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Analytic expressions for alpha decay half-lives and potential barriers

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Abstract

From an adjustment on a recent selected data set of partial α-decay half-lives of 344 ground state to ground state transitions, analytical formulas are proposed for $log_{10}T_{1/2}(s)$ depending or not on the angular momentum of the α particle. In particular, an expression allows to reproduce precisely the partial α-decay half-lives of even-even heavy nuclei and, then, to predict accurately the partial α-decay half-lives of other very heavy elements from the experimental or predicted Q_α. Comparisons have been done with other empirical approaches. Moreover, the potential barrier against α-decay or α-capture has been determined within a liquid drop model including a proximity energy term. Simple expressions are provided to calculate the potential barrier radius and height.

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1 Introduction

In 1911 Geiger and Nuttal [1] observed a simple dependence of the α decay constant on the mean α particle range in air for a fixed radioactive family. Later on, in 1928 [2,3], the spontaneous α decay was described as a quantum tunnelling effect through the potential barrier separating the initial state of the parent nucleus and the final state formed by the separated α particle and daughter nucleus. Often, in a first approximation, this potential barrier standing against α decay is taken as the combination of a square-well and after of a pure Coulomb barrier to make easier the integration of the Schrödinger equation. Such a barrier is naturally unsufficient to reproduce accurately the α-decay exit channel and the α capture entrance channel and, more generally, the fusion barriers. In the quasi-molecular deformation valley investigated by the α decay or capture the neck between the two nuclei is very deep and, consequently, the surfaces in regard are very close to each other and the proximity forces between the nucleons at the surfaces lower the barrier, smooth it and shift it towards a more external position [4,5].

The experimental data are regularly enlarged [6], particularly in the super-heavy nucleus region and near the proton and neutron drip lines. Indeed, isotopes of the elements 112, 113, 114, 115, 116, 117 and 118 have been synthetized recently in fusion-evaporation reactions and observed mainly via their α decay cascades [7,8,9,10,11,12]. These recent data have led to new theoretical studies on the α emission process, for example within the DDM3Y interaction [13,14], the relativistic mean field theory [15], the Skyrme-Hartree-Fock mean-field model [16], the superasymmetric fission model [17] and the generalized liquid drop model (GLDM) [18,19].

Modern Geiger-Nuttal plots are expressed as $log_{10}T_\alpha = aZQ^{-1/2} + b$. Since, different expressions have been proposed [4,20,21,22,23,24,25,26,27] to calculate $log_{10}T_\alpha$ from A, Z and the Q_α value. Formulas taking into account the proximity of the magic numbers have also been provided [17]. The adjustment of the formula coefficients is generally realized on the total α decay half-lives and the spins and possible excitations of parent and daughter nuclei are not taken into account. New expressions containing l-dependent terms have been recently proposed [28,29,30] and the dependence on the excitation energy has been investigated [29,30,31,32].

The ability of the formulas proposed in Ref. [4] to describe the whole data set of total α decay half-lives and its predicted power on new data, particularly for the heaviest elements, has been verified recently [19,26,27].

In a recent paper [5], a carefully updated and selected partial α decay half-life data set of 344 ground-state-to-ground-state α transitions has been studied. The purpose of the present work is, firstly, to adjust the coefficients of the above-mentioned formulas [4] on this ground-state-to-ground-state decay data [5] in incorporating a l-dependence and to test and compare the efficiency of these new adjusted formulas and, secondly, to provide simple expressions to determine the alpha-decay or capture barriers.

2 Alpha decay half-lives

In a previous study [4] the α decay half-lives deduced from the WKB barrier penetration probability through the GLDM potential barriers were compared with the total α decay half-lives of 373 emitters having an α branching ratio close to one. The rms deviation between the theoretical and experimental values of $log_{10}T_\alpha(s)$ was 0.63 and only 0.35 for the subset of 131 even-even nuclides.

In this previous paper a fitting procedure on this data set led to accurate empirical expressions (formulas(18)-(21) in [4] depending on three parameters in each subset) respectively for the 131 even-even, 106 even(Z)-odd(N), 86 odd-even and 50 odd-odd nuclei, the rms deviation being respectively 0.285, 0.39, 0.36 and 0.35.

A good agreement between the predictions using these formulas and the experimental data obtained after 2000 is shown in Ref. [19] confirming the predictability of these formulas. Very recently [10] the isotopes ^{293}117 and ^{294}117 were produced in fusion reactions between ^{48}Ca and ^{249}Bk. Two decay chains involving 11 new nuclei were identified. 5 events correspond to the isotope ^{293}117 and 1 event to the isotope ^{294}117. The uncertainties on the α-decay half-lives and Q values are important. In the Table 1 the characteristics of the two cascades are given : the range of the experimental Q value and the experimental α-decay half-life and the values predicted using the above-mentioned formulas. There is a very good agreement for the cascade starting from the ^{293}117 nucleus and for four nuclei of the other cascade. The disagreement is very important for the ^{290}115 and ^{282}111 nuclei. In these two cases the experimental Q value is lower than expected.

The preceding formulas are related to the total α decay half-life and, consequently, to all possible transitions from the ground state of the parent nucleus into both the ground and excited states of the daughter nucleus. The only parameter in these formulas is the Q value for the ground state to ground state transition. For some nuclei the transition to excited states is also important [30] which, partially, might explain the remaining differences between the experimental and theoretical data for some specific nuclei.

Table 1
Comparison between the experimental and calculated α-decay half-lives for the recent observed decay-chains originated from the isotopes A = 293 and A = 294 of the new element Z=117.

$^{A}_{Z}$	Q(MeV)	T_{exp}	T_{form}	$^{A}_{Z}$	Q(MeV)	T_{exp}	T_{form}
$^{293}_{117}$	11.1-11.26	10-25 ms	9.7-24 ms	$^{289}_{115}$	10.35-10.55	0.14-0.48 s	0.15-0.54 s
$^{285}_{113}$	9.65-9.85	3.7-10.5 s	3.1-12.0 s	$^{294}_{117}$	10.86-11.06	0.042-0.45 s	0.15-0.54 s
$^{290}_{115}$	10.05-10.13	0.016 s	1.18-323 s	$^{286}_{113}$	9.66-9.86	19.6 s	16.7-71.3 s
$^{282}_{111}$	9.03-9.23	0.51 s	314-1513 s	$^{278}_{109}$	9.5-9.88	7.6 s	0.48-7.1 s
$^{274}_{107}$	8.83-9.03	54 s	41-194 s				

Recently [5], a carefully updated and selected partial α decay half-life data set of 344 ground-state-to-ground-state α transitions has been extracted. The same fitting procedure applied to this new data set leads to the following empirical formulas respectively for the 136 even-even, 84 even(Z)-odd(N), 76 odd-even and 48 odd-odd nuclei, the rms deviation being respectively 0.3280, 0.9559, 0.8891 and 0.9080.

$$log_{10}\,[T] = -25.752 - 1.15055 A^{\frac{1}{6}}\sqrt{Z} + \frac{1.5913 Z}{\sqrt{Q}}, \tag{1}$$

$$log_{10}\,[T] = -34.156 - 0.87487 A^{\frac{1}{6}}\sqrt{Z} + \frac{1.6923 Z}{\sqrt{Q}}, \tag{2}$$

$$log_{10}\,[T] = -32.623 - 1.0465 A^{\frac{1}{6}}\sqrt{Z} + \frac{1.7495 Z}{\sqrt{Q}}, \tag{3}$$

$$log_{10}\,[T] = -31.186 - 0.98047 A^{\frac{1}{6}}\sqrt{Z} + \frac{1.6744 Z}{\sqrt{Q}}. \tag{4}$$

The Q_α values have been determined from the atomic mass data of Audi et al [6]. This new data set relative only to 344 partial α decay half-lives of ground to ground state transitions is more difficult to reproduce than the one relative to 373 emitters [4] and to total α decay half-lives.

For the even-odd, odd-even and odd-odd nuclei the ground-state-to-ground-state transitions may occur for different spins and parities of the parent and daughter nuclei and, consequently, the α particle may take away an angular momentum l. According to the selection rules the minimal orbital angular momentum of the emitted α particle has been evaluated in [5] assuming that $l = 0$ for all even-even nuclei. From these l values and for improving the accuracy of the preceding formulas an explicit dependence on l has been researched and the following empirical formulas are proposed. They lead respectively for the 84 even-odd, 76 odd-even and 48 odd-odd nuclei to a rms deviation of 0.5552, 0.6661 and 0.6807.

$$log_{10}[T] = -27.750 - 1.1138A^{\frac{1}{6}}\sqrt{Z} + \frac{1.6378Z}{\sqrt{Q}} + \frac{1.7383\ 10^{-6}ANZ[l(l+1)]^{\frac{1}{4}}}{Q} + 0.002457A[1-(-1)^l], \tag{5}$$

$$log_{10}[T] = -27.915 - 1.1292A^{\frac{1}{6}}\sqrt{Z} + \frac{1.6531Z}{\sqrt{Q}} + \frac{8.9785\ 10^{-7}ANZ[l(l+1)]^{\frac{1}{4}}}{Q} + 0.002513A[1-(-1)^l], \tag{6}$$

$$log_{10}[T] = -26.448 - 1.1023A^{\frac{1}{6}}\sqrt{Z} + \frac{1.5967Z}{\sqrt{Q}} + \frac{1.6961\ 10^{-6}ANZ[l(l+1)]^{\frac{1}{4}}}{Q} + 0.00101A[1-(-1)^l]. \tag{7}$$

The agreement with experimental data is better due to the introduction of two new additional empirical terms depending on l and simulating the centrifugal effects and the hindrance of α emission with odd values of l. The Q_α values, the evacuated angular momentum, the experimental ground state to ground state α-decay half-lives and values evaluated from formulas (1,5-7) are given in Tables (2-5). For most of the nuclei, the difference between the experimental and theoretical data is relatively weak but for only some nuclei such as $^{113}_{53}$I, $^{149}_{64}$Gd, $^{206}_{85}$At, $^{218}_{91}$Pa and $^{235}_{95}$Am the difference is very important and increases strongly the rms deviation. The extracted experimental data on these specific nuclei seem perhaps questionable.

Additionally for the 59 heavy ($N > 126$ and $Z > 82$) e-e nuclei of this data set the following formula

$$log_{10}[T] = -27.690 - 1.0441A^{\frac{1}{6}}\sqrt{Z} + \frac{1.5702Z}{\sqrt{Q}} \tag{8}$$

leads to a very small rms deviation of 0.1867 while for the 77 remaining lighter e-e nuclei the expression

Table 2
Comparison between the decimal logarithms of the experimental and calculated with the formula (1) ground state to ground state α-decay half-lives (in s) for 136 even-even nuclei.

$\frac{A}{Z}$	Q	$logT_{exp}$	$logT_{form}$	$\frac{A}{Z}$	Q	$logT_{exp}$	$logT_{form}$	$\frac{A}{Z}$	Q	$logT_{exp}$	$logT_{form}$	$\frac{A}{Z}$	Q	$logT_{exp}$	$logT_{form}$
${}^{106}_{52}$	4.290	-4.15	-3.85	${}^{108}_{52}$	3.445	0.49	0.73	${}^{112}_{54}$	3.33	2.53	2.78	${}^{114}_{56}$	3.53	1.77	2.72
${}^{144}_{60}$	1.905	22.86	23.02	${}^{146}_{62}$	2.528	15.51	15.51	${}^{148}_{62}$	1.986	23.34	23.42	${}^{148}_{64}$	3.271	9.37	9.39
${}^{150}_{64}$	2.808	13.75	13.81	${}^{152}_{64}$	2.203	21.53	21.60	${}^{150}_{66}$	4.351	3.08	3.05	${}^{152}_{66}$	3.726	6.93	7.07
${}^{154}_{66}$	2.946	13.98	13.80	${}^{152}_{68}$	4.934	1.06	1.05	${}^{154}_{68}$	4.28	4.68	4.59	${}^{154}_{70}$	5.474	-0.36	-0.43
${}^{156}_{70}$	4.811	2.42	2.699	${}^{158}_{70}$	4.172	6.63	6.40	${}^{156}_{72}$	6.028	-1.63	-1.74	${}^{158}_{72}$	5.405	0.81	0.83
${}^{160}_{72}$	4.902	2.77	3.25	${}^{162}_{72}$	4.417	5.8	5.97	${}^{174}_{72}$	2.497	22.8	23.7	${}^{160}_{74}$	6.065	-0.99	-1.00
${}^{162}_{74}$	5.677	0.46	0.56	${}^{164}_{74}$	5.278	2.38	2.35	${}^{166}_{74}$	4.856	4.74	4.48	${}^{180}_{74}$	2.508	25.75	25.09
${}^{162}_{76}$	6.767	-2.73	-2.68	${}^{166}_{76}$	6.139	-0.52	-0.45	${}^{168}_{76}$	5.818	0.62	0.83	${}^{170}_{76}$	5.539	1.79	2.03
${}^{172}_{76}$	5.227	3.98	3.49	${}^{174}_{76}$	4.872	5.34	5.34	${}^{186}_{76}$	2.823	22.8	22.3	${}^{168}_{78}$	6.997	-2.7	-2.7
${}^{170}_{78}$	6.708	-1.85	-1.74	${}^{174}_{78}$	6.184	0.03	0.15	${}^{176}_{78}$	5.885	1.22	1.36	${}^{178}_{78}$	5.573	2.45	2.73
${}^{180}_{78}$	5.24	4.24	4.33	${}^{188}_{78}$	4.008	12.53	11.93	${}^{190}_{78}$	3.251	19.31	18.72	${}^{174}_{80}$	7.233	-2.7	-2.73
${}^{176}_{80}$	6.897	-1.69	-1.64	${}^{180}_{80}$	6.258	0.73	0.69	${}^{182}_{80}$	5.997	1.86	1.74	${}^{184}_{80}$	5.662	3.44	3.21
${}^{186}_{80}$	5.205	5.71	5.46	${}^{188}_{80}$	4.705	8.72	8.31	${}^{186}_{82}$	6.47	0.68	0.66	${}^{188}_{82}$	6.109	2.06	2.11
${}^{190}_{82}$	5.697	4.25	3.94	${}^{192}_{82}$	5.221	6.57	6.33	${}^{194}_{82}$	4.738	9.99	9.13	${}^{210}_{82}$	3.792	16.57	15.86
${}^{190}_{84}$	7.693	-2.59	-2.84	${}^{192}_{84}$	7.319	-1.48	-1.67	${}^{194}_{84}$	6.987	-0.38	-0.55	${}^{196}_{84}$	6.657	0.77	0.64
${}^{198}_{84}$	6.309	2.27	2.01	${}^{200}_{84}$	5.981	3.66	3.41	${}^{202}_{84}$	5.701	5.13	4.69	${}^{204}_{84}$	5.485	6.28	5.74
${}^{206}_{84}$	5.327	7.14	6.54	${}^{210}_{84}$	5.407	7.08	6.02	${}^{212}_{84}$	8.954	-6.52	-6.83	${}^{214}_{84}$	7.833	-3.78	-3.78
${}^{216}_{84}$	6.906	-0.84	-0.72	${}^{218}_{84}$	6.115	2.27	2.43	${}^{198}_{86}$	7.349	-1.18	-1.03	${}^{204}_{86}$	6.545	2.01	1.85
${}^{206}_{86}$	6.384	2.74	2.48	${}^{208}_{86}$	6.261	3.37	2.97	${}^{210}_{86}$	6.159	3.95	3.38	${}^{212}_{86}$	6.385	3.16	2.35
${}^{214}_{86}$	9.208	-6.57	-6.75	${}^{216}_{86}$	8.2	-4.35	-4.10	${}^{218}_{86}$	7.263	-1.46	-1.15	${}^{220}_{86}$	6.405	1.75	2.11
${}^{222}_{86}$	5.59	5.52	5.88	${}^{206}_{88}$	7.415	-0.62	-0.56	${}^{210}_{88}$	7.152	0.57	0.30	${}^{212}_{88}$	7.032	1.18	0.70
${}^{214}_{88}$	7.273	0.39	-0.22	${}^{216}_{88}$	9.526	-6.74	-6.82	${}^{218}_{88}$	8.546	-4.59	-4.33	${}^{220}_{88}$	7.592	-1.74	-1.45
${}^{222}_{88}$	6.679	1.59	1.88	${}^{224}_{88}$	5.789	5.52	5.85	${}^{226}_{88}$	4.871	10.73	11.06	${}^{216}_{90}$	8.071	-1.57	-2.08
${}^{218}_{90}$	9.849	-6.96	-6.89	${}^{220}_{90}$	8.953	-5.01	-4.71	${}^{222}_{90}$	8.127	-2.69	-2.37	${}^{224}_{90}$	7.298	0.12	0.36
${}^{226}_{90}$	6.45	3.39	3.70	${}^{228}_{90}$	5.52	7.93	8.23	${}^{230}_{90}$	4.77	12.49	12.81	${}^{232}_{90}$	4.082	17.76	18.08
${}^{226}_{92}$	7.701	-0.57	-0.23	${}^{228}_{92}$	6.803	2.9	3.10	${}^{230}_{92}$	5.993	6.43	6.74	${}^{232}_{92}$	5.414	9.5	9.81
${}^{234}_{92}$	4.858	13.04	13.28	${}^{236}_{92}$	4.573	15.0	15.28	${}^{238}_{92}$	4.27	17.25	17.62	${}^{232}_{94}$	6.716	4.13	4.32
${}^{234}_{94}$	6.31	5.89	6.11	${}^{236}_{94}$	5.867	8.11	8.27	${}^{238}_{94}$	5.593	9.59	9.73	${}^{240}_{94}$	5.256	11.45	11.69
${}^{242}_{94}$	4.985	13.18	13.40	${}^{244}_{94}$	4.666	15.5	15.61	${}^{238}_{96}$	6.62	5.51	5.56	${}^{240}_{96}$	6.398	6.52	6.54
${}^{242}_{96}$	6.216	7.28	7.38	${}^{244}_{96}$	5.902	8.87	8.95	${}^{246}_{96}$	5.475	11.26	11.32	${}^{248}_{96}$	5.162	13.16	13.23
${}^{240}_{98}$	7.719	2.03	1.99	${}^{246}_{98}$	6.862	4.21	5.27	${}^{248}_{98}$	6.361	7.56	7.53	${}^{250}_{98}$	6.128	8.69	8.66
${}^{252}_{98}$	6.217	8.01	8.17	${}^{254}_{98}$	5.927	9.31	9.64	${}^{246}_{100}$	8.378	0.17	0.43	${}^{248}_{100}$	8.002	1.66	1.66
${}^{250}_{100}$	7.557	3.38	3.26	${}^{252}_{100}$	7.153	5.04	4.83	${}^{254}_{100}$	7.308	4.14	4.16	${}^{256}_{100}$	7.027	5.14	5.29
${}^{252}_{102}$	8.55	0.74	0.55	${}^{254}_{102}$	8.226	1.82	1.60	${}^{256}_{102}$	8.581	0.53	0.38	${}^{260}_{106}$	9.92	-2.04	-2.12

$$log_{10}\left[T\right] = -28.786 - 1.0329A^{\frac{1}{6}}\sqrt{Z} + \frac{1.6127Z}{\sqrt{Q}} \qquad (9)$$

leads to a rms deviation of only 0.2659.

For comparison, the coefficients of the formulas proposed in [28] have

Table 3

Same as Table 2 but using the formula (5) for 84 e-o nuclei.

$^{A}_{Z}$	Q	l	$logT_{exp}$	$logT_{form}$	$^{A}_{Z}$	Q	l	$logT_{exp}$	$logT_{form}$	$^{A}_{Z}$	Q	l	$logT_{exp}$	$logT_{form}$	$^{A}_{Z}$	Q	l	$logT_{exp}$	$logT_{form}$
$^{107}_{52}$	4.008	0	-2.35	-2.71	$^{109}_{52}$	3.23	0	2.06	2.08	$^{113}_{54}$	3.09	0	3.89	4.57	$^{147}_{62}$	2.3105	0	18.52	18.91
$^{149}_{64}$	3.099	0	13.27	11.28	$^{151}_{64}$	2.6522	0	15.03	16.05	$^{151}_{66}$	4.1795	0	4.28	4.24	$^{153}_{66}$	3.559	0	8.39	8.62
$^{153}_{68}$	4.8023	0	1.85	1.83	$^{155}_{68}$	4.118	0	6.16	5.84	$^{155}_{70}$	5.3376	0	0.3	0.28	$^{157}_{70}$	4.621	0	3.89	3.94
$^{157}_{72}$	5.88	0	-0.91	-1.07	$^{159}_{74}$	6.45	0	-2.09	-2.33	$^{163}_{74}$	5.52	2	0.83	1.97	$^{169}_{76}$	5.716	0	1.59	1.48
$^{171}_{76}$	5.371	2	2.69	3.71	$^{171}_{78}$	6.61	0	-1.35	-1.24	$^{173}_{76}$	5.055	0	5.03	4.69	$^{173}_{78}$	6.35	2	-0.36	0.27
$^{175}_{78}$	6.1781	2	1.73	0.96	$^{175}_{80}$	7.06	2	-1.96	-1.49	$^{177}_{78}$	5.6428	0	2.33	2.72	$^{177}_{80}$	6.74	0	-0.82	-0.89
$^{181}_{78}$	5.15	0	4.86	5.15	$^{183}_{78}$	4.82	0	7.48	6.98	$^{183}_{80}$	6.039	0	1.95	1.83	$^{185}_{80}$	5.774	0	2.93	3.00
$^{185}_{82}$	6.695	2	2.32	0.71	$^{195}_{84}$	6.746	0	0.79	0.64	$^{195}_{86}$	7.69	0	-2.22	-1.83	$^{197}_{84}$	6.412	0	2.09	1.96
$^{199}_{84}$	6.074	0	3.44	3.41	$^{201}_{84}$	5.7989	0	4.77	4.67	$^{201}_{86}$	6.86	0	0.95	1.03	$^{203}_{84}$	5.496	2	8.3	7.19
$^{203}_{86}$	6.6298	0	1.83	1.91	$^{205}_{84}$	5.324	0	7.18	7.09	$^{205}_{86}$	6.39	2	4.61	3.78	$^{205}_{88}$	7.49	0	-0.66	-0.46
$^{207}_{84}$	5.2158	0	8.0	7.66	$^{207}_{86}$	6.2511	0	3.43	3.46	$^{207}_{88}$	7.27	0	0.42	0.29	$^{209}_{84}$	4.9792	2	10.21	10.24
$^{209}_{86}$	6.1555	0	4.0	3.86	$^{209}_{88}$	7.144	0	0.67	0.72	$^{211}_{84}$	7.5945	5	-0.28	-0.49	$^{211}_{86}$	5.9654	2	5.75	5.75
$^{211}_{88}$	7.043	0	1.15	1.06	$^{213}_{84}$	8.5361	0	-5.38	-5.61	$^{213}_{86}$	8.243	5	-1.71	-1.74	$^{213}_{88}$	6.861	2	2.66	2.67
$^{213}_{90}$	7.84	0	-0.85	-0.93	$^{215}_{84}$	7.5263	0	-2.75	-2.59	$^{215}_{86}$	8.839	0	-5.64	-5.66	$^{215}_{88}$	8.864	5	-2.79	-2.75
$^{215}_{90}$	7.665	2	0.48	0.49	$^{217}_{86}$	7.8871	0	-3.27	-2.92	$^{217}_{88}$	9.161	0	-5.79	-5.75	$^{217}_{90}$	9.433	5	-3.62	-3.52
$^{219}_{86}$	6.9461	2	0.7	1.32	$^{219}_{88}$	8.138	2	-1.48	-2.04	$^{219}_{90}$	9.51	0	-5.98	-5.89	$^{219}_{92}$	9.86	5	-4.26	-3.86
$^{221}_{86}$	6.147	2	3.92	4.80	$^{221}_{88}$	6.8804	2	1.97	2.53	$^{221}_{90}$	8.626	2	-2.37	-2.72	$^{223}_{88}$	5.9790	2	7.99	6.67
$^{223}_{90}$	7.567	2	0.78	0.77	$^{225}_{92}$	8.014	2	-1.14	0.06	$^{227}_{90}$	6.1466	2	6.82	6.85	$^{229}_{92}$	6.475	0	4.43	5.04
$^{233}_{92}$	4.9085	0	12.77	13.76	$^{235}_{92}$	4.6783	1	17.65	17.90	$^{237}_{94}$	5.7484	1	12.12	11.91	$^{241}_{96}$	6.1852	3	11.28	11.19
$^{245}_{98}$	7.2585	0	3.94	4.24	$^{247}_{96}$	5.353	1	15.55	15.47	$^{249}_{98}$	6.296	1	11.65	10.99	$^{251}_{98}$	6.1758	5	12.04	12.86
$^{251}_{100}$	7.4251	1	7.85	6.67	$^{253}_{100}$	7.199	5	8.22	8.71	$^{255}_{102}$	8.442	5	4.2	4.59	$^{257}_{100}$	6.8635	2	9.18	8.28

Table 4
Same as Table 3 but using the formula (6) for 76 o-e nuclei.

$^{A}_{Z}$	Q	l	$logT_{exp}$	$logT_{form}$	$^{A}_{Z}$	Q	l	$logT_{exp}$	$logT_{form}$	$^{A}_{Z}$	Q	l	$logT_{exp}$	$logT_{form}$	$^{A}_{Z}$	Q	l	$logT_{exp}$	$logT_{form}$
$^{113}_{53}$	2.71	0	9.3	7.23	$^{145}_{61}$	2.322	0	17.3	18.04	$^{147}_{63}$	2.9903	0	10.98	11.72	$^{149}_{65}$	4.0775	2	4.97	4.62
$^{151}_{63}$	1.9639	2	26.2	26.32	$^{151}_{65}$	3.496	2	8.82	8.88	$^{153}_{69}$	5.2481	0	0.21	0.18	$^{155}_{69}$	4.572	0	3.06	3.69
$^{159}_{73}$	5.681	5	0.11	1.43	$^{163}_{75}$	6.017	0	-0.22	-0.23	$^{169}_{77}$	6.151	0	-0.11	0.11	$^{175}_{77}$	5.4	2	3.02	3.77
$^{177}_{77}$	5.08	0	4.7	5.08	$^{177}_{81}$	7.067	0	-1.61	-1.63	$^{179}_{81}$	6.718	0	-0.57	-0.38	$^{181}_{79}$	5.7513	2	3.39	3.02
$^{183}_{79}$	5.4656	0	4.15	4.03	$^{185}_{79}$	5.18	0	4.99	5.51	$^{191}_{83}$	6.778	5	2.85	1.59	$^{193}_{83}$	6.304	5	4.5	3.56
$^{195}_{83}$	5.832	5	6.79	5.76	$^{197}_{85}$	7.1	0	-0.44	-0.30	$^{199}_{85}$	6.78	0	0.9	0.89	$^{201}_{85}$	6.4732	0	2.08	2.11
$^{201}_{87}$	7.52	0	-1.21	-0.96	$^{203}_{85}$	6.2101	0	3.16	3.23	$^{203}_{87}$	7.26	0	-0.24	-0.07	$^{205}_{85}$	6.0195	0	4.2	4.08
$^{205}_{87}$	7.0549	0	0.59	0.65	$^{207}_{85}$	5.872	0	4.88	4.75	$^{207}_{87}$	6.9	0	1.19	1.22	$^{209}_{83}$	3.1372	5	26.78	27.00
$^{209}_{85}$	5.7571	0	5.68	5.28	$^{209}_{87}$	6.777	0	1.75	1.67	$^{209}_{89}$	7.73	0	-1.04	-0.95	$^{211}_{85}$	5.9824	0	4.79	4.13
$^{211}_{87}$	6.66	0	2.37	2.11	$^{211}_{89}$	7.62	0	-0.67	-0.61	$^{213}_{83}$	5.982	5	5.15	4.92	$^{213}_{85}$	9.254	0	-6.9	-7.17
$^{213}_{87}$	6.9049	0	1.54	1.08	$^{213}_{89}$	7.5	0	-0.14	-0.23	$^{215}_{85}$	8.178	0	-4.0	-4.26	$^{215}_{87}$	9.54	0	-7.07	-7.13
$^{215}_{89}$	7.744	0	-0.77	-1.12	$^{217}_{87}$	8.469	0	-4.77	-4.32	$^{217}_{89}$	9.832	0	-7.16	-7.11	$^{217}_{91}$	8.489	0	-2.45	-2.69
$^{219}_{87}$	7.4485	0	-1.69	-1.08	$^{219}_{89}$	8.83	0	-4.93	-4.56	$^{219}_{91}$	10.08	0	-7.28	-6.98	$^{221}_{87}$	6.4578	2	2.55	3.34
$^{221}_{89}$	7.78	0	-1.13	-1.36	$^{221}_{91}$	9.25	0	-5.23	-4.94	$^{223}_{89}$	6.7832	2	2.6	2.89	$^{223}_{91}$	8.33	0	-2.03	-2.32
$^{225}_{89}$	5.9351	2	6.23	6.85	$^{225}_{91}$	7.39	2	0.39	1.38	$^{227}_{89}$	5.04219	0	11.02	11.29	$^{227}_{91}$	6.5804	0	3.73	4.12
$^{229}_{91}$	5.835	1	10.03	9.39	$^{231}_{91}$	5.1499	0	12.97	11.69	$^{235}_{93}$	5.194	1	13.94	14.31	$^{235}_{95}$	6.61	1	5.17	7.51
$^{237}_{93}$	4.9583	1	16.19	15.91	$^{239}_{95}$	5.9224	1	11.11	10.99	$^{241}_{95}$	5.63782	1	12.6	12.61	$^{243}_{95}$	5.4388	1	14.16	13.82
$^{245}_{97}$	6.4545	2	9.37	8.15	$^{245}_{99}$	7.909	3	3.52	4.15	$^{249}_{97}$	5.525	2	13.61	13.34	$^{251}_{99}$	6.5967	0	7.48	7.58
$^{253}_{99}$	6.73916	0	6.29	6.87	$^{257}_{101}$	7.5576	1	7.57	6.07	$^{257}_{105}$	9.23	1	0.51	1.81	$^{261}_{107}$	10.56	0	-1.47	-3.01

Table 5
Same as Table 3 but using the formula (7) for 48 o-o nuclei.

$^{A}_{Z}$	Q	l	$logT_{exp}$	$logT_{form}$	$^{A}_{Z}$	Q	l	$logT_{exp}$	$logT_{form}$	$^{A}_{Z}$	Q	l	$logT_{exp}$	$logT_{form}$	$^{A}_{Z}$	Q	l	$logT_{exp}$	$logT_{form}$
$^{112}_{53}$	2.99	4	5.45	5.29	$^{148}_{63}$	2.694	0	14.7	14.72	$^{152}_{67}$	4.507	0	3.13	3.10	$^{154}_{67}$	4.041	0	6.57	5.88
$^{156}_{69}$	4.344	0	5.12	5.17	$^{162}_{73}$	5.01	1	3.68	4.39	$^{160}_{75}$	6.715	0	-2.02	-2.48	$^{162}_{75}$	6.24	0	-0.96	-0.80
$^{166}_{77}$	6.724	0	-1.95	-1.71	$^{170}_{77}$	6.11	0	0.08	0.52	$^{176}_{77}$	5.24	0	2.6	4.36	$^{170}_{79}$	7.168	0	-2.55	-2.39
$^{174}_{79}$	6.699	0	-0.81	-0.86	$^{212}_{83}$	6.207	5	4.57	4.10	$^{214}_{83}$	5.621	5	7.16	6.96	$^{196}_{85}$	7.2	0	-0.57	-0.36
$^{198}_{85}$	6.893	0	0.64	0.71	$^{200}_{85}$	6.596	0	1.88	1.82	$^{202}_{85}$	6.354	0	3.01	2.78	$^{204}_{85}$	6.07	0	4.15	3.98
$^{206}_{85}$	5.888	0	7.36	4.79	$^{208}_{85}$	5.751	0	6.04	5.41	$^{210}_{85}$	5.631	2	7.73	7.02	$^{212}_{85}$	7.824	5	-0.42	-1.15
$^{214}_{85}$	8.987	0	-6.25	-6.03	$^{204}_{87}$	7.171	0	0.39	0.48	$^{206}_{87}$	6.923	0	1.28	1.36	$^{208}_{87}$	6.79	0	1.82	1.83
$^{210}_{87}$	6.65	2	2.43	3.25	$^{212}_{87}$	6.529	2	4.1	3.75	$^{214}_{87}$	8.589	5	-2.27	-2.67	$^{216}_{87}$	9.175	0	-6.15	-5.77
$^{218}_{87}$	8.014	0	-2.97	-2.60	$^{220}_{87}$	6.801	1	1.62	2.76	$^{206}_{89}$	7.94	0	-1.6	-1.29	$^{208}_{89}$	7.73	0	-1.01	-0.65
$^{214}_{89}$	7.35	2	1.23	1.40	$^{216}_{89}$	9.235	5	-3.31	-3.67	$^{218}_{89}$	9.38	0	-5.97	-5.56	$^{222}_{89}$	7.137	0	0.73	1.16
$^{224}_{89}$	6.327	1	5.73	5.73	$^{226}_{89}$	5.536	2	9.25	9.60	$^{212}_{91}$	8.43	4	-2.1	-1.09	$^{218}_{91}$	9.815	0	-3.76	-5.87
$^{226}_{91}$	6.987	0	2.45	2.57	$^{228}_{91}$	6.264	3	7.6	7.51	$^{230}_{91}$	5.439	2	11.31	11.25	$^{252}_{99}$	6.79	1	7.83	8.29

been recalculated using the Q_α values of Audi et al [6]. They are given below and the rms deviation is respectively 0.3283, 0.6158, 0.6748 and 0.6792 for the 136 even-even, 84 even-odd, 76 odd-even and 48 odd-odd nuclei.

$$log_{10}\left[T\right] = -25.926 - 1.1471(A-4)^{\frac{1}{6}}\sqrt{Z} + \frac{1.5917Z}{\sqrt{Q}}, \tag{10}$$

$$log_{10}\left[T\right] = -29.989 - 1.0670(A-4)^{\frac{1}{6}}\sqrt{Z} + \frac{1.6750Z}{\sqrt{Q}} + \frac{0.7182A^{\frac{1}{6}}\sqrt{l(l+1)}}{Q} + 0.6932[1-(-1)^l], \tag{11}$$

$$log_{10}\left[T\right] = -29.769 - 1.0817(A-4)^{\frac{1}{6}}\sqrt{Z} + \frac{1.6758Z}{\sqrt{Q}} + \frac{0.2322A^{\frac{1}{6}}\sqrt{l(l+1)}}{Q} + 0.6476[1-(-1)^l], \tag{12}$$

$$log_{10}\left[T\right] = -30.597 - 0.9839(A-4)^{\frac{1}{6}}\sqrt{Z} + \frac{1.6439Z}{\sqrt{Q}} + \frac{0.5901A^{\frac{1}{6}}\sqrt{l(l+1)}}{Q} + 0.28890[1-(-1)^l]. \tag{13}$$

The dependence on A or A-4 in the second term does not change the accuracy of the formulas. The form of the two additional terms depending on l is empiric and tested on only a small number of nuclei and a very limited set of l values. It has been found that the l dependence assumed in the formulas (5-7) is particularly efficient to lower the rms deviation.

A separated adjustment of all the coefficients of the Viola-Seaborg formulas [20,21] which do not take into account the l dependence leads to the following expressions and an accuracy of 0.349, 0.950, 0.912 and 0.870 for the e-e, e-o, o-e and o-o nuclei. The accuracy is almost of the same order than the precision of the formulas (1-4) but the expressions depend on four parameters in each subset.

$$log_{10}\left[T\right] = \frac{1.5872Z - 1.3456}{\sqrt{Q}} - 0.22783Z - 31.392, \tag{14}$$

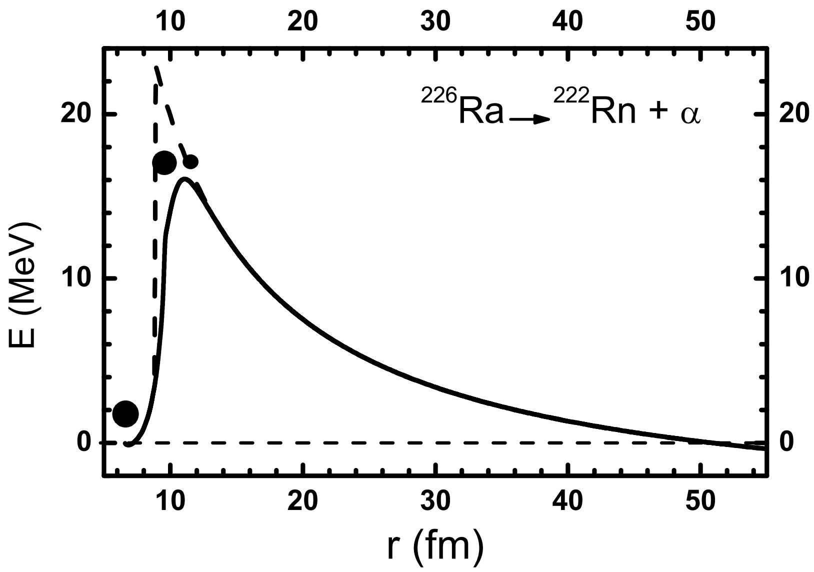

Fig. 1. α-decay barrier for the ^{226}Ra nucleus. The full and broken curves correspond to the deformation energy with and without taking into account the nuclear proximity effects.

$$log_{10}\,[T] = \frac{1.4125Z + 20.649}{\sqrt{Q}} - 0.04826Z - 48.445, \tag{15}$$

$$log_{10}\,[T] = \frac{1.69185Z + 3.3357}{\sqrt{Q}} - 0.17826Z - 40.248, \tag{16}$$

$$log_{10}\,[T] = \frac{1.5344Z + 9.463}{\sqrt{Q}} - 0.13858Z - 40.410. \tag{17}$$

3 Alpha emission or capture barrier

Within the GLDM the l-dependent potential energy governing the stability of the nuclear rotating system is the sum of the volume, surface, Coulomb, proximity and rotational energies (see Refs [4,24]). The selected quasi-molecular shape sequence leads from one spherical nucleus to two unequal tangent spherical nuclei in keeping almost spherical ends.The formation of a deep neck and its rupture is ensured before the elongation of the system.

As an example, the alpha decay barrier of ^{226}Ra is displayed in Fig. 1. The Q value has been introduced empirically in adding at the macroscopic energy of the mother nucleus the difference between the experimental and theoretical Q value with a linear attenuation factor vanishing at the contact point between the nascent fragments. The proximity energy lowers the barrier height by around 7 MeV and moves the barrier top to a more external position corresponding to two separated spheres maintained in unstable equilibrium by the balance between the repulsive Coulomb forces and the attractive nuclear proximity forces. The selected one-body shape plays a minor role since the distance between the two parts increases only slightly and the main part of the barrier corresponds to two-body shapes. For two body-shapes an analytic formula reproduces accurately the proximity energy [24]:

$$E_{Prox}(r) = 4\pi\gamma e^{-1.38(r-R_\alpha-R_d)}[0.6584A^{\frac{2}{3}} - (\frac{0.172}{A^{\frac{1}{3}}} + 0.4692A^{\frac{1}{3}})r \\ -0.02548A^{\frac{1}{3}}r^2 + 0.01762r^3] \ \ MeV. \qquad (18)$$

The following expression allows to determine rapidly and accurately the distance between the mass centers at the α barrier top. A and Z are the mass and charge of the mother nucleus.

$$R = 2.536 + 1.1157\ [4^{\frac{1}{3}} + (A-4)^{\frac{1}{3}}] \ \ fm. \qquad (19)$$

The height of the barrier against α decay can be determined using:

$$E = -1.43 + \frac{e^2 \times 2 \times (Z-2)}{2.536 + 1.1157[4^{\frac{1}{3}} + (A-4)^{\frac{1}{3}}]} - Q \ \ MeV, \qquad (20)$$

from which the alpha-capture barrier height can be deduced in adding Q.

The coefficients of these two expressions have been adjusted on a whole set of α decay or capture potential barriers calculated within the GLDM. It has been shown that the radius and height of these barriers are in agreement with the experimental data and the results obtained within the Krappe-Nix-Sierk potential [33,34] relative to fusion data. The calculations of the α emission half-lives deduced directly from the WKB barrier penetration probability through these barriers and without preformation factor lead to a RMS deviation of $log_{10}T_{1/2}(s)$ of only 0.63 [4]. The introduction of quadrupole deformations of the fragments should allow to diminish this error. This has been investigated in several works [5,35]. Semi-microscopic optical alpha-nucleus potentials based on the double folding model have been advanced also recently [36].

4 Summary and conclusion

Empirical expressions (1,5-7) depending on the angular momentum of the α particle for the even-odd, odd-even and odd-odd nuclei are proposed to determine $log_{10}T_{1/2}(s)$. The coefficients have been adjusted on a recent proposed data set of partial α-decay half-lives of 344 ground state to ground state transitions. The introduction of two new terms simulating the centrifugal effects and the hindrance of α emission with odd values of l improves strongly the efficiency of the formulas even though the terms are semi-empirical. An accurate expression (8) is provided to evaluate the partial α-decay half-lives of even-even heavy and superheavy elements from the experimental or predicted Q_α. The accuracy of these new formulas is slightly better than the precision of other proposed expressions readjusted to this new considered data set and using the Q_α values given in [6]. The predictability of these expressions for other exotic nuclei is linked to the precision of the evaluation of the Q_α value and then of the nuclear masses. The potential α-decay or capture barrier has also been calculated within a liquid drop model including the proximity effects between the α particle and its daughter nucleus. Analytic expressions are given to evaluate rapidly the α-decay or capture barrier radius and height.

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