) + (m_2^2-m_1^2)^2)/(4s)$ (see eq. (31)), the expression $s = (m_1^2+k_\perp^2)/(1-x) + (m_2^2+k_\perp^2)/x$ (see eq. (27)), and the resulting expressions $k^z = \Big(x(m_1^2+k_\perp^2)/(1-x) - (1-x)(m_2^2+k_\perp^2)/x\Big)/(2\sqrt{s})$, $dk^z(e_{1k}+e_{2k})/(e_{1k}\, e_{2k}) = dx/(x(1-x))$, one can also cast the above expression for the norm into the following one:

$$F_1(0) = \frac{16\pi^2}{(2\pi)^3 N} \int \frac{d^2k_\perp\; dx}{2x\,(1-x)}\; \tilde{\phi}^2(\vec{k}^2)\,. \tag{34}$$

4.4 Pseudoscalar-meson decay constant

The expression of the pion decay constant obtained in the case of an hyperplane with arbitrary orientation ξ^μ [45] can be generalized as follows to a pseudoscalar meson composed of quarks with different mass:

$$f_P = \frac{\sqrt{3}}{(2\,\pi)^3}\, \frac{4\pi}{\sqrt{N}} \int \frac{d\vec{p}}{e_1\; \xi\!\cdot\! p_2}\; \frac{m_1\, \xi\cdot p_2 + m_2\, \xi\!\cdot\! p_1}{\sqrt{s-(m_1-m_2)^2}}\; \tilde{\phi}(\vec{k}^2)\,. \tag{35}$$

For the instant form, it reads:

$$f_P^{IF} = \frac{\sqrt{3}}{(2\,\pi)^3} \frac{4\pi}{\sqrt{N}} \int \frac{d\vec{p}}{e_1\; e_2}\; \frac{m_1\, e_2+m_2\, e_1}{\sqrt{s-(m_1-m_2)^2}}\; \tilde{\phi}(\vec{k}^2)\,. \tag{36}$$

This expression can be expressed in terms of the internal variable $\vec{k}$ and the total momentum $\vec{P}$, using eqs. (2, 3) of ref. [10] (see also footnote 2). Taking into account in particular that the integration of the quantity $\vec{k}\cdot\vec{P}$ over the orientation of $\vec{k}$ gives 0, the expression is found to be equal to:

$$f_P^{IF} = \frac{\sqrt{3}}{(2\,\pi)^3} \frac{4\pi}{\sqrt{N}} \int \frac{d\vec{k}}{e_{1k}\, e_{2k}}\; \frac{m_1\, e_{2k}+m_2\, e_{1k}}{\sqrt{(e_{1k}+e_{2k})^2-(m_1-m_2)^2}}\; \tilde{\phi}(\vec{k}^2). \tag{37}$$

The expression is independent of the momentum of the pseudoscalar meson, $\vec{P}$, and therefore is Lorentz invariant. The expression in the "point form" can be obtained from eq. (35) by using the relevant definition of ξ^μ, $\xi^\mu = \frac{P^\mu}{M}$:

$$f_P^{\text{“P.F.”}} = \frac{\sqrt{3}}{(2\,\pi)^3}\, \frac{4\pi}{\sqrt{N}} \int \frac{d\vec{p}}{e_1\; P\!\cdot\! p_2}\; \frac{m_1\, P\cdot p_2 + m_2\, P\!\cdot\! p_1}{\sqrt{s-(m_1-m_2)^2}}\; \tilde{\phi}(\vec{k}^2)\,. \tag{38}$$

This expression is explicitly Lorentz invariant, in contrast to the instant-form one where this property is implicit. Similarly to this approach however, the "P.F." expression of the pion decay constant can be expressed in terms of the internal variable $\vec{k}$. The result so obtained is given by:

$$f_P^{\text{“P.F.”}} = \frac{\sqrt{3}}{(2\,\pi)^3}\frac{4\pi}{\sqrt{N}}\int \frac{d\vec{k}}{e_{1k}\,e_{2k}}\;\frac{m_1\,e_{2k}+m_2\,e_{1k}}{\sqrt{(e_{1k}+e_{2k})^2-(m_1-m_2)^2}}\;\tilde{\phi}(\vec{k}^2). \tag{39}$$

The expression in the front-form case can be found in the literature [40] but can also be obtained from eq. (35) with a standard change of variable. It reads:

$$f_P^{FF} = \frac{\sqrt{3}}{(2\,\pi)^3}\frac{4\pi}{\sqrt{N}}\int \frac{d^2k_\perp\;dx}{x\,(1-x)}\;\frac{m_1\,x+m_2\,(1-x)}{\sqrt{s-(m_1-m_2)^2}}\;\tilde{\phi}(\vec{k}^2), \tag{40}$$

where the argument of the wave function, $\vec{k}^2$, has been given previously (see eqs. (31, 32) and text after eq. (33)). Not surprisingly, the above expression can be recovered from the instant-form one, eq. (36), in the limit $\vec{P}\to\infty$. One has therefore:

$$f_P = f_P^{IF} = f_P^{\text{“P.F.”}} = f_P^{FF}. \tag{41}$$

An expression of f_P different from the above one, but nevertheless equivalent, has been given in the literature [40][3]. With our notations, it reads:

$$\begin{aligned} f_P &= \frac{\sqrt{3}\,(m_1+m_2)}{(2\,\pi)^3}\frac{4\pi}{\sqrt{N}}\int \frac{d\vec{k}}{e_{1k}\,e_{2k}}\;\frac{\sqrt{(e_{1k}+e_{2k})^2-(m_1-m_2)^2}}{2\,(e_{1k}+e_{2k})}\;\tilde{\phi}(\vec{k}^2) \\ &= \frac{\sqrt{3}\,(m_1+m_2)}{(2\,\pi)^3}\frac{4\pi}{\sqrt{N}}\int \frac{d^2k_\perp\;dx}{x\,(1-x)}\;\frac{\sqrt{s-(m_1-m_2)^2}}{2s}\;\tilde{\phi}(\vec{k}^2)\,. \end{aligned} \tag{42}$$

The equivalence of the two expressions can be checked by making the change of variable already mentioned above, $k^z = \big(x(m_1^2+k_\perp^2)/(1-x) - (1-x)(m_2^2+k_\perp^2)/x\big)/(2\sqrt{s})$.

Apart from its interest in itself, the decay constant of pseudoscalar-mesons is relevant for the description of their form factors. In the pion case, a relation to the squared charge radius has been proposed in the chiral symmetry limit [55, 56, 57, 58]:

$$r_\pi^2 = \frac{3}{4\pi^2 f_\pi^2}\,. \tag{43}$$

The relation, which does not explicitly refer to the pion description, holds in the point-like limit. A second relation concerns the asymptotic behavior of the pion form factor [59, 60]:

$$F_1(Q^2)_{Q^2\to\infty} = 16\pi f_\pi^2\,\frac{\alpha_s}{Q^2}\,. \tag{44}$$

Corrections are expected from a non-perturbative calculation [61], which could also be found in a RQM approach [45].

[3] We are grateful to the author for confirming that the absence of a factor $\sqrt{x(1-x)}$ in the denominator of his expression for f_P (eq. (2.65) of the arXiv reference) is a misprint, without consequence for numerical results presented in his paper.

5 Quantitative effect of constraints for form factors

We consider in this section the quantitative effect of constraints related to covariant space-time translations for the charge form factor of pseudoscalar mesons. This is done for both the pion and the kaon mesons. For our purpose, it is sufficient to consider the simplest description of the mesons. It is not totally arbitrary however and we assume that it is given by a Gaussian wave function with a parameter as obtained from a standard confining potential [62]:

$$\phi_0(\vec{k}) \propto exp(-\vec{k}^2(1+m_</m_>)/(2\sigma_{st})) \tag{45}$$

where σ_{st} represents the string tension ($\sigma_{st} = 0.2$ GeV$^2\simeq$1 GeV/fm), while $m_<$ and $m_>$ respectively represent the smallest and largest constituent-quark masses. The quark masses are determined by requiring that they allow one to reproduce approximately the pion or kaon decay constants (respectively 0.0924 GeV and 0.113 GeV in our conventions for the definition of these constants). As mentioned at the end of the previous section, there is in the pion case some approximate relation of this constant to the charge radius (see eq. (43)). The above relationship may be therefore relevant for describing the pion charge form factor at small Q^2. The values we use for the quark masses are $m_{u,d} = 0.25$ GeV and $m_s = 0.47$ GeV, corresponding to pion and kaon decay constants of 0.092 GeV and 0.113 GeV. To the extent where the string tension we are using is determined by fitting Regge trajectories, results presented in this section are essentially parameter free.

A Gaussian wave function has been used in many works [34, 58, 35, 37, 39, 40, 41, 42, 6, 43, 8] but, in most cases, the parameter on which it depends and the quark masses were fitted to the measured pion (or kaon) form factor. Though we show calculated form factors and measurements in the same figure, our intent in this section is not to compare them in detail. We stress that our main intent here is to examine the role of constraints related to space-time translations and determine which approach is, ultimately, the most appropriate for a comparison to measurements. Nevertheless, a comparison at this point may be useful to tell us how good are estimates based on a wave function whose parameters are obtained from considerations different from the measured form factors. It can determine whether there is a large space for improvement from considering a better wave function to be considered in the next section.

Charge form factors of the charged pions and kaons are calculated for both a small range of Q^2 ($Q^2 < 0.2$ GeV^2), which is mainly sensitive to the charge radius, and an intermediate range which could be of relevance for present and future measurements, ($0 < Q^2 < 10$ GeV^2). In this case, the form factors are multiplied by a factor Q^2, as it is expected that this product should tend asymptotically to a constant (up to log terms). The contribution that could account for this asymptotic behavior within a RQM framework [45] is however ignored in the present section but will be considered in the next one.

Different RQM approaches are considered but, in all cases, calculations are performed in the Breit frame. They involve the standard front-form one with $q^+ = 0$ (denoted F.F. (perp.), the instant-form one (denoted I.F.), a front-form one with the momentum transfer oriented along the front direction, $\vec{n}$ (denoted F.F. (parallel)) and a point-form one (denoted "P.F."). We notice that this point form differs from the Dirac one based on a hyperboloid surface [52]. As mentioned by Bakamjian [51], it is rather some kind of instant-form approach with the symmetry properties of Dirac's point-form. Contrary to some previous papers [10, 11], we do not present results for an approach inspired from Dirac's point-form [63]. These ones, which involve front-form ones with a summation over all directions (making their calculation rather lengthy), drop between the two cases shown here for a perpendicular and a parallel configuration of the momentum transfer, $\vec{q}$, and the front direction, $\vec{n}$. We could also add that the two front-form calculations presented here respectively coincide with the instant-form ones in an infinite momentum frame with a momentum transfer perpendicular and parallel to this infinite momentum.

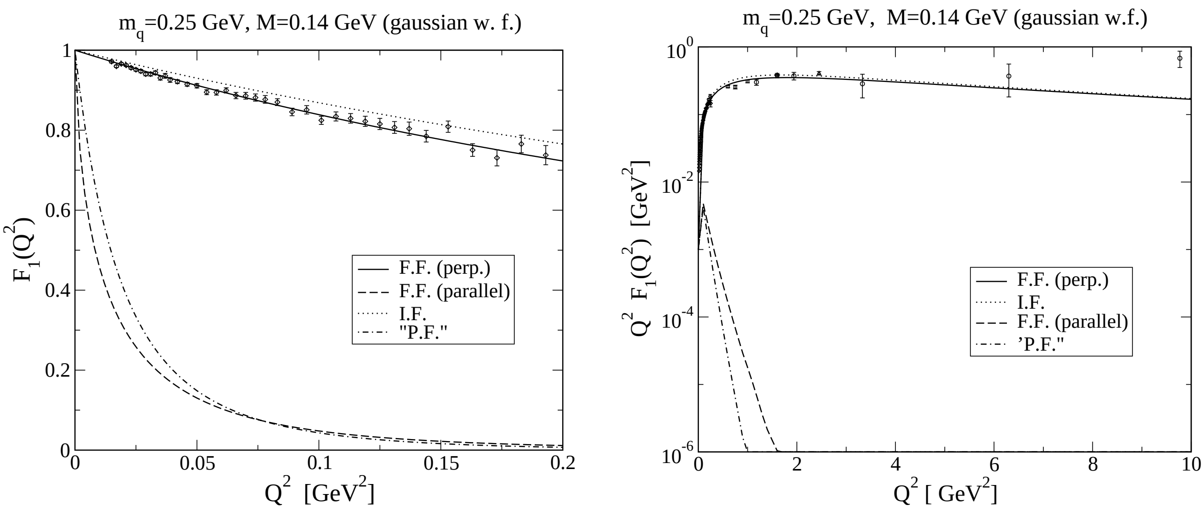

Figure 2: Pion charge form factor, $F_1(Q^2)$, at small and high Q^2 (left and right panels respectively). In the latter case, the form factor is multiplied by Q^2 and represented on a logarithmic scale to emphasize the asymptotic behavior. The different curves represent form factors without accounting for constraints from covariant space-time translations except for the F.F. (perp.) one (continuous line) that satisfies these properties. When constraints are accounted for, all curves coincide with the F.F. (perp.) one. All form factors are calculated in the Breit frame.

Expressions of form factors that are used here have been given in a paper summarizing the technical aspects [53]. It is reminded that these expressions ensure that form factors are boost and rotation independent and lead to the same results after accounting for constraints related to the covariant transformations of currents under space-time translations. As these results were given for quite general cases, we give in the appendix B the explicit expressions for the Breit-frame case considered in the present work. In this respect, we note that expressions of form factors used here differ from the ones used in ref. [45], except for the front-form case with $q^+ = 0$. This work, mainly devoted to the asymptotic behavior of the pion charge form factor, was not concerned with accounting for the constraints related to covariant space-time translations. Though the numerical effect is a relatively minor one, the choice of the current adopted there differs from the one used here at $Q^2 \neq 0$. This could allow one to get results close to each other for different approaches after taking into account the constraints discussed in this work but would prevent one to get identical results for the different approaches as here. Let's add that the form of matrix elements of the single-particle currents used here was suggested by an analysis of expressions obtained with the simplest Feynman triangle diagram [11] (and taking into account that we are dealing in this work with fermions instead of bosons).

Results for the pion and kaon charge form factors in absence of the constraints related to space-time translations are shown in figs. 2 and 3 respectively, together with the corresponding measurements (refs. [64, 65, 66, 67, 68, 69] for the pion and refs. [70, 71] for the kaon). Examination of the figures shows that front-form form factors in the perpendicular configuration ($q^+ = 0$) and the instant-form ones are close to each other. They strongly differ from the other ones (front-form ones in the parallel configuration and point-form ones), especially for the pion where discrepancies can reach many orders of magnitude in the high-Q^2 range. This result can be related to the departure of the momentum transferred to the constituents, $(p_i - p_f)^2$, from the one transferred to the system under consideration, q^2, preventing one to satisfy eq. (18) that is expected for Poincaré covariant space-time translations. Looking at eq. (20), it is found that the discrepancy in the first case, due to the vanishing of the factor $q \cdot \xi$, involves terms that are of the second order in interaction effects. In the second case, this factor does not vanish and first order effects are therefore involved. The steep slope of the pion form factor at low Q^2 can be then related to the appearance of a factor of the order $4e_k^2/M^2$ in front of the Q^2 term in the argument of wave functions entering the corresponding expressions for form factors. As the pion mass is small in comparison of the sum of the constituent masses, the effect due to this factor is necessarily large. In comparison, the kaon mass is closer to the sum of the constituent masses and, as a result, discrepancies are significantly smaller.

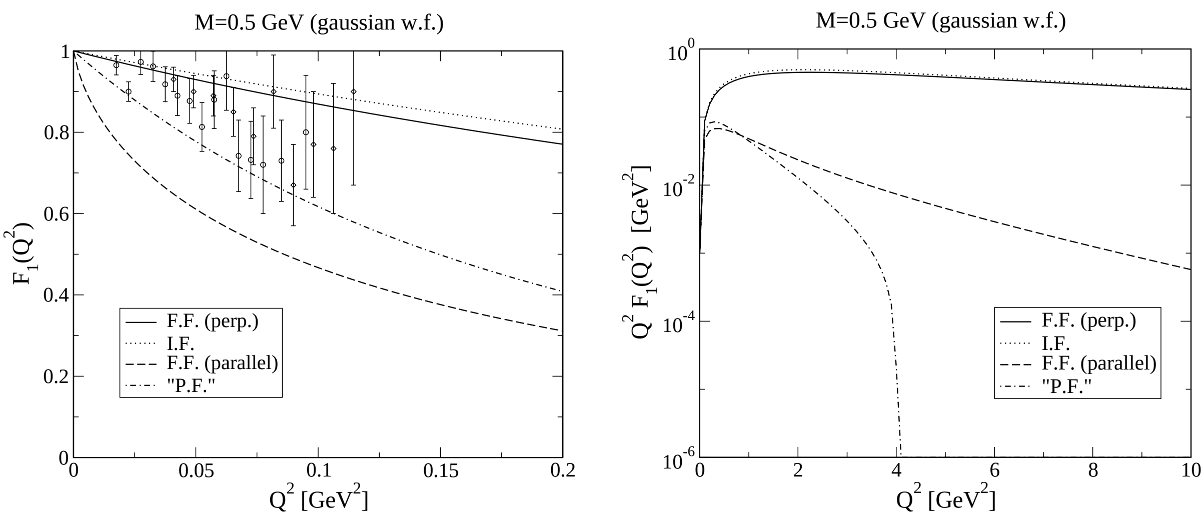

Figure 3: Same as in fig. 2, but for the kaon meson.

Not surprisingly, present results show a strong qualitative similarity with those for scalar constituents [11]. The effects mentioned here essentially involve the wave functions and are relatively insensitive to the current operator. There is however one effect that the restriction of the present study to charge form factors does not allow us to evidence. In comparison with the scalar constituent case, the charge form factor is systematically suppressed with respect to the scalar form factor at high Q^2. This effect, which involves the current operator, is roughly given by a factor $1/Q^2$.

When constraints related to covariant space-time translations are accounted for, form factors as those shown in figs. 2 and 3 tend to get closer to each other [10]. In the present case, the choice of the currents in different forms also ensures Lorentz invariance

[53]. Thus, accounting for the above constraints makes form factors shown in figs. 2 and 3 identical to the front-form ones in the perpendicular configuration ($q^+ = 0$, denoted F.F. (perp.)). There is therefore no need to present new figures. It is worth noticing that accounting for the constraints has removed tremendous discrepancies at both low and large Q^2. In particular, the steep slope of the pion charge form factor at low Q^2, which could be infinite when the pion mass vanishes, has disappeared. In this case, the constraints allow one to get rid of the paradox where the charge radius becomes infinite while the mass of the system goes to zero, what is generally obtained by increasing the attraction. More generally, the constraints can reduce or even remove ambiguities like the one which leads to get very different form factors from employing a unique wave function, depending on the mass of the system, while one would expect a unique result from a non-relativistic calculation. To some extent, the constraints restore fundamental symmetry properties that are missed from using some particular (truncated) approaches, somewhat similarly to what occurs in other fields of physics.

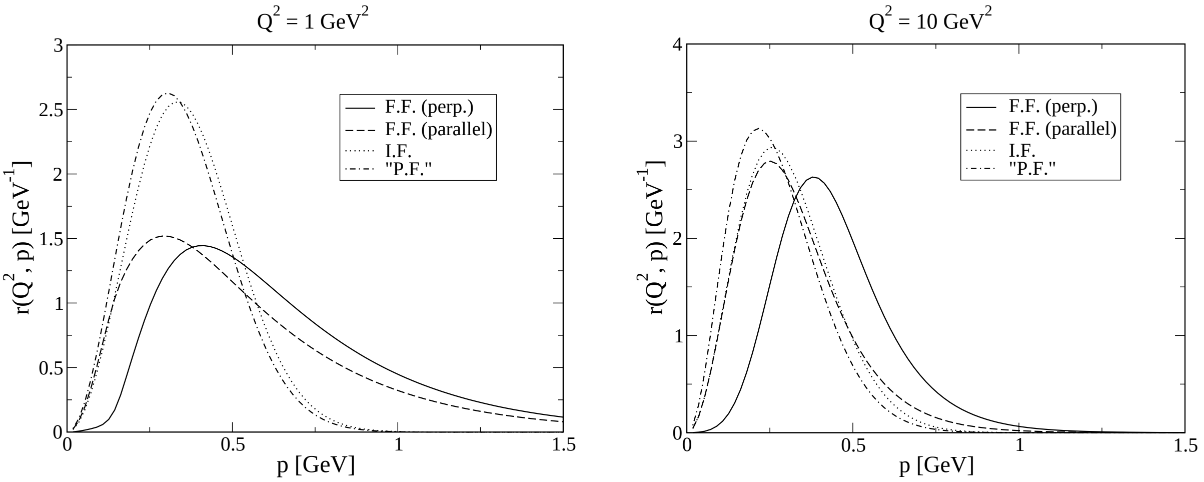

Figure 4: Contribution to the pion charge form factor as a function of the spectator momentum, p. Results represent the ratio $r(Q^2, p)$ of the integrands to the integrated form factor (the same for all cases). The integral of these ratios over p is equal to 1.

It has been shown that accounting for the constraints related to covariant space-time translations could allow one to cast the form factors calculated in different forms into a unique expression that is suggested by using a dispersion-relation approach. While expressions for form factors then involve the same integrand, we would like to stress that this result is far from being trivial. It is obtained by using a change of variables that differs from one form to the other. This is evidenced by looking at the contribution to form factors corresponding to given values of the momentum of the spectator constituent, which in the present work, is chosen as the integration variable. For illustrating this result, we considered the integrand, $f(Q^2, p)$, obtained in different forms after integrating on the components of the spectator momentum corresponding to a total momentum, $p = \sqrt{p_\parallel^2 + p_\perp^2}$. The ratios of these integrands to the form factors, which are defined as:

$$r(Q^2, p) = \frac{f(Q^2, p)}{\int dp\, f(Q^2, p)}\,, \tag{46}$$

are shown in fig. 4 as a function of the p variable for two momentum transfers, $Q^2 = 1$ and

$Q^2 = 10$ GeV^2. As the examination of the two panels shows, results depend significantly on the approach, as well as the momentum transfer. It is noticed that results at low Q^2 fall in two sets that are indicative of what occurs at the value $Q^2 = 0$ (not shown in the figure) where the two front-form results and the instant- and point-form separately coincide with each other. The contribution to the form factor extends to large momenta in the first case while it is rather concentrated at relatively small momenta in the second case. At larger Q^2, the contribution to the form factor tends to concentrate to the lower p range in all cases. It sounds that the results for the instant-form and front-form one with the parallel momentum configuration are getting closer to each other and could coincide in the infinite Q^2 limit. As the first approach in an infinite-momentum frame coincides with the second one, such a result may not be so surprising.

In this section, we concentrated on the role of constraints related to covariant space-time translations. Having shown that accounting for them amounts to obtain form factors equal to the front-form ones with $q^+ = 0$ (F.F. (perp.)), we can consider these last form factors for a first comparison with measurements. While, there is essentially no free parameter, it is found that there is a rough agreement in the pion case. It is noticeable that the calculated pion charge form factor evidences some kind of plateau in the range 1–5 GeV^2. As mentioned previously, this plateau has nothing to do with the theoretically expected one as the corresponding physics has been ignored. It simply results from the fact that the product of the form factor by Q^2 has a maximum in the above range. With this respect, the departure for the point around 10 GeV^2 probably provides hint for missing physics. In the kaon case, there could be some trend to slightly overestimate the experimental data. In view of this first comparison with measurements, improving the description of pseudoscalar meson form factors could reveal to be difficult.

6 Reproducing measurements

In this section, we intend to make some comparison with measurements, taking into account that form factors can be identified to those obtained in the standard front-form case with $q^+ = 0$ after the effect of constraints related to covariant space-time translations has been incorporated. The comparison mainly concerns the pion charge form factor, which has been measured in a momentum-transfer range larger than for the kaon case. We consider successively: further physical ingredients that should be accounted for, the perturbative calculation of the solution of the mass operator, and results so obtained once the QCD coupling α_s is given some standard value. The discussion essentially involves the value of the string tension, σ_{st}, and the quark mass, m_q. In most cases, we impose that the pion-decay constant be reproduced, which determines the value of the last quantity in terms of σ_{st} and the physical ingredients under consideration. The pion decay constant indeed enters in a chiral-symmetry calculation that provides a large part of the squared pion charge radius.

6.1 Further physical ingredients

To make a relevant comparison with measurements, further physical ingredients should be considered besides the contribution to the mass operator of the only confining force we accounted for in the previous section for simplicity. We thus expect some contribution due to the one-gluon exchange force. Some was considered in the past [35, 36, 45] but as mentioned in the introduction, it tends to overestimate the pion charge form factor at high Q^2 with the currently used value of the QCD coupling $\alpha_s \simeq 0.4$. This suggests that the underlying non-perturbative calculation needs corrections. In the present work, we perform a perturbative calculation with respect to this one-gluon exchange, which can be therefore considered as providing a minimal effect. It is expected that this contribution enhances the high-momentum tail of the solution of the mass operator and, consequently, produce an increase of the form factor at very high Q^2 in comparison to the Gaussian solution. The former has a power-law behavior $(1/Q^4)$ [72, 18] while the latter vanishes exponentially.

There is a second contribution involving one-gluon exchange. As is known, a standard estimate of the pion charge form factor from a single-particle like current misses the expected asymptotic behavior of this form factor [72, 18]. To reproduce this behavior $(1/Q^2)$, a two-particle one-gluon exchange current, which implies the off-mass shell behavior of the one-gluon exchange interaction, has to be considered. Its expression has been recently determined for a RQM calculation of the pion charge form factor [45]. Interestingly, its contribution, which involves the low-momentum component of the meson wave function, is not depending too much on its description. It provides an increase of the charge form factor corresponding to a small decrease of the squared charge radius ($\Delta r^2_{ch} \simeq -0.07$ fm^2) and an increase of the product $Q^2\,F_1(Q^2)$ (of the order of 0.2 GeV^2 at $Q^2 = 10$ GeV^2). The first contribution looks small in comparison to the total squared charge radius but is not so small if it is compared to the part of this quantity that could be attributed to the matter squared radius (as roughly obtained from the difference between the measured one, 0.43 fm^2, and the part given by a chiral-symmetry calculation in the point-like limit, $3/(4\pi^2 f_\pi^2) = 0.34$ fm^2. The second contribution is definitively relevant at the highest values of Q^2 where the pion charge form factor has been measured.

6.2 Perturbative calculation for the solution of the mass operator

To obtain a perturbative solution of the mass operator, we start from eq. (1) of ref. [45], which was given for a non-perturbative case. Due to the non-local character of the interaction, the solution is more easily calculated in momentum space than in configuration space. The correction to the unperturbed Gaussian solution, $\phi_0(\vec{k})$, is given for the free Green's function case by:

$$\Delta\phi_0(\vec{k}) = \frac{1}{4e_k^2 - M^2} \int \frac{d\vec{k}'}{(2\,\pi)^3}\, \frac{4\, g^2_{eff}\, \frac{4}{3}\, e_k\, e_{k'} \Big(2 - \frac{m_q^2\,(e_k^2+e_{k'}^2)}{2\,e_k^2\,e_{k'}^2}\Big)}{\sqrt{e_k}\;\,(\vec{k}-\vec{k}')^2\;\sqrt{e_{k'}}}\, \phi_0(\vec{k}')\,, \tag{47}$$

where g^2_{eff} is related to the strong coupling α_s by the relation $g^2_{eff}/(4\pi) = \alpha_s$. The model implicitly used for describing the confinement interaction in the previous section (harmonic-oscillator-type interaction) allows one to do better however by incorporating its effect in the Green's function. The solution then reads:

$$\Delta\phi_0(\vec{k}) = \sum_n \frac{\bar{\phi}_n(\vec{k})}{(6{+}8n)\sigma_{st} + 4m_q^2 {-} M^2}$$

$$\times \int\int \frac{d\vec{k}"}{(2\,\pi)^3}\,\frac{d\vec{k}'}{(2\,\pi)^3}\,\bar{\phi}_n(\vec{k}')\,\frac{4\,g_{eff}^2\,\frac{4}{3}\,e_{k"}\,e_{k'}\Big(2-\frac{m_q^2\,(e_{k"}^2+e_{k'}^2)}{2\,e_{k"}^2\,e_{k'}^2}\Big)}{\sqrt{e_{k"}}\;\;(\vec{k}"-\vec{k}')^2\;\sqrt{e_{k'}}}\,\phi_0(\vec{k}')\,, \quad (48)$$

where $\bar{\phi}_n(\vec{k})$ describes the radial excitations given by a harmonic oscillator model consistently with the Gaussian solution employed for the ground state (normalization $\int \frac{d\vec{k}}{(2\pi)^3}\;\bar{\phi}_n^2(\vec{k}) = 1$). The modification of the Green's function mainly affects the low-k behavior of the correction to the solution of the mass operator, which it tends to reduce. In practice, we add to the correction given in the free Green's function case the correction implied by the more complete Green's function with $n \leq 500$.

6.3 Results and discussion

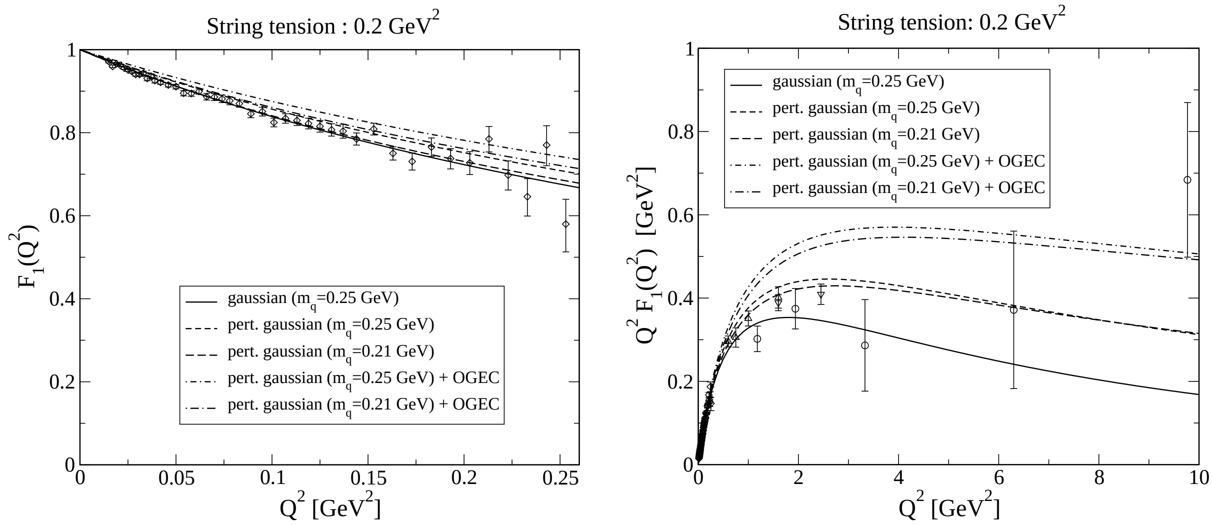

Figure 5: Pion charge form factor, $F_1(Q^2)$, at small and high Q^2 (left and right panels respectively). In the latter case, the form factor is multiplied by Q^2 and represented on a linear scale to emphasize a possible $1/Q^2$ asymptotic behavior. All curves correspond to the string tension $\sigma_{st} = 0.2$ GeV^2. The different curves represent form factors corresponding to a Gaussian solution for the mass operator, the solution including the perturbative one-gluon exchange in the interaction, this last solution with including the contribution of the two-particle current and, finally, these last results with a quark mass fitted to reproduce the pion decay constant $m_q = 0.21$ GeV instead of $m_q = 0.25$ GeV in the previous results.

We first present results by adding to those for the pion charge form factor given in the previous section the contribution involving the one-gluon exchange contribution from a perturbative calculation as described above and the contribution due to two-particle currents as described in ref. [45] for the standard front-form case. The intent is to provide us with a qualitative illustration of the effect of these contributions. This is done in fig. 5 where we show the form factors obtained with a Gaussian solution of the mass operator (previous section, continuous line), and with adding successively the effect of a one-gluon exchange in the quark-quark interaction (short-dash line) and of a two-particle current (short-dash dotted line, denoted OGEC). As these last two calculations correspond to a different value for the pion decay constant, f_π, we also show the results with the right pion decay constant (long-dash and long-dash dotted lines), which supposes to modify the quark mass from $m_q = 0.25$ GeV to $m_q = 0.21$ GeV. The results without the two-particle current could be compared to the results involving the confinement interaction only. As in the previous section, results for the form factor extending up to $Q^2 = 10$ GeV^2 are multiplied by the factor Q^2, in relation with the expectation of a plateau in the asymptotic limit, but, as the logarithmic scale does not justify any more here, we adopt a linear scale.

Corrections due to the one-gluon exchange in the interaction provide *a priori* a sizeable effect. However, at low Q^2, the correction is almost proportional to the form factor without the one-gluon exchange and, as examination of the corresponding panel shows, not much effect is seen after renormalizing the form factor at $Q^2 = 0$ to its conventional value 1 (more than a factor 2). The slight discrepancy that can be observed between the continuous and short-dash lines can be traced back to the fact that including the one-gluon exchange in the interaction modifies the value of f_π from 0.092 to 0.1015 GeV. As there is some indication that part of the squared charge radius is proportional to the inverse of the square of the pion decay constant, see the relation given by eq. (43) and ref. [58] for some probing of this relation, we examined the consequences due to a change of this value. As the comparison of the continuous and long-dash lines indicates, it is found that the discrepancy can be largely removed by using in the calculation that includes corrections due to the one-gluon exchange in the interaction a quark mass consistent with the value $f_\pi = 0.092$ GeV, ($m_q = 0.21$ GeV instead of 0.25 GeV). The correction due to the one-gluon exchange current, being proportional to Q^2, does not represent much in the low-Q^2 range. Its size nevertheless compares to the discrepancies between different calculations of the contributions to form factor due to the single-particle current.

Significant effects begin to show up at higher Q^2 as examination of the corresponding panel in fig. 5 indicates. The correction due to the one-gluon exchange in the interaction (discrepancy between the continuous and short- or long-dash lines) tends to show a plateau in the range (5–10) GeV^2 while the form factor without this contribution begins to decrease. Actually, this plateau is misleading and it rather represents a maximum as calculations at much higher Q^2 shows. The appearance of a plateau in the product of the form factor by Q^2, as expected from QCD, is produced by the one-gluon exchange current contribution (OGEC), which, due to its proportionality to Q^2 at small values of this quantity, is still increasing in the range considered in the figure (the value at $Q^2 = 10$ GeV^2 is off the asymptotic value by about 20%). Considering the total form factor, it is found to significantly overshoot measurements as in the low-Q^2 domain but, contrary to this case, the discrepancy cannot be alleviated by modifying the quark mass so that to reproduce the pion decay constant, as shown in the figure. With this respect, it is noticed that the slight discrepancy between the two calculations of the contribution of the two-particle current around $Q^2 = 10$ GeV^2 has some relationship with the discrepancy between the corresponding pion decay constants, as expected from the expression of the asymptotic form factor given by eq. (44). Thus, the consideration of the various contributions due to one-gluon exchange on top of the confinement interaction improves to some extent the description of the pion form factor at high Q^2 but probably provides a too large effect.

The second set of results is intended to remedy the above high-Q^2 discrepancy and

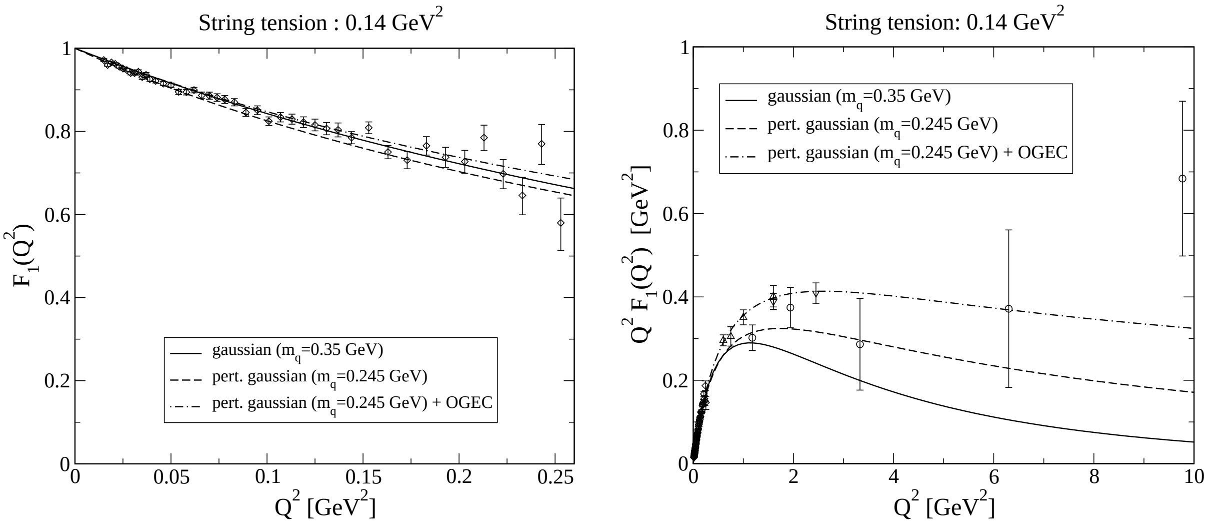

Figure 6: Pion charge form factor, $F_1(Q^2)$, at small and high Q^2 (left and right panels respectively). In the latter case, the form factor is multiplied by Q^2 and represented on a linear scale to emphasize a possible $1/Q^2$ asymptotic behavior. All curves correspond to the string tension $\sigma_{st} = 0.14$ GeV^2 and assume the same value of the pion decay constant. The different curves represent form factors corresponding to a Gaussian solution for the mass operator ($m_q = 0.35$ GeV), the solution incorporating the perturbative one-gluon exchange in the interaction ($m_q = 0.245$ GeV), and this last solution with including the contribution of the two-particle current (OGEC).

implies a change of the string tension σ_{st} so that to approximately reproduce measurements. In this case, the quark mass is always fitted to reproduce the pion decay constant, f_π, and corresponding results are presented in fig. 6. We considered many values of the string tension and decrease it down to 0.14 GeV^2, so that to approximately reproduce the most accurate measurements in the range Q^2 =1–2 GeV^2. As the corresponding panel shows, the one-gluon exchange contributions in both the interaction and in the current begin to be relevant in this range before becoming essential at higher values of Q^2. Due to large error bars, it is difficult to say whether the present calculation disagrees with the measurement at the point close to $Q^2 = 10$ GeV^2. At low Q^2 (left panel of fig. 6), it is found that the three calculations are close to measurements at first sight. A closer examination nevertheless shows that the calculation with a Gaussian wave function or the one incorporating contributions from one-gluon exchange in both the interaction and in the current are doing better as far as the squared charge radius is concerned. In units of fm^2, the numbers are 0.41 and 0.42 respectively instead of 0.49 in the third case while the usually value quoted from measurements is 0.43. The comparison with the theoretical numbers for the string tension 0.20 GeV^2, 0.45, 0.39 and 0.46 respectively, shows a significant effect from one-gluon exchange contributions but, also, a significant dependence on the strength of the confinement. This one implies a different balance of contributions due to the one-gluon exchange in the interaction and in the current.

To complement the above results, we also give in fig. 7 the different solutions of the mass operator used here: Gaussians with σ_{st} =0.2 GeV^2 and 0.14 GeV^2, and the corresponding perturbative calculations. In all cases, the quark mass is fixed to reproduce the pion decay constant, $f_\pi = 0.092$ GeV, with the result m_q =0.25, 0.35, 0.21 and 0.245 GeV

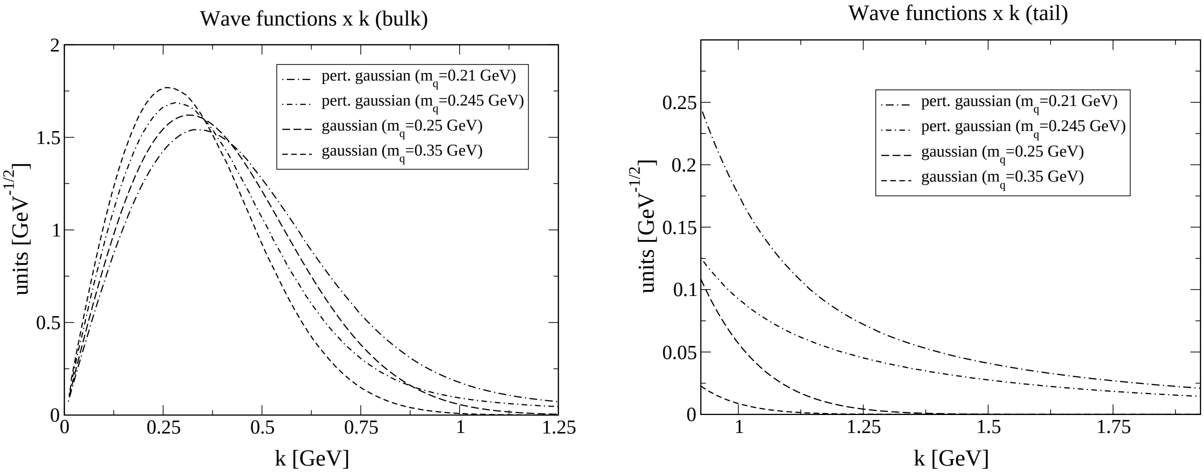

Figure 7: Products of the solutions of the pion mass operator considered in this work with the factor k, $k\,\phi_0(k)\sqrt{8/N}$ (normalization $\int dk\,\left(k\,\phi_0(k)\sqrt{8/N}\right)^2 = 1$). Results are shown as a function of the internal k variable for the bulk contribution ($k \leq 1.25$ GeV, left panel) and for the tail contribution ($k \geq 0.925$ GeV, right panel). They involve different values of the string tension ($\sigma_{st} = 0.2$ GeV^2 and $\sigma_{st} = 0.14$ GeV^2), without and with incorporating the effect of a perturbative one-gluon exchange contribution. In all cases, the quark mass is fitted to reproduce the pion decay constant.

respectively. The left panel shows the bulk contribution, which is roughly the same for all cases. Part of the differences can be ascribed to the choice of the string tension and to the one-gluon exchange contribution that slightly shifts the wave function to higher values of the internal variable k. The role of the one-gluon exchange contribution is better seen on the right panel. In its absence, as can be seen on the figure, wave functions drop exponentially. When it is considered, the tail of the wave function shows a slow decrease given by the power-law behavior $k^{-7/2}$ ($k^{-5/2}$ for the quantity shown in the figure). This tail is responsible for a slower decrease of form factors in comparison to the pure Gaussian case.

Due to a limited experimental information, we did not consider the kaon charge form factor. Let's first mention that a lower value of the string tension, as suggested by the pion results, would provide a better account of measurements in comparison to what was shown in sect. 5. The main question is whether the one-gluon exchange contribution to the current would cancel most of the effect as it does for the pion. In absence of a detailed study of this contribution for the kaon, it is difficult to answer. We only note that a calculation with an average quark mass, $m = (m_{u,d} + m_s)/2$ leads to an effect slightly less than a half of the pion one.

We provided in the text of the present paper a discussion of ingredients allowing us to make a comparison with other RQM calculations, especially with respect to the choice of the mass operator, possible quark form factors and non-perturbative effects (introduction and sect. 2), and two-particle currents motivated by the asymptotic behavior of the pion charge form factor (this section). One could also compare present results with those of field-theory or lattice calculations. In making such a comparison, one should however take into account that calculations to some order in a given framework are not necessarily identical to the ones in other frameworks. Considering first the contribution that allows one to reproduce the asymptotic behavior of the pion charge form factor, the present one is found to be in good agreement with the field-theory one calculated over a large Q^2 range in ref. [26] for instance and with the one calculated differently at high Q^2 in ref. [18] (the comparison is not possible at small and moderate Q^2 in this case). It qualitatively agrees with the one obtained in lattice calculations by comparing unquenched and quenched calculations [47]. In either case, this contribution is not essential to explain the bulk contribution at small and moderate Q^2 (below $3-4$ GeV^2). In this range, it looks as if our results agree with the field-theory or lattice ones but we should stress that our results are calculated up to the first order in the QCD coupling, $\alpha_s = 0.4$, while the other ones are likely to include many orders. We mentioned possible problems with accounting for the one-gluon exchange non-perturbatively in a RQM calculation together with the standard value $\alpha_s = 0.4$. The comparison of RQM calculations with field-theory or lattice ones probably suggests that the former ones should use an effective coupling with a value smaller than the standard one. This could account for retardation effects or, perhaps also, the running character of the QCD coupling. A further suggestion would consist in introducing quark form factors.

7 Conclusion

In this paper we considered the consequences of covariant Poincaré space-time translations for the description of the form factor of pseudoscalar mesons in a RQM framework. This property is generally used to factorize out the dependence on the space-time position in the current matrix element, allowing one to obtain the energy-momentum conservation. For the calculation of form factors, one can thus use the current at $x = 0$, which, most often, is taken as a single-particle current, independently of the form of relativity that is employed. We showed that the energy-momentum conservation does not exhaust all properties stemming from the covariant character of Poincaré space-time translations. Looking at relations derived in the vicinity of the point $x = 0$ [9], and implying the squared momentum transferred to the constituents and to the one transferred to the system, we found that they are generally inconsistent with the single-particle assumption for the current. Only the standard front-form approach with the condition $q^+ = 0$ could fulfill the expected relations. For other forms or other kinematic conditions, we worked out a way to account for the further relations. It amounts to introduce interaction effects in the relation of the constituents momenta to the momentum transfer while keeping the single-particle structure of the current. These effects, that are here or there depending on the approach, correspond to take into account many-particle currents at all orders in the interaction as an expansion would show. In field-theory based approaches, they would correspond to Z-type diagram ones. When these effects are introduced, the equivalence of different approaches can be restored provided that the matrix element of the single-particle current in each form is appropriately chosen.

Interestingly, present theoretical results for the pseudoscalar-meson charge form factor reproduce those expected in a dispersion-relation approach [40, 41] (with some correction for the last reference). Except for the standard front-form case (with $q^+ = 0$), such a result could not be anticipated. Wondering about this result, we feel that two ingredients play an essential role. To a large extent, the effects of constraints considered here tend to restore the equality of the squared momenta transferred to the constituents and the one transferred to the system. This relation is always fulfilled in a field-theory approach and, *a fortiori*, in a dispersion-relation approach which is based on this framework, but it is generally not satisfied in the simplest RQM approach. The second ingredient concerns the matrix element of the single-particle current to start with. In absence of interaction between constituents, the scattering amplitude of the system with the electromagnetic field which underlies implicitly present RQM results, is the same as in the dispersion-relation approach. Besides this contribution, the matrix element may also contain extra contributions. These ones have an interaction character and ensure that its value at $Q^2 = 0$, which is associated to the charge and is not concerned by the above constraints, be Lorentz invariant. The effective matrix elements so obtained could differ from the ones naively expected. It thus appears that getting reliable form factors, which should be independent of the form chosen to implement relativity, implies all aspects of the Poincaré group: boosts, rotations but also space-time translations.

The above effects have been illustrated in the case of the pion and kaon mesons, using a description of these mesons accounting essentially for the confinement interaction. The effects can be especially large for some calculations in the pion case, in relation with the fact that the pion mass is significantly smaller than the sum of the relativistic kinetic energies of the constituents in comparison with the kaon case. Noting that in absence of these effects, these calculations would produce a charge radius that would go to infinity while the mass of the system vanishes as well as other paradoxes [50], it is probably not overstated to say that the above effects remove violations of space-time translations covariance properties that these calculations imply. To a very large extent, present results agree qualitatively with those obtained for scalar constituents [11] but they offer the advantage to concern a physical system. In comparison with experiment, this first set of results provides a good account of measurements with a string tension equal to 0.2 GeV^2. It represents a reasonable starting point for improvements.

Necessary improvements involve the contributions of the one-gluon exchange in both the interaction and in the current. The first one has been estimated from a perturbative-type calculation, taking into account that a complete non-perturbative calculation could lead to some difficulties with the QCD coupling $\alpha_s = 0.4$. The second one ensures that the expected QCD asymptotic behavior for the charge form factor is recovered. It is found in the pion case that the two contributions tend to increase the form factor, especially at high Q^2. A quite reasonable account of measurements is obtained by lowering the string tension from 0.2 GeV^2 to 0.14 GeV^2, a value that is probably at the lower limit of what is acceptable. Interestingly, however, the change in the string tension has provided a better description of measurements in both the low-Q^2 and the intermediate-Q^2 domains. We could probably improve the description by playing with the different ingredients, by requiring for instance that the mass operator reproduces the mass difference between pseudoscalar and vector mesons or between the non-strange and strange mesons. We nevertheless refrain to do it as we believe that many questions could be raised at this point concerning the theoretical inputs.

From this study, we can take for granted a number of features. The contributions of one-gluon exchange in both the interaction and in the current become important ingredients of a realistic description of the pion charge form factor around and beyond $Q^2 = 1\mathrm{GeV}^2$, due to their increasing role with Q^2. The role of the contribution in the current could extend to the low-Q^2 domain where it compares to other effects. Remarkably, this contribution was found to be rather stable and not depending much on which calculation we were performing. In comparing different results, we found it was important that they correspond to the same value of the pion decay constant, what was achieved by fitting the quark mass. This prevented us to make biased conclusions in many circumstances. While the above contributions seem to go in the right direction, questions however may arise from the limitation of their study.

As remembered in the introduction, an unrestricted calculation of the one-gluon exchange contribution to the interaction would produce a too large form factor. This could be remedied by using a smaller value for the string tension but this one is already at the lower limit of what is acceptable. We should add that describing the confining force as a sum of a kinetic energy and a linear r-dependent term could have similar effects to some extent and would not *a priori* alleviate the problem. This, in turn, raises the question of the precise form of the confinement interaction and its interplay with the one-gluon exchange contribution to the interaction. Let's mention that retardation effects, which have been considered in the scalar constituent case, could decrease this last contribution but it is not clear whether they would be sufficient to remove the above problem. They could also affect the calculation of the meson mass spectrum but there is no indication that this one will be necessarily improved. As for the one-gluon exchange contribution to the current, the question that may be raised concerns higher-order corrections in the QCD coupling α_s, which have been discarded for simplicity in the expression we used. The question also concerns possible corrections related to the confinement interaction, which have no reason to be negligible in comparison to the contribution retained here but are completely unknown. Ultimately, one could invoke quark form factors [35, 36] though these ones are not quite consistent at first sight with the expression of the asymptotic behavior of the pion charge form factor. An improved model would consist in considering the coupling to the photon of quark-antiquark pairs generated by one-gluon exchange. It can incorporate the vector-meson phenomenology [73] and in particular the ρ-vector meson dominance one [74] that could be important for the range $Q^2 \leq 1$ GeV^2. Part of this last effect is accounted for in the present work [45] but there is another part corresponding to quark-antiquark correlations which is certainly not. How much space is left for such an effect in the present work is not clear but some could allow one to correct or to alleviate part of the drawbacks mentioned above.

We started this work with the idea that an implementation of relativity free of uncertainties relative to the choice of a particular form should be used for studying the form factor of pseudoscalar mesons and, thus, for getting information on the mass operator describing these particles. From a phenomenological viewpoint, it sounds that this task should not be too difficult, partly due to the fact that the simplest description is already doing well. The main question is whether this information is consistent with what is theoretically expected. This part of the task could be more difficult. It would probably require more elaborate descriptions of both the mass operator and the current within the RQM framework but, also, more accurate measurements of the pion charge form factor for $Q^2 \geq 3$ GeV^2 to discriminate between different schemes. Clarifying the role of the measurement at around $Q^2 = 10$ GeV^2 by making it more accurate would be specially useful. It seems difficult to accommodate its present central value within descriptions of the form factor which do relatively well at smaller momentum transfers. A complementary study would involve the consideration of the pseudoscalar meson spectrum. Though it is likely very approximate, the mass operator obtained here could be used for predictions of this spectrum. In turn, the comparison with experiment could lead to improve the mass operator and help to discriminate effects that are due to its description from the ones that are due to the description of the electromagnetic interaction.

Acknowledgements
This work is partly supported by the National Sciences Foundations of China under grants No. 10775148, 10975146, 11035006 (Y.B.). The author is also grateful to the CAS for grant No KJCX3-SYW-N2. B.D. is very grateful to IHEP and to LPSC for offering hospitality allowing him to achieve this work.

A Relation to the pion charge form factor expressed using Wigner rotation angles

In this section, we provide details that allow one to make a comparison with results for the pion form factor given in ref. [41]. We first give expressions for form factors that include both the Dirac current and an anomalous magnetic moment so that a more complete comparison can be made with the above work that contains the two terms (with a different parametrization). As this work is based on the use of Wigner rotation angles while ours does not consider them explicitly, we also provide expressions for quantities involving the sum of these Wigner rotation angles ω_1 and ω_2, that are relevant for a comparison. They allow one to get closer expressions for form factors while considerably simplifying the comparison which shows a discrepancy factor in the integrand. We finally show how our results, which agree with those obtained by Melikhov [40], can be checked from the consideration of the simplest Feynman diagram.

For definiteness, we write the quark current as:

$$< p_i|j^\mu|p_f >= \bar{u}(p_i)\Big(\tilde{F}_1^q\,\gamma^\mu - \frac{\tilde{F}_2^q}{2m}\,\sigma^{\mu\nu}(p_i{-}p_f)_\nu\Big)u(p_f)\,, \tag{49}$$

where $\sigma^{\mu\nu} = \frac{1}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$. For our main purpose here, it is sufficient to consider one kind of quark. It would be straightforward to generalize to different quarks expressions given below if necessary. For on mass-shell quarks, the above current can be cast into the Gordon form:

$$< p_i|j^\mu|p_f >= \bar{u}(p_i)\Big(\frac{G_E^q}{2m}\,\frac{(p_i{+}p_f)^\mu}{1+\frac{Q^2}{4m^2}} + \frac{G_M^q}{(2m)^2}\,\frac{\epsilon^{\mu\nu\rho\sigma}\gamma^\nu\gamma_5(p_i{-}p_f)_\rho(p_i{+}p_f)_\sigma}{1+\frac{Q^2}{4m^2}}\Big)u(p_f)\,, \tag{50}$$

where:

$$G_E^q = \tilde{F}_1^q + \frac{q^2}{4m^2}\tilde{F}_2^q, \qquad G_M^q = \tilde{F}_1^q + \tilde{F}_2^q\,. \tag{51}$$

The corresponding dispersion-relation expression for the pion charge form factor is given by:

$$F_1(Q^2) = \frac{1}{N}\int d\bar{s}\; d(\frac{s_i - s_f}{Q})\;\phi(s_i)\;\phi(s_f)$$

$$
\begin{aligned}
&\times \frac{4\tilde{F}_1^q \, s_i \, s_f - \tilde{F}_2^q \, D \, Q^2}{2D\sqrt{D}\sqrt{s_i \, s_f}} \, \theta(\cdots) \\
&= \frac{1}{N} \int d\bar{s} \; d(\frac{s_i - s_f}{Q}) \; \phi(s_i) \; \phi(s_f) \\
&\times \frac{G_E^q \, (s_i + s_f + Q^2)^2 + \frac{G_M^q}{m^2} \left(s_i \, s_f \, Q^2 - m^2 D \, Q^2 \right)}{2D\sqrt{D}\sqrt{s_i \, s_f} \, (1 + \frac{Q^2}{4m^2})} \, \theta(\cdots) \, . \qquad (52)
\end{aligned}
$$

Though our approach does not explicitly refer to the Wigner rotation angles, it is convenient to express some of the factors appearing in the above expression in terms of these quantities so that to facilitate the comparison with the expression given in ref. [41]. In this aim, the following relations are useful:

$$
\begin{aligned}
&\tan(\omega_1 + \omega_2) = \frac{\xi}{m \, (s_i + s_f + Q^2)} \, , \\
&\cos(\omega_1 + \omega_2) = \frac{m \, (s_i + s_f + Q^2)}{\sqrt{s_i \, s_f \, (4 \, m^2 + Q^2)}} \, , \\
&\sin(\omega_1 + \omega_2) = \frac{\xi}{\sqrt{s_i \, s_f \, (4 \, m^2 + Q^2)}} \, , \\
&\text{with } \; \xi = \sqrt{s_i \, s_f \, (4 \, m^2 + Q^2) - m^2 \, (s_i + s_f + Q^2)^2} \, . \qquad (53)
\end{aligned}
$$

One thus gets:

$$
\begin{aligned}
F_1(Q^2) = &\frac{1}{N} \int d\bar{s} \; d(\frac{s_i - s_f}{Q}) \; \phi(s_i) \; \phi(s_f) \\
&\times \frac{G_E^q \, (s_i + s_f + Q^2) \cos(\omega_1 + \omega_2) \; + G_M^q \, \frac{\xi}{m} \sin(\omega_1 + \omega_2)}{D\sqrt{D} \, \sqrt{1 + \frac{Q^2}{4m^2}}} \, \theta(\cdots) \, . \qquad (54)
\end{aligned}
$$

The contribution to the pion charge form factor for the term $\tilde{F}_1^q$ alone, considered in the main text, is easily recovered by using the relation:

$$
\cos(\omega_1 + \omega_2) + \frac{\xi}{m \, (s_i + s_f + Q^2)} \sin(\omega_1 + \omega_2) = 2 \, \frac{\sqrt{s_i \, s_f \, (1 + \frac{Q^2}{4m^2})}}{s_i + s_f + Q^2} \, . \qquad (55)
$$

The comparison of the above expression for the form factor, eq. (54), with the one given in ref. [41] shows that this last one contains an extra Q^2 dependent factor, $s_i{+}s_f{+}Q^2$ (first line of their eq. (88)), that is not supported neither by our approach, nor by Melikhov's one [40] (see eqs. (2.50, 2.53) of the web version of the paper).

The appearance of a discrepancy factor similar to the above one was already observed for the scalar-constituent case [11]. Actually the full factor is given by $(s_i{+}s_f{+}Q^2)/(2\sqrt{s_i \, s_f})$ so that the charge form factor at $Q^2 = 0$ is unchanged. It could be checked by considering the simplest Feynman triangle diagram [58]. Due to divergences, the consideration of this diagram is not so useful in the spin-1/2 constituent case. It nevertheless provides some information for converging contributions like the one proportional to the factor $\tilde{F}_2^q$ in eq. (49). It is thus found that the contribution to the pion charge form factor can be cast into the form of the term proportional to $\tilde{F}_2^q$ in eq. (52) by using the function

$\phi(s) = \sqrt{s}/(s-M^2)$. This result is inconsistent with the appearance of the extra Q^2 dependent factor, $s_i{+}s_f{+}Q^2$, as found in ref. [41]. A similar conclusion could be obtained for some contributions proportional to $\tilde{F}_1^q$ in eq. (49) but caution is required in this case in separating various diverging terms. The appearance of the extra Q^2 dependent factor, $s_i+s_f+Q^2$, in ref. [41] originates from a factor $P_0P'_0$ (their eqs. (8) and (28)), which has been calculated by the authors in the lab frame where $P_0P'_0 = \frac{1}{2}(s_i+s_f+Q^2)$. Of course, this result is not Lorentz invariant despite its form. The formalism developed by the authors implies in principle other factors, N_C, N_{CG}, that depend on P_0 ($N_{CG} \propto P_0^2$, their eq. (7); $N_C \propto \sqrt{N_{CG}}$, their eq. (10)). For a reason unknown to us, these factors have been ignored in their final expression for the form factors. We only notice that the combined effect of these factors, $(N_C N'_C)/(N_{CG}N'_{CG})$, in their eqs. (34, 38, 39) provides an extra factor $1/(P_0P'_0)$ that could cancel the above undesirable factor.

B Expressions of form factors used in this work

Expressions of form factors used in this work are obtained from general ones given in ref. [53]. These last ones are somewhat involved and getting explicit expressions to be used for calculating form factors, including the effect of constraints related to covariant space-time translations, may not be straightforward. Giving them here may be therefore useful. In principle, results do not depend on the frame used for calculations after the effect related to the above constraints is accounted for but, in practice, it is necessary to choose some. Results presented in this work have been obtained in the Breit-frame where some simplification occurs. This is done successively for the front form with a momentum transfer perpendicular to the front orientation (F.F. (perp.), the front form with a momentum transfer parallel to the front orientation (F.F. (parallel)), the instant form (I.F.) and the point form ("P.F."). Expressions are given as integrals over the momentum of the spectator constituent. In the front-form case, we consider components, $p_{\parallel}$ and $\vec{p}_{\perp}$, that are parallel and perpendicular to the front orientation, $\vec{n}$, (defined in terms of the 4-vector [11] ω as $\vec{n} = -\vec{\omega}/\omega^0$). In the other cases, we consider components that are parallel and perpendicular to the momentum transfer, $\vec{q}$. The integrand is given as the product of the one for scalar constituents multiplied by a factor that represents the ratio of currents for spin 1/2 and spin 0 constituents. It involves the variables relative to the spectator constituent, $p_{\parallel}$ and $\vec{p}_{\perp}$, as well as other quantities that depend on them, such as the argument entering wave functions, $k_{i,f}^2$ (or $s_{i,f}$), the energy of the system, E, and the energies of the interacting constituents, $e_{i,f}$. In all cases, the relation of $k_{i,f}^2$ to $s_{i,f}$, see eq. (26), may be written as:

$$k_{i,f}^2 = \vec{k}_{i,f}^2 = \frac{1}{4}\Big(s_{i,f} - 2(m_1^2+m_2^2) + \frac{(m_1^2-m_2^2)^2}{s_{i,f}}\Big)\,. \tag{56}$$

The energy of the system in the Breit frame is given by:

$$E = \sqrt{M^2 + \frac{Q^2}{4}}\,. \tag{57}$$

The other quantities, which generally depend on the approach, are given below for each of them.

• Front form with a momentum transfer perpendicular to the front orientation

$$F_1(Q^2) = \frac{1}{(2\pi)^3} \int d^2p_\perp \, dp_\parallel \; \tilde{\phi}(\vec{k}_i^{\,2}) \, \tilde{\phi}(\vec{k}_f^{\,2}) \; \frac{E \;\; \theta(E - e_p - p_\parallel)}{2e_p(E - e_p - p_\parallel)}$$
$$\times \frac{\frac{s_i + s_f}{2} - (m_1 - m_2)^2 - \frac{Q^2\,(e_p + p_\parallel)}{2(E - e_p - p_\parallel)}}{\sqrt{s_i - (m_1 - m_2)^2} \sqrt{s_f - (m_1 - m_2)^2}} \, , \tag{58}$$

where

$$s_{i,f} = \frac{2e_pE \pm \vec{p}_\perp \cdot \vec{q} - \frac{e_p + p_\parallel}{E}(E^2 - \frac{Q^2}{4}) + m_1^2 - m_2^2}{1 - \frac{e_p + p_\parallel}{E}} \, . \tag{59}$$

Using the quantity $x = \frac{e_p + p_\parallel}{E}$, it is noticed that the above expression for the form factor can be cast into the one given by eq. (26) while $s_{i,f}$ takes the form given by eq. (27).

• Front form with a momentum transfer parallel to the front orientation

$$F_1(Q^2) = \frac{1}{(2\pi)^3} \int d^2p_\perp \, dp_\parallel \; \tilde{\phi}(\vec{k}_i^{\,2}) \, \tilde{\phi}(\vec{k}_f^{\,2}) \; \frac{E \;\; \theta(E - \frac{\alpha Q}{2} - e_p - p_\parallel)}{2e_p(E - e_p - p_\parallel)}$$
$$\times \frac{\frac{s_i + s_f}{2} - (m_1 - m_2)^2 - \frac{-\alpha Q \frac{s_i - s_f}{2} - (e_p + p_\parallel)(p_i - p_f)^2}{2(E - e_p - p_\parallel)}}{\sqrt{s_i - (m_1 - m_2)^2} \sqrt{s_f - (m_1 - m_2)^2}} \, , \tag{60}$$

where

$$s_{i,f} = \frac{(E \mp \frac{\alpha Q}{2})\Big(2e_pE \pm \alpha p_\parallel Q + m_1^2 - m_2^2\Big) - (e_p + p_\parallel)(E^2 - \frac{\alpha^2 Q^2}{4})}{E \mp \frac{\alpha Q}{2} - e_p - p_\parallel} \, ,$$
$$\alpha = \frac{E - e_p - p_\parallel}{\sqrt{e_p^2 - p_\parallel^2 + \frac{Q^2}{4} + m_1^2 - m_2^2}} \, ,$$
$$-(p_i - p_f)^2 = \frac{\alpha^2 Q^2\,(m_1^2 + p_\perp^2)}{(E - \frac{\alpha Q}{2} - e_p - p_\parallel)(E + \frac{\alpha Q}{2} - e_p - p_\parallel)} \, . \tag{61}$$

• Instant form

$$F_1(Q^2) = \frac{1}{(2\pi)^3} \int d^2p_\perp \, dp_\parallel \; \tilde{\phi}(\vec{k}_i^{\,2}) \, \tilde{\phi}(\vec{k}_f^{\,2}) \; \frac{e_i + e_f + 2e_p}{2e_p\,(e_i + e_f)}$$
$$\times \frac{\frac{s_i + s_f}{2} - (m_1 - m_2)^2 - \frac{(e_i - e_f)\frac{s_i - s_f}{2} - e_p\,(p_i - p_f)^2}{e_i + e_f}}{\sqrt{s_i - (m_1 - m_2)^2} \sqrt{s_f - (m_1 - m_2)^2}} \, , \tag{62}$$

where

$$s_{i,f} = (e_{i,f} + e_p)^2 - \frac{\alpha^2 Q^2}{4} \, ,$$
$$e_{i,f} = \sqrt{m_1^2 + p_\perp^2 + (p_\parallel \mp \frac{\alpha Q}{2})^2} \, ,$$
$$\alpha = \sqrt{1 + \frac{p_\parallel^2}{m_1^2 + p_\perp^2 + \frac{Q^2}{4}}} \, ,$$
$$-(p_i - p_f)^2 = \alpha^2 Q^2 - (e_i - e_f)^2 \, . \tag{63}$$

• "Point form"
The expression for form factors in the "point-form" approach is perhaps more naturally written in terms of the velocity of the initial or final system in the Breit frame, $\mp\vec{v} = \vec{Q}/\sqrt{4M^2+Q^2}$ with our conventions:

$$F_1(Q^2) = \frac{1}{(2\pi)^3}\int d^2p_\perp\, dp_\parallel\, \tilde{\phi}(\vec{k}_i^{\,2})\,\tilde{\phi}(\vec{k}_f^{\,2})\;\frac{1}{e_p\left(1-\frac{m_2^2-m_1^2}{2s_i}-\frac{m_2^2-m_1^2}{2s_f}\right)}$$
$$\times\frac{\frac{s_i+s_f}{2}-(m_1-m_2)^2-\frac{(e_i-e_f)\frac{s_i-s_f}{2}-e_p\,(p_i-p_f)^2}{e_i+e_f}\Big)}{\sqrt{s_i-(m_1-m_2)^2}\sqrt{s_f-(m_1-m_2)^2}}\,, \tag{64}$$

where

$$\begin{aligned}
s_{i,f} &= (e_{i,f}+e_p)^2(1-\beta^2v^2)\,,\\
e_{i,f} &= \frac{e_\mp \mp \beta v\,(p_\parallel \mp \beta v\, e_p)}{1-\beta^2v^2}\,,\\
e_\pm &= \sqrt{(m_1^2-m_2^2)(1-\beta^2v^2)+(e_p\pm\beta v\, p_\parallel)^2}\,,\\
\beta v &= \frac{\frac{Q}{2}}{\sqrt{m_2^2+p_\perp^2+\delta}+\sqrt{m_2^2+p_\perp^2+\frac{Q^2}{4}+\delta}}\,,\\
-(p_i-p_f)^2 &= \frac{4(\beta v)^2}{(1-\beta^2v^2)^2}\,\frac{(2e_p+e_++e_-)^2}{4(e_++e_-)^2}\Big((e_++e_-)^2-4p_\parallel^2\Big)\,.
\end{aligned} \tag{65}$$

The very last equation can be cast into a form implying the factor multiplying the quantity Q as in the other cases. The relevant equation is the following one:

$$\begin{aligned}
&\beta v = \frac{\alpha Q}{\sqrt{4M^2+\alpha^2Q^2}}\,,\\
\text{with}\quad &\alpha = \frac{M}{\sqrt{2\sqrt{m_2^2+p_\perp^2+\delta}\Big(\sqrt{m_2^2+p_\perp^2+\delta}+\sqrt{m_2^2+p_\perp^2+\frac{Q^2}{4}+\delta}\Big)}}\,.
\end{aligned} \tag{66}$$

The quantity δ appearing in the above equation is a small correction that vanishes for equal mass constituents. It is defined as follows:

$$4(m_2^2+p_\perp^2+\delta) = \frac{(2e_p+e_++e_-)^2}{4(e_++e_-)^2}\Big((e_++e_-)^2-4p_\parallel^2\Big)\,. \tag{67}$$

It however depends on βv, which complicates the practical derivation of this last quantity.

The full equation to be solved reads:

$$Q^2\frac{(1-\beta^2v^2)^2}{4(\beta v)^2} = 4(m_2^2+p_\perp^2+\delta)\,. \tag{68}$$

It supposes to solve an equation of the 4rth degree in the variable β^2. The correction has been accounted for in the present work. Actually, we found simpler to solve the equation by using an iterative process (a few iterations were sufficient). Moreover, as the quantity δ involves terms depending on $(m_1^2-m_2^2)$, one could wonder about the appearance of m_2^2

instead of m_1^2 or any combination of the masses in eq. (67). The choice made here sounds to provide a faster convergence of the iterative process.

Despite differences in expressions, all form factors obtained in different approaches at $Q^2 = 0$ can be reduced to a common expression given in terms of the internal variable $\vec{k}$:

$$F_1(Q^2=0) = \frac{16\pi^2}{N} \int \frac{d\vec{k}}{(2\pi)^3} \frac{(e_{1k}+e_{2k})}{2\,e_{1k}\,e_{2k}} \tilde{\phi}^2(\vec{k}^2) = \frac{16\pi^2}{N} \int \frac{d\vec{k}}{(2\pi)^3} \phi_0^2(\vec{k}) = 1\,. \qquad (69)$$

It was mentioned in the main text that strong effects, before accounting for constraints related to covariant space-time translations, were due to the smallness of the pion mass in comparison with the sum of the quark masses. Looking at the expressions for form factors, it is seen that only the instant-form result (Breit frame) does not depend on the mass of the system. For the front form with a momentum transfer perpendicular to the front orientation ($q^+ = 0$), the dependence on M, through the dependence on E, can be shown to have no effect by using the alternative variable $x = \frac{e_p + p_\parallel}{E}$ instead of $p_\parallel$. For the front form with a momentum transfer parallel to the front orientation, a similar trick can be used but only after accounting for the above mentioned constraints. For the "point form", the dependence on M appears through the quantity βv. By inserting in the first of eqs. (66), the value of α obtained from the second one, it is easily seen that the dependence of βv on M cancels. The way the dependence on M disappears in this last case differs from the one for the front-form case.

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